

ENGINEERING TRIPOS PART IIB

Friday 23 April 2010 2.30 to 4

Module 4D5

FOUNDATION ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: 4D5 Supplementary Databook (14 pages).

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A jackup rig is founded on two strip foundations, each of width b , with the distance between their centres given by a , as shown in Fig. 1.

(a) Describe with sketches, four possible mechanisms for failure of the foundation system. What factors will determine which of these mechanisms will occur? [30%]

Considering failure modes that do not involve interaction between the foundations:

(b) The total vertical load on the foundations due to the weight of the structure, V , is equal to $4b l s_u$ where l is the length of the strip foundation and s_u is the undrained shear strength of the soil. If the foundation to leg connections are idealised as pin-joints and the foundations cannot sustain tension, calculate the horizontal load H , applied at a distance $3a$ above the seabed, which will cause failure. You may assume that the foundations mobilise equal horizontal reactions. State the mode of failure. [30%]

(c) Calculate the horizontal load at failure if the foundations can sustain tension. [20%]

(d) Now assume a failure mode that involves interaction between foundations. Calculate the horizontal load that will cause failure on a circular slip surface passing through the outer edge of each foundation. If $b = a/2$, will this be a critical failure mode compared to those identified in parts (b) and (c)? [20%]

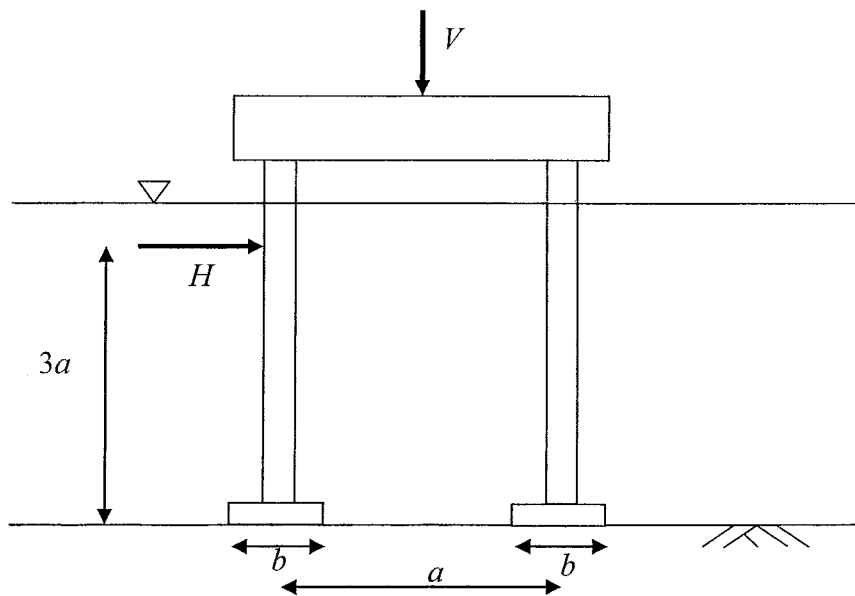


Fig. 1 (not to scale)

2 Figure 2 shows the plan of a flexible raft foundation for a new building, to be built alongside an existing structure. The net bearing pressure (allowing for the weight of the excavated soil) at the foundation level is to be 500 kPa for the new building and 250 kPa for the existing building. The foundation level for both buildings and the water table are each 1 m below ground level. The bulk unit weight of the clay is 20 kN m^{-3} .

(a) Assuming the subsoil to be clay of shear modulus 5 MPa, estimate the immediate (undrained) settlement at points A and B, stating any assumptions you make. [30%]

(b) The borehole data from the site showed that the subsoil is over-consolidated clay, from ground surface to a depth of 31 m, overlying bedrock. Laboratory oedometer tests of the over-consolidated clay showed that the compressibility of the clay can be given by:

$$v = 1.2 - \kappa \ln \sigma'_v$$

where v is the specific volume, σ'_v is the vertical effective stress and the compressibility index κ is equal to 0.062.

(i) Estimate the increase in vertical stress 5 m below the centre of the foundation of the existing building. [20%]

(ii) Estimate the drained settlement of the existing building by dividing the subsoil below the foundation into two layers of 10 m and 20 m thickness. [50%]

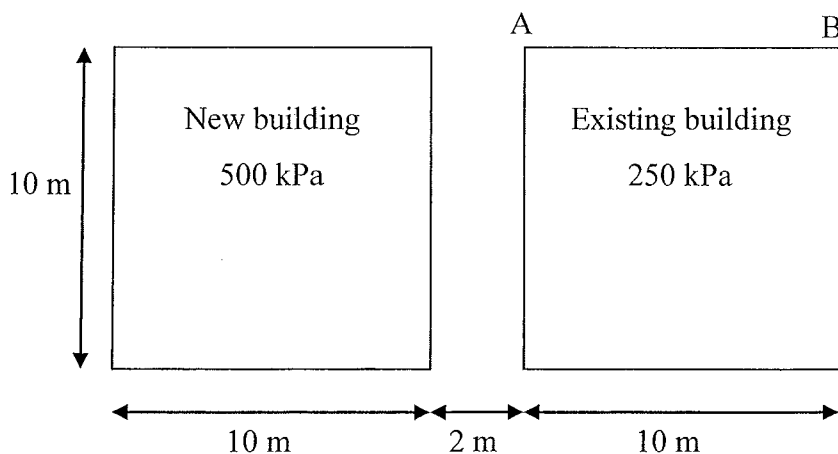


Fig. 2 (not to scale)

3 (a) A site has a very deep saturated sand of bulk density 20 kN m^{-3} , relative density 50% and friction angle ϕ_{crit} of 35° . A tubular close-ended pile of outer diameter 0.5 m and length 30 m was hammer-driven in the site. Calculate the vertical capacity of the pile using the API (2000) design method. The pile-soil interface friction angle can be taken as 25° . [25%]

(b) An offshore wind turbine in shallow water is to be supported on a single open-ended monopile of diameter 4 m and wall thickness 40 mm. The site comprises normally-consolidated soft clay with an undrained strength $s_u = 2.5z \text{ kPa}$, where z (m) is the depth below the mudline. The effective unit weight γ' is 7 kN m^{-3} and the coefficient of horizontal consolidation c_h is $20 \text{ m}^2 \text{ year}^{-1}$. The design storm load comprises of a vertical compressive force of 6 MN and a horizontal force of 4 MN applied 15 m above the mudline.

Consolidation around a driven pile in clay is 90% complete after an equivalent dimensionless time $T_{eq} = c_h t / D_{eq}^2 = 10$, where D_{eq} is the equivalent pile diameter

- (i) Calculate D_{eq} (assuming that the pile was unplugged during driving) and estimate the *set-up* period that should be allowed after pile installation before the structure is attached. [10%]
- (ii) By considering only the vertical load, make a preliminary calculation of the required pile length using the API design method. Ignore the self weight of the monopile itself but assume that the pile fails in a plugged manner and allow for the weight of the soil within the pile. [40%]
- (iii) Evaluate whether the pile length calculated in (ii) is sufficient to resist the horizontal load. You may ignore the possibility of structural failure of the pile. [10%]
- (c) Estimate the length of pile which would be able to carry the design loads. [15%]

4 (a) A steel tubular pile with an external diameter of 1 m and a wall thickness of 20 mm is to be installed in stiff clay. In-situ testing indicated that the undrained strength, s_u , of the clay is 100 kPa. The shear modulus, G , of the clay can be taken as $150s_u$, the Young's modulus of the steel is 210 GPa and the yield stress of the steel is 250 MPa.

(i) Calculate the equivalent axial stiffness E_p and plastic moment capacity M_p of the pile. [20%]

(ii) If the pile length in the clay is 15 m, calculate the maximum horizontal load that can be safely applied at the ground surface. Recalculate the maximum safe horizontal load if it is to be applied 4 m above the ground surface. [35%]

(iii) If the pile is restrained from rotation at the ground surface by the presence of a pile cap, what is the maximum horizontal load that can be applied safely at the ground surface [15%]

(b) A bored pile, 300 mm in diameter and 20 m in length, is constructed in stiff clay. The undrained strength, s_u , of the clay increases linearly from 80 kPa at the ground level up to 250 kPa at the pile base. Assume that the equivalent axial stiffness, E_p , of the pile is 15 GPa. The shear modulus, G , of the soil is to be taken as equal to $150s_u$, and its Poisson's ratio is 0.2.

(i) Assuming the pile to be rigid, calculate the settlement of the pile head at the design vertical load of 500 kN. [20%]

(ii) What compression of the pile might be expected at this load if the load is assumed to be constant along the pile length? [10%]

END OF PAPER

Cambridge University Engineering Department
Supplementary Databook

Module 4D5: Foundation Engineering

IT. January 2008

Section 2: Bearing capacity of shallow foundations

2.1 Tresca soil, with undrained strength s_u .

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \quad (\text{Prandtl, 1921})$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2 B / L$$

The exact solution for a rough circular foundation ($B/L=1$) is $q_f = 6.05s_u$, hence $s_c = 1.18 \sim 0.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h , is:

$$d_c = 1 + 0.33 \tan^{-1} (h/D) \quad (\text{or } h/B \text{ for a strip or rectangular foundation})$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

$$\text{If } V/V_{ult} > 0.5: \quad \frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2 \frac{V}{V_{ult}} - 1 \right)^2$$

$$\text{If } V/V_{ult} < 0.5: \quad H = H_{ult} = B s_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof (V'_{ult} = bearing capacity of effective area $B-e$)

$$\text{If } V/V'_{ult} < 0.5: \quad \frac{H}{H_{ult}} = \left(1 - 2 \frac{M}{VB} \right)$$

$$\text{Without lift-off: } \left(\frac{V}{V_{ult}} \right)^2 + \left[\frac{M}{M_{ult}} \left(1 - 0.3 \frac{H}{H_{ult}} \right) \right]^2 + \left| \left(\frac{H}{H_{ult}} \right)^3 \right| - 1 = 0 \quad (\text{Taiebat \& Carter 2000})$$

2.2 Frictional (Coulomb) soil, with friction angle ϕ .

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2) e^{(\pi \tan \phi)} \quad (\text{Prandtl 1921})$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2 (N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

$$\text{Rough base: } N_\gamma = 0.1054 e^{9.6\phi}$$

$$\text{Smooth base: } N_\gamma = 0.0663 e^{9.3\phi}$$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$

$$s_\gamma = 1 - 0.3 B / L$$

For circular footings assume $L = B$.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

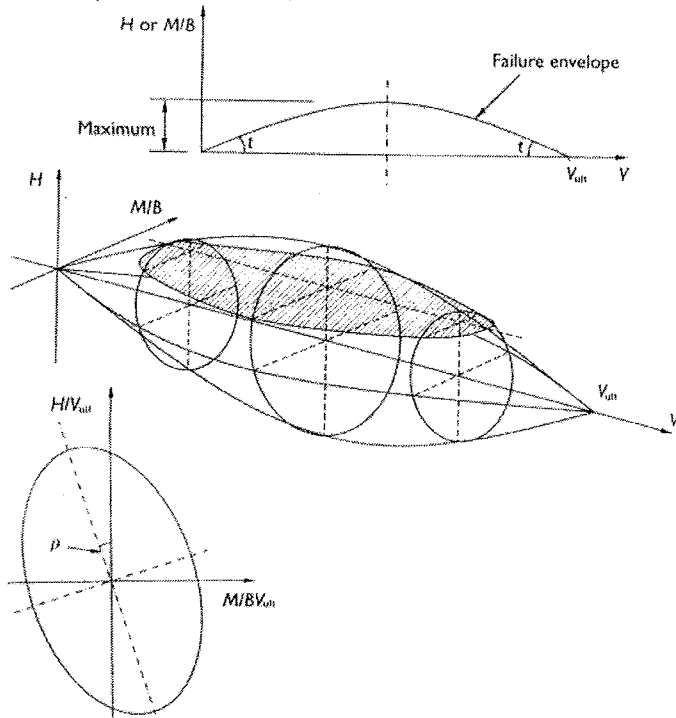
Combined V-H-M loading

(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h} \right]^2 + \left[\frac{M/BV_{ult}}{t_m} \right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m} \right] = \left[\frac{V}{V_{ult}} \left(1 - \frac{V}{V_{ult}} \right) \right]^2$$

$$\text{where } C = \tan \left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2t_h t_m} \right) \quad (\text{Butterfield \& Gottardi 1994})$$

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15^\circ$. t_h is the friction coefficient, $H/V = \tan \phi$, during sliding.



Section 3: Settlement of shallow foundations

3.1 Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)

Vertical stress $\sigma_z = \frac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \frac{P}{2\pi R^2} \left[\frac{3r^2z}{R^3} - \frac{(1-2\nu)R}{R+z} \right]$

Tangential stress $\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2} \left[\frac{R}{R+z} - \frac{z}{R} \right]$

Shear stress $\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$



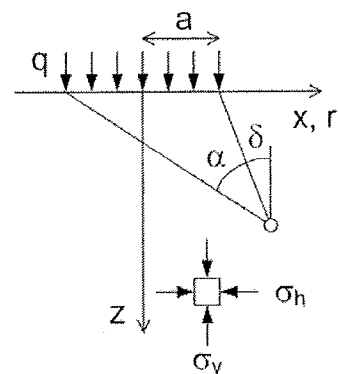
Uniformly-loaded strip

Vertical stress $\sigma_v = \frac{q}{\pi} [\alpha + \sin \alpha \cos(\alpha + 2\delta)]$

Horizontal stress $\sigma_h = \frac{q}{\pi} [\alpha - \sin \alpha \cos(\alpha + 2\delta)]$

Shear stress $\tau_{vh} = \frac{q}{\pi} \sin \alpha \sin(\alpha + 2\delta)$

Principal stresses



$$\sigma_1 = \frac{q}{\pi} (\alpha + \sin \alpha) \quad \sigma_3 = \frac{q}{\pi} (\alpha - \sin \alpha)$$

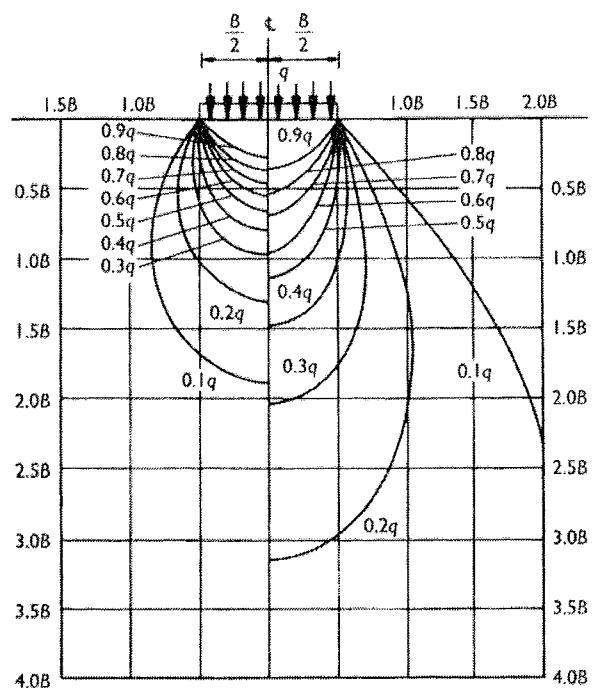
Uniformly-loaded circle (on centerline, r=0)

Vertical stress

$$\sigma_v = q \left[1 - \left(\frac{1}{1 + (a/z)^2} \right)^{\frac{3}{2}} \right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2} \left[(1 + 2\nu) - \frac{2(1 + \nu)z}{(a^2 + z^2)^{1/2}} + \frac{z^3}{(a^2 + z^2)^{3/2}} \right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

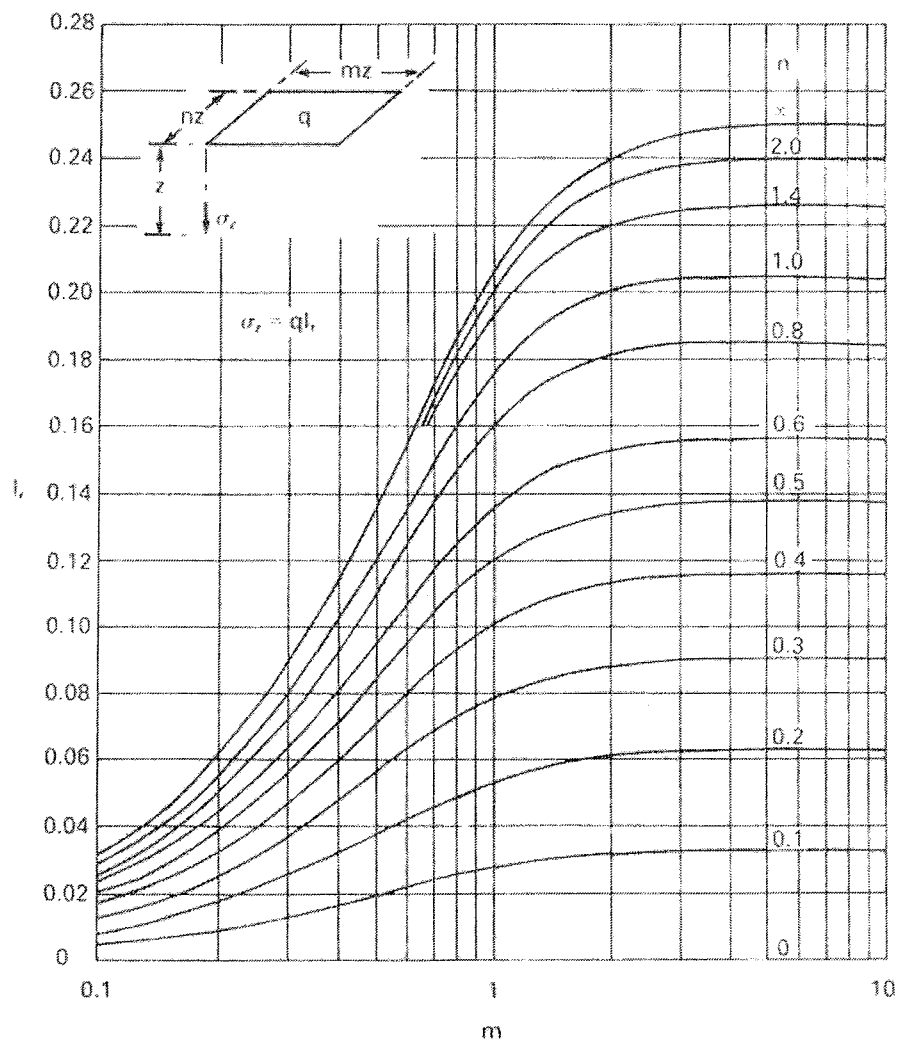
3.2 Elastic stress distribution below rectangular area

The vertical stress, σ_z , below the corner of a uniformly-loaded rectangle ($L \times B$) is:

$$\sigma_z = I_r q$$

I_r is found from m ($=L/z$) and n ($=B/z$) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi} \left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) + \tan^{-1} \left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1} \right) \right]$$



Influence factor, I_r , for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

3.3 Elastic solutions for surface settlement

3.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w , at distance s :
$$w(s) = \frac{1}{2\pi} \frac{(1-\nu) P}{G s}$$

Circular area (radius a), uniform soil

Uniform load: central settlement:
$$w_o = \frac{(1-\nu)}{G} qa$$

edge settlement:
$$w_e = \frac{2(1-\nu)}{\pi G} qa$$

Rigid punch: ($q_{avg} = V/\pi a^2$)

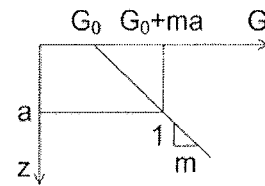
$$w_r = \frac{\pi(1-\nu)}{4G} q_{avg} a$$

Circular area, heterogeneous soil

For $G_0 = 0$, $\nu = 0.5$:

$$w = q/2m \quad \text{under loaded area of any shape}$$

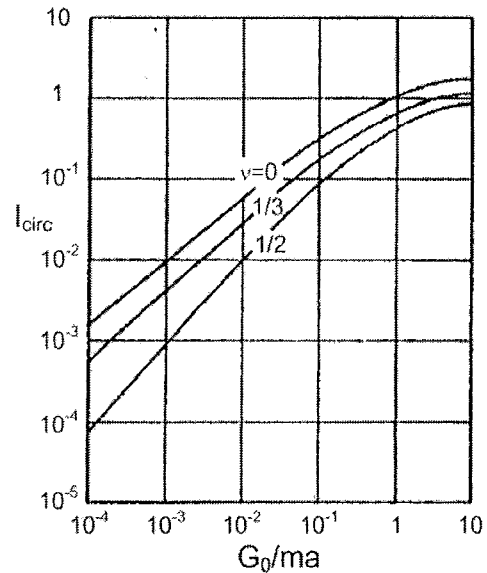
$$w = 0 \quad \text{outside loaded area}$$



For $G_0 > 0$, central settlement:

$$w_o = \frac{qa}{2G_0} I_{circ}$$

For $\nu = 0.5$, $w_o \approx \frac{qa}{2(G_0 + ma)}$



Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G} \frac{qB}{2} I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B :

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G} \frac{q_{avg} \sqrt{BL}}{2} I_{rgd}$ where I_{rgd} varies from 0.9 → 0.7 for $L/B = 1-10$.

Note: $G = \frac{E}{2(1+\nu)}$ where ν = Poisson's ratio, E = Young's modulus.

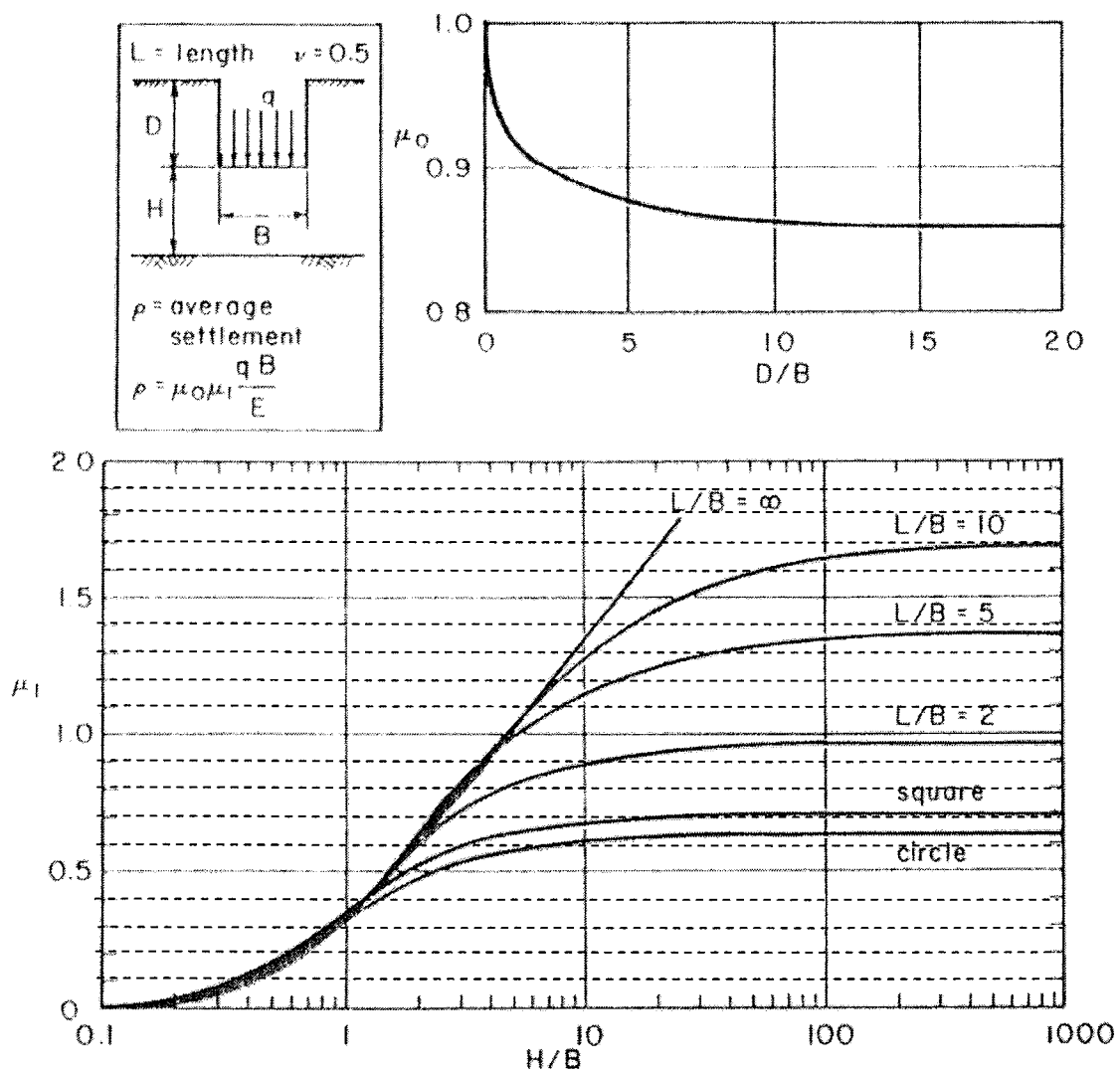
3.3.2 Isotropic, homogeneous, elastic finite space

Elastic layer of finite thickness

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$W_{avg} = \mu_0 \mu_1 \frac{qB}{E} \quad E = 2G(1 + \nu)$$

The influence factor μ_1 accounts for the finite layer thickness. The influence factor μ_0 accounts for the embedded depth.



Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Janbu, Bjerrum and Kjaernslis' chart reinterpreted. Canadian Geotechnical Journal (15) 123-128.

Section 4: Bearing capacity of deep foundations

4.1 Axial capacity: API (2000) design method for driven piles

Sand

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf} \tan \delta = K \sigma'_{v0} \tan \delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$

Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q \sigma'_{v0} < q_{b,limit}$

Soil category	Soil density	Soil type	Soil-pile friction angle, δ (°)	Limiting value $\tau_{s,lim}$ (kPa)	Bearing capacity factor, N_q	Limiting value, $q_{b,lim}$ (MPa)
1	Very loose Loose Medium	Sand Sand-silt Silt	15	50	8	1.9
2	Loose Medium Dense	Sand Sand-silt Silt	20	75	12	2.9
3	Medium Dense	Sand Sand-silt	25	85	20	4.8
4	Dense Very dense	Sand Sand-silt	30	100	40	9.6
5	Dense Very dense	Gravel Sand	35	115	50	12

API (2000) recommendations for driven pile capacity in sand

Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

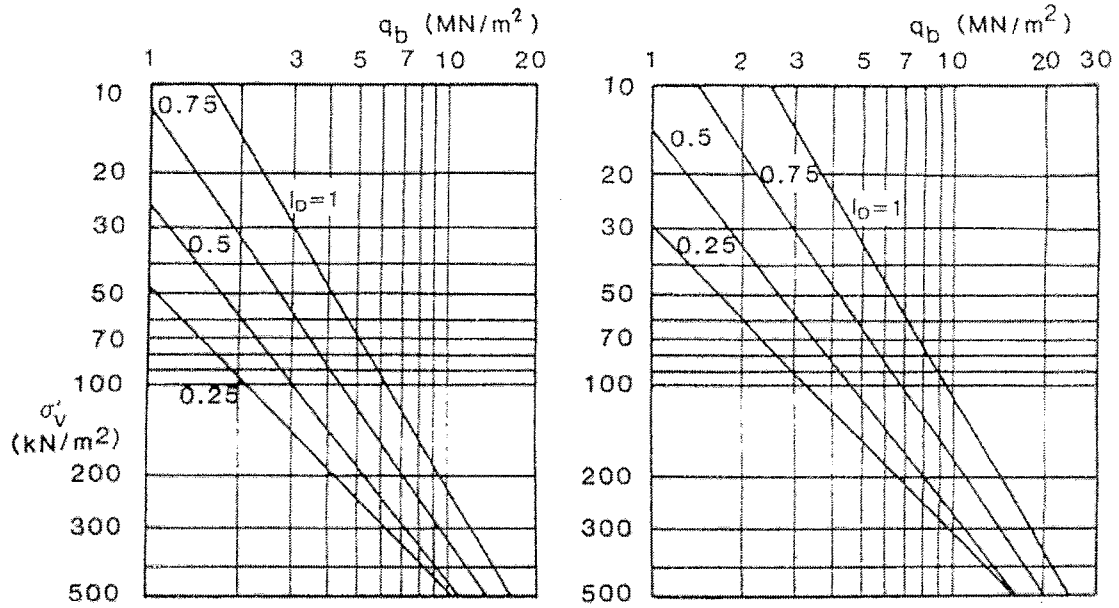
Unit shaft resistance: $\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \text{Max} \left[\left(\frac{\sigma'_{v0}}{s_u} \right)^{0.5}, \left(\frac{\sigma'_{v0}}{s_u} \right)^{0.25} \right]$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

Unit base resistance: $q_b = N_c s_u$ $N_c = 9.$

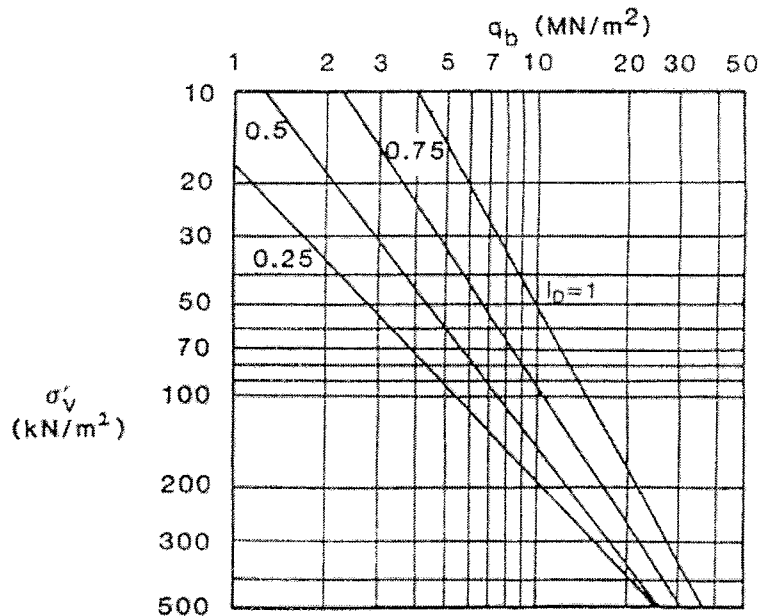
4.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, q_b , is expressed as a function of relative density, I_D , constant volume (critical state) friction angle, ϕ_{cv} , and in situ vertical effective stress, σ'_v .



(a) $\phi_{cv} = 27^\circ$

(b) $\phi_{cv} = 30^\circ$



(c) $\phi_{cv} = 33^\circ$

Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

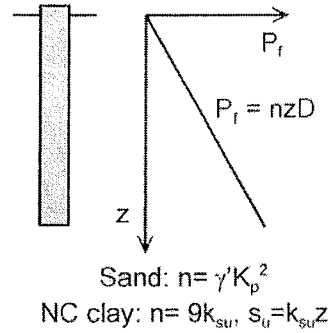
4.3 Lateral capacity: linearly increasing lateral resistance with depth

Lateral soil resistance (force per unit length), $P_u = nzD$

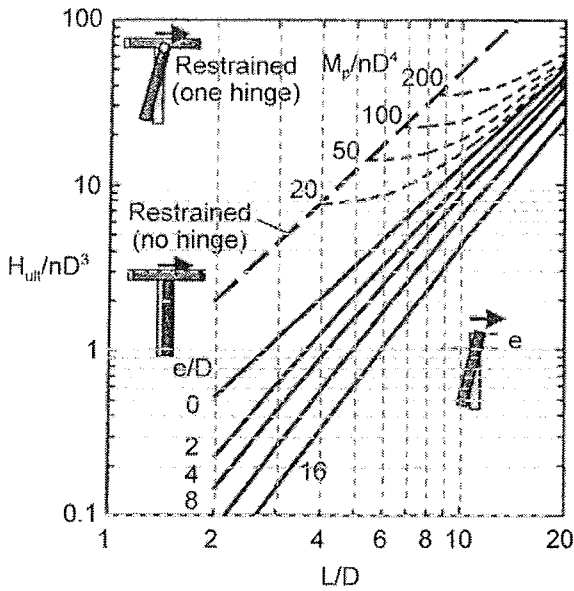
In sand, $n = \gamma'K_p^2$

In normally consolidated clay with strength gradient k ; $s_u = kz$; $n=9k$

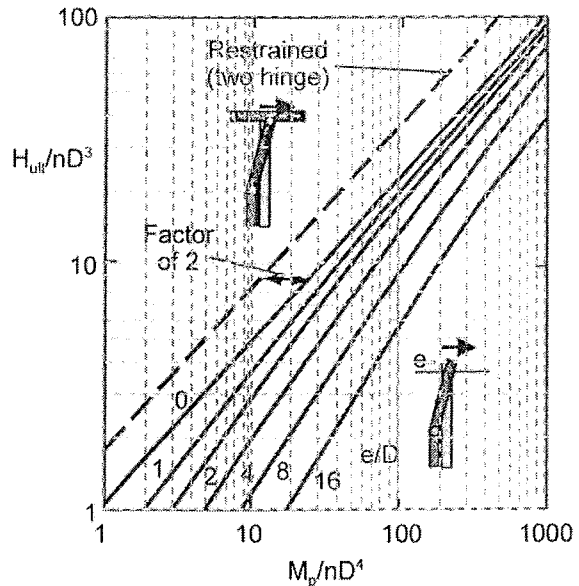
- H_{ult} ultimate horizontal load on pile
- M_p plastic moment capacity of pile
- D pile diameter
- L pile length
- e load level above pile head
(=M/H for H-M pile head loading)
- γ' effective unit weight
- K_p passive earth pressure coefficient,
 $K_p = (1 + \sin \phi) / (1 - \sin \phi)$



Sand or normally-consolidated clay



Short pile failure mechanism



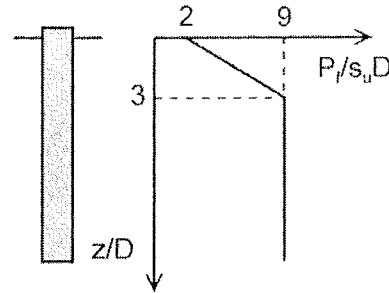
Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

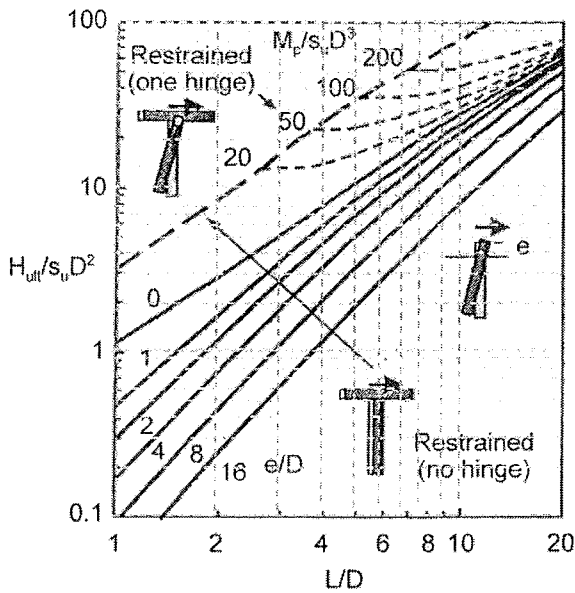
4.4 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), P_u , increases from $2s_uD$ at surface to $9s_uD$ at $3D$ depth then remains constant.

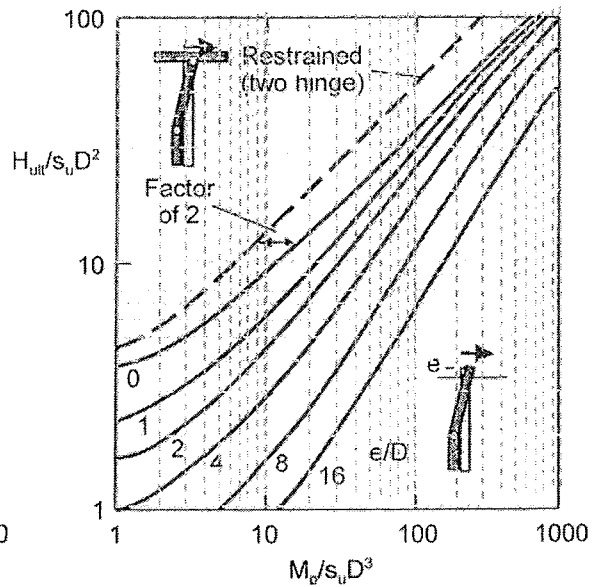
- H_{ult} ultimate horizontal load on pile
- M_p plastic moment capacity of pile
- D pile diameter
- L pile length
- e load level above pile head
($=M/H$ for H-M pile head loading)
- s_u undrained shear strength



Uniform clay



Short pile failure mechanism



Long pile failure mechanism

Lateral pile capacity
(uniform clay lateral resistance profile)

Section 5: Settlement of deep foundations

5.1 Settlement of a rigid pile

Shaft response:

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

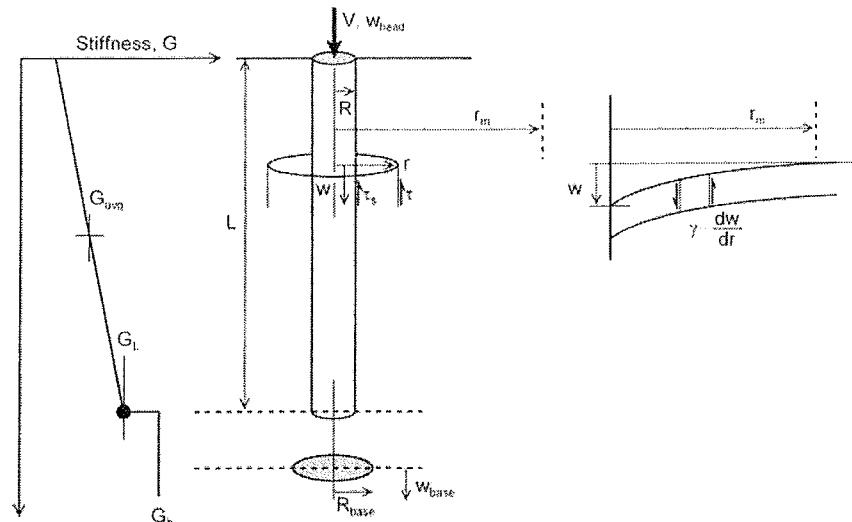
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, r_m , for shaft stiffness, τ_s/w .



Nomenclature for settlement analysis of single piles

Combined response of base (rigid punch) and shaft:

$$\frac{V}{w_{\text{head}}} = \frac{Q_b}{w_{\text{base}}} + \frac{Q_s}{w}$$

$$\frac{V}{w_{\text{head}}} = \frac{4R_{\text{base}} G_{\text{base}}}{1-\nu} + \frac{2\pi L G_{\text{avg}}}{\zeta}$$

$$\frac{V}{w_{\text{head}} D G_L} = \frac{2}{1-\nu} \frac{G_{\text{base}} D_{\text{base}}}{G_L D} + \frac{2\pi}{\zeta} \frac{G_{\text{avg}} L}{G_L D}$$

$$\frac{V}{w_{\text{head}} D G_L} = \frac{2}{1-\nu} \frac{\eta}{\xi} + \frac{2\pi}{\zeta} \rho \frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{\text{base}}/R = D_{\text{base}}/D$ Slenderness ratio L/D

Stiffness gradient ratio, rho $\rho = G_{\text{avg}}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{\text{base}}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln \left\{ \left[0.5 + (5\rho(1-\nu) - 0.5)\xi \right] \frac{L}{D} \right\} \quad \text{for } \xi=1: \quad \zeta = \ln \left\{ 5\rho(1-\nu) \frac{L}{D} \right\}$$

5.2 Settlement of a compressible pile

$$\frac{V}{w_{\text{head}} D G_L} = \frac{2\eta}{(1-\nu)\xi} + \rho \frac{2\pi \tanh \mu L}{\zeta} \frac{L}{\mu L} \frac{L}{D}$$

$$1 + \frac{1}{\pi\lambda(1-\nu)\xi} \frac{\tanh \mu L}{\mu L} \frac{L}{D}$$

$$\text{where } \mu = \frac{\sqrt{8/\zeta\lambda}}{D}$$

Pile compressibility

$$\lambda = E_p/G_L$$

Pile-soil stiffness ratio

Pile head stiffness, $\frac{V}{w_{\text{head}}}$, is maximum when $L \geq 1.5D\sqrt{\lambda}$

Numerical Answers for 4D5 Exam 2010

Q1

- b) $H=2/3 \text{ bl}_{su}$
- c) $H=0.84 \text{ bl}_{su}$
- d) $H=1.18 \text{ bl}_{su}$

Q2

- a) $w_A = 101 \text{ mm}$
 $w_B = 46 \text{ mm}$
- b) i) 10 kPa
ii) 162 mm

Q3

- a) $Q=3732\text{kN}$
- b) i) $D_{eq}=0.8 \text{ m}$
 $T = 117 \text{ days}$
ii) $L = 15.2\text{m}$
- c) $L= 24\text{m}$

Q4

- a) i) $E_p=16.8 \text{ GPa}$
 $M_p= 5 \text{ MNm}$
ii) $H= 2200 \text{ kN}$
 $H= 1000 \text{ kN}$
iii) $H= 3500 \text{ kN}$
- b) i) 0.79mm
ii) 9.4 mm

Assessor's Comments 4DS

- Q1. A fairly straightforward question which was not very popular. Many candidates appeared to have tackled this last and to have run out of time. Several candidates failed to check other plausible mechanisms in order to find the lowest bound on failure in part b.
- Q2. Another unpopular question which was well dealt with by a couple of candidates, but in general was poorly answered. Many candidates neglected to state their assumptions in part a) and several candidates appeared to be confused by the influence of the existing building in the problem. This contributes to the existing stress state, not to any increase due to construction.
- Q3. A very popular and well answered question with one perfect solution. The design method taught in the course was in general followed well, with most errors being fairly trivial or numerical slips.
- Q4. Another popular and well answered question. Several candidates assumed a value of 4 for the dimensionless influence zone rather than calculating its size, but otherwise most of the errors were numerical.