ENGINEERING TRIPOS PART IIB

Monday 26 April 2010 9 to 10.30

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

mspg03

1 This question investigates the poor performance of "soft storeys" during earthquakes. Consider a three-storey concrete frame building which has masonry infill walls on the upper two floors only. Assume that all storeys have equal mass and that the lateral stiffness of the upper two floors is equal to k_2 as shown in Fig. 1.

(a) Use the static deflected shape of the structure when exposed to a constant horizontal ground motion, $\ddot{u}_g$, (the resulting forces are shown in Fig. 1), to estimate the first mode shape of the structure. Normalise the mode shape so that the floor 1 has a magnitude of 1. Find the mode shapes and corresponding natural frequencies of the frame. [25%]

(b) If the design engineer has assumed that all storeys have equal stiffness ($k_1 = k_2$), then using part (a), determine an approximate mode shape and natural frequency for the first mode. Is the actual natural frequency higher or lower? [25%]

(c) In reality, the masonry infill walls at the upper storeys may significantly increase lateral stiffness. Assume that the upper floors are actually five times as stiff as the lower floor ($5k_1 = k_2$). Again, determine an approximate mode shape and natural frequency for the first mode. [20%]

(d) Using parts (b) and (c) above and considering only the first mode, explain why a "soft" floor 1 often collapses during an earthquake. Note that the maximum lateral force transmitted from the ground to floor 1 during an earthquake can be taken as: $f_1 = \Gamma m \bar{u}_1 A$, where Γ is the modal participation factor, $\bar{u}_1$ is the modal displacement of floor 1 and A is the spectral acceleration from the response spectrum. [30%]

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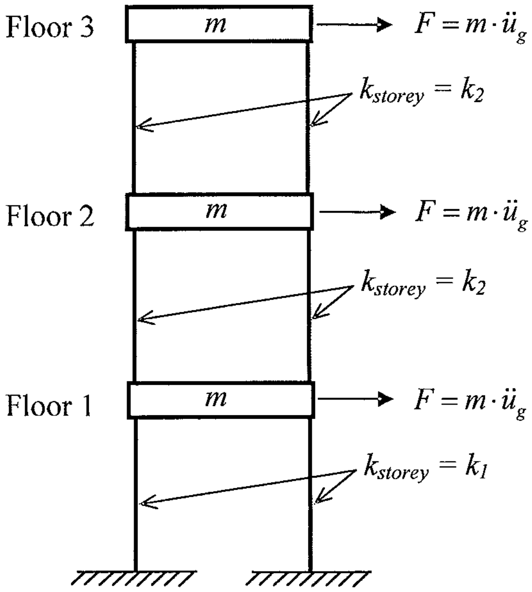

Fig. 1

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2 A foot bridge is constructed using a steel beam that is simply supported at its ends A and B as shown in Fig. 2a. The beam is a 254 × 102 × 28 Universal Beam, and its web lies in the plane of the paper.

(a) Show that the natural frequency of the 1st mode for small, flexural oscillations is around 6 Hz. Also calculate the natural frequency for the 2nd mode of flexural oscillations. [30%]

(b) A vertical force $f_1(t)$ is applied at point P which is 3 m from the left hand support A. This force varies with time in a triangular manner, as shown in Fig. 2b. Estimate the maximum dynamic deflection that will occur at point P due to this load considering only the 1st mode of vibration. [30%]

(c) Estimate the maximum dynamic deflection at the point Q which is 4 m from the right hand support B, due to the load $f_1(t)$ acting at point P. Consider both 1st and 2nd modes of vibration and comment on the effect of considering the 2nd mode. [40%]

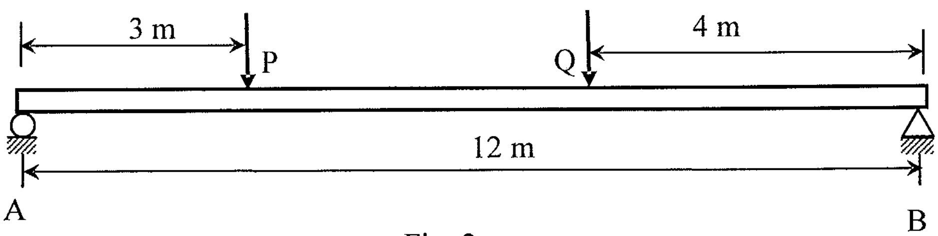

Fig. 2a

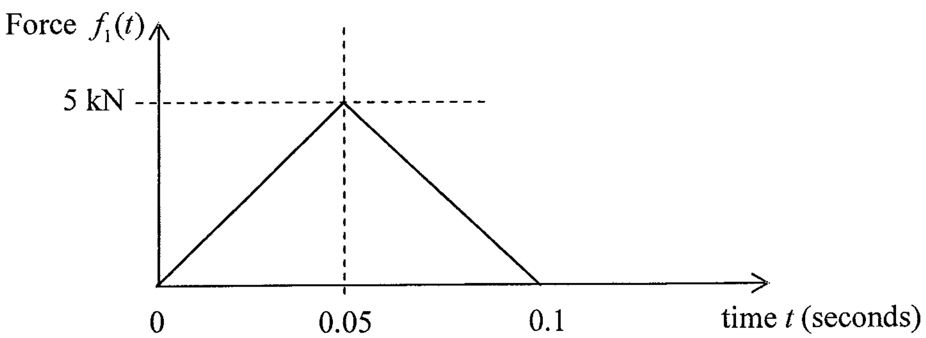

Fig. 2b

3 (a) Explain briefly how the finite element method can be used to solve boundary value problems with materials that have solid and fluid phases. [10%]

(b) The soil medium is generally considered to be semi-infinite. Describe two strategies that may be used to simulate this semi-infinite extent of the soil medium while using the finite element method. [20%]

(c) A concrete foundation block for a heavy machine has dimensions of 2 m × 2 m in plan and a depth of 1 m. The foundation block is embedded into a saturated sand layer such that the top surface of the block is level with the sand surface. The unit weight of concrete is 24 kN m^{-3} and that of the saturated sand is 19.5 kN m^{-3}. The voids ratio and the Poisson's ratio of sand may be taken as 0.9 and 0.3 respectively. The water table is at the surface of the sand layer. By considering a reference plane 0.5 m below the foundation block, calculate the horizontal and rotational stiffness of the foundation-sand system. [30%]

(d) The mass of a machine supported on the concrete foundation block is 2000 kg and this mass may be assumed to be concentrated at a distance of 1 m above the top surface of the foundation block. Calculate the natural frequencies for the horizontal and rocking modes of vibration. Assume that the whole system will rock about an axis in the reference plane considered in part (c) above. [20%]

(e) When the machine operates it is discovered that the natural frequency in the horizontal mode of vibration is actually 20% smaller than the value calculated in part (d) above. Give two reasons why this may be the case and calculate the change in the natural frequency of the rocking mode of vibration. [20%]

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4 (a) Describe the wind engineering phenomena that would have to be considered in the design of:

(i) a long-span suspension bridge;

(ii) a tall chimney.

In each case describe measures that may be introduced to reduce the response. [70%]

(b) Describe measures that may be taken to make a building design more resistant to the effects of bomb attacks. [30%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

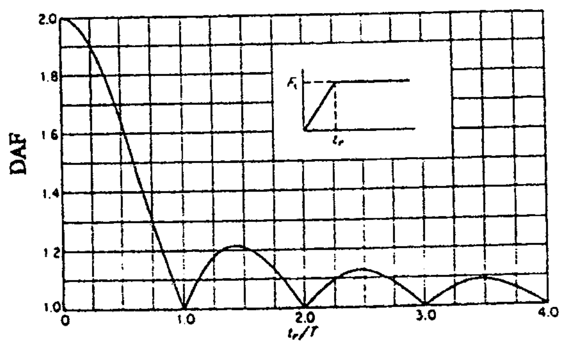
Modal analysis of simply-supported uniform beams

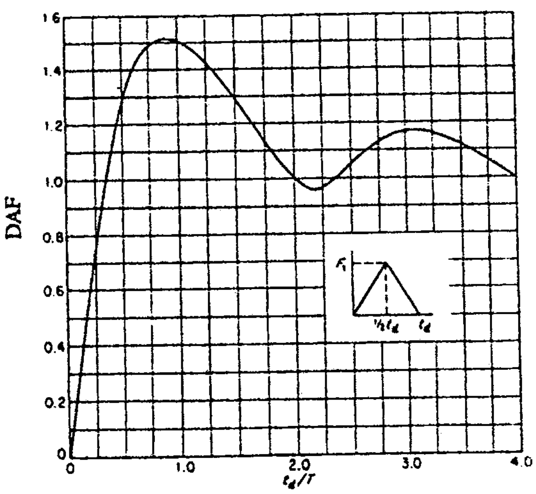
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

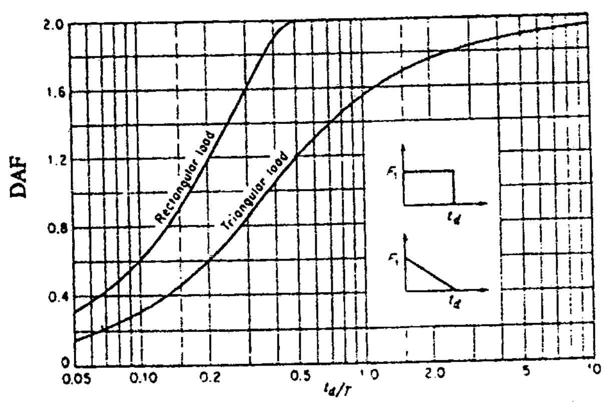
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

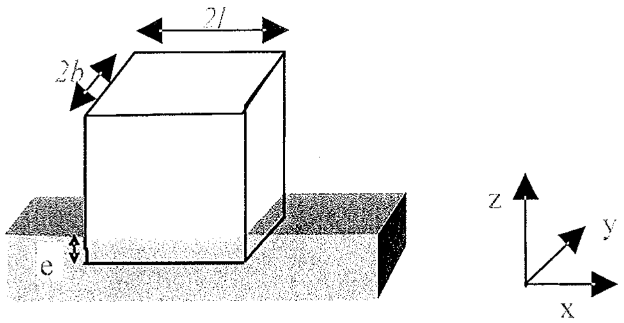
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^{3}}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^{3}}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^{3}\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_S is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3-e)^2}{(1+e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1+\gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \cdot e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2} \sigma'_v \sin \phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma'_v \right)^2 \right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006