

ENGINEERING TRIPOS PART IIB

---

Tuesday 20 April 2010 2.30 to 4

---

Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than three questions.*

*All questions carry the same number of marks.*

*The approximate percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet  
(4 pages).*

*(ii) The Cumulative Normal Distribution Function (1 page).*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

1 (a) You are employed as a forensic engineering specialist to assess four reinforced concrete bridges that have each been subjected to one of the extreme scenarios listed below:

- (i) severe flooding by a river passing under the bridge;
- (ii) fire resulting from a petrol lorry crashing into one reinforced concrete pier supporting the bridge above;
- (iii) earthquake;
- (iv) spalling of several large chunks of concrete from the deck soffit onto the road underneath.

The bridges all have three spans. Each span consists of a simply supported reinforced concrete slab on reinforced concrete piers and abutments.

For each scenario, describe the causes and type of damage that might result and the effect each of these incidents might have on the longer term durability of the structures. Discuss remedial measures you might employ to refurbish the bridges and reduce the risk of a similar event happening again in the future. List any test techniques you might employ to assist.

[80%]

(b) In practice, structural failure is extremely rare. Comment on how limit state design accounts for these extremely rare events and the validity of the assumptions involved. Give one example of a structural collapse, with details of the primary cause or causes.

[20%]

2 (a) Describe the principles that underlie the Whole Life Costing approach to investment decision-making in construction. Give an example of how these might be applied to choose between two options for building reinforced concrete structures on a new section of motorway. [30%]

(b) A designer specifies that steel bars in a particular project should have a *characteristic* strength  $f_{yk}$  of 460 MPa. The steel yield strength may be assumed normally distributed with a coefficient of variation of 10%. The bars are all 16 mm diameter and the variability in this diameter can be assumed negligible.

(i) Determine the *design* yield strength  $f_{yd}$  for the bars and hence calculate the probability that a bar chosen at random will have less than this design strength. [10%]

(ii) The *design* value of tensile load to be applied to the bars is specified to equal the design value of the strength of the bars calculated in (i) above. The load, which is composed of dead load only, can be assumed to be normally distributed with standard deviation of 10 kN. What is the reliability index  $\beta$  and hence probability of failure in tension for any randomly selected bar subjected to this loading? [20%]

(iii) When the bars arrive on site the mill certificate for the steel states that the mean strength is actually 500 MPa with a standard deviation of 23 MPa. Do the bars supplied comply with the specification? A site survey indicates that the actual dead loads will have both mean and standard deviation 20% greater than that assumed in (ii) above. What is the reliability index and probability of failure for the actual bars subjected to this revised loading? The bar strengths and loading are assumed to remain normally distributed. [20%]

(iv) As an alternative to specifying that the design load should equal the design strength, the engineer decides to specify a target reliability index of  $\beta = 3.5$  in order to determine the dead loads that could be carried safely by the steel bars. If the standard deviation of the loads is 10 kN and the originally specified steel properties are assumed, what would be the value of the *characteristic* load and the *design* load? What is the probability of failure for the bars under this loading? [20%]

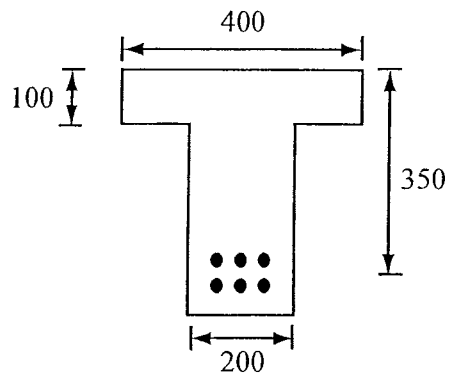
3 (a) Plot a schematic stress versus axial-strain curve for a normal strength concrete cylinder subjected to uniaxial compression identifying salient features. How would the behaviour of a high strength concrete specimen differ from that of a normal strength concrete specimen? Add a typical stress-strain curve for high strength concrete to your plot. [20%]

(b) A 425 mm deep reinforced concrete T-beam section with a web width of 200 mm and a flange width of 400 mm is shown in Fig. 1. The flange depth is 100 mm and the effective depth to the centroid of the longitudinal reinforcing steel is 350 mm. The steel reinforcement consists of  $6 \times 20$  mm diameter bars with a Young's modulus of 210 GPa, and factored design yield stress of  $f_{yd} = 430$  MPa. The concrete has a Young's modulus of 30 GPa and a factored design compressive cube strength of  $f_{cd} = 30$  MPa. The concrete is assumed to fail in compression at a uniform stress of  $0.6 f_{cd}$ .

(i) Assuming under-reinforced behaviour, calculate the maximum bending moment that the cross-section can sustain at the ultimate limit state. [35%]

(ii) There are problems with the quality of the concrete in the top surface of the flange. As a result, there is a zone of lower strength concrete, with a lower stiffness, that extends the full flange width to a depth of 50 mm measured from the top surface. The concrete in the rest of the cross-section is of good quality. Taking into account the zone of lower strength concrete, make a sketch of the longitudinal bending stress and strain distribution through the cross-section at both the serviceability limit state and the ultimate limit state. Discuss the implications in terms of the curvature of the section under working load, and the ultimate limit state failure load, when compared to a section with good quality concrete throughout. Do not carry out any further calculations. [35%]

(iii) Briefly discuss three factors that might lead to poor quality concrete in the top surface of a concrete section. [10%]



all dimensions in mm

Fig. 1

4 (a) A  $400 \text{ mm} \times 300 \text{ mm}$  rectangular reinforced concrete section is shown in Fig. 2. The beam is reinforced with 10 mm diameter vertical stirrups at 125 mm spacings. The factored design yield stress of the steel stirrups is 430 MPa. The beam can be modelled as a hollow section with an equivalent wall thickness of 80 mm and it can be assumed that any steel present yields. A factored torque of  $T = 60 \text{ kNm}$  is applied to the section.

(i) What additional longitudinal reinforcement, with a design yield stress of 430 MPa, is required to sustain the applied torque? [15%]

(ii) Find the minimum design concrete strength required to ensure that the beam can sustain the applied torque. The effectiveness factor for the concrete can be taken as 0.5 and assume that the additional longitudinal reinforcement is that found in (i). [30%]

(iii) How would the expressions used in (i) and (ii) change if the design yield stress of the additional longitudinal reinforcement was 500 MPa instead of 430 MPa? [25%]

(b) Discuss four important design considerations in the design of loadbearing masonry buildings. Why is it that the compressive strength of a masonry blockwork wall is greater than the intrinsic compressive strength of the mortar? [30%]

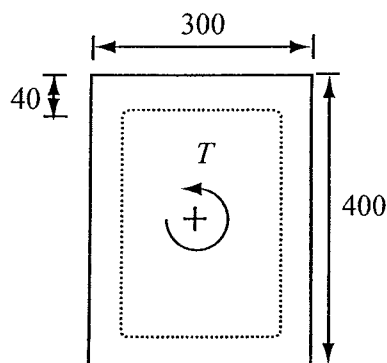


Fig. 2

**END OF PAPER**

**Module 4D7 : Concrete and masonry structures****Formula and Data Sheet**

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

**Material variability, partial safety factors and probability of failure**

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually  $\gamma_m = 1.0$  on all material strengths,  $\gamma_f = 1.0$  on all loads.

At ULS, usually  $\gamma_m$  is 1.15 for steel, 1.5 for concrete; and  $\gamma_f$  is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Convolution integral 
$$P_f = \int_{-\infty}^{+\infty} f_S(x) F_R(x) dx$$

**Cement paste**

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$$\left[ 3.15 \frac{W}{C} - 1.14h \right] / \left[ 1 + 3.15 \frac{W}{C} \right]$$

for hydration degree  $h$  :

and gel/space ratio (gel volume / gel + capillaries)  $2.14h / \left[ h + 3.15 \frac{W}{C} + a \right]$ .

**Mechanical properties of concrete**

Cracking strain typically  $150 \times 10^{-6}$ , strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

**Durability considerations**

Present value of some future good:  $S_i / (1 + r)^i$  for stepped,

or  $S_i / \exp(r_c t_i)$  for continuous discounting

where  $(1 + r) = \exp(r_c)$

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is  $\{2k(H + h_c)/\Delta n\}^{1/2} t^{1/2}$ .

Uniaxial diffusion into homogeneous material :  $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$

solution  $c = c_o (1 - \text{erf}(z)), z = x/2\sqrt{Dt}$

Table of erf (z) :

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
erf(z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	
z	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	$\infty$
erf(z)	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Passivation for pH > 12 and Cl<sup>-</sup> < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2  $\mu\text{A}/\text{cm}^2$ , resistivity > 100 k  $\Omega$  cm, half-cell potential > -200 mV (but probable for < -350 mV).

#### SLS : cracking

Steel minimum area  $A_s f_y \geq f_t A_{ct}$

for pure tension, to produce multiple cracks.

Then, limitation of crack width to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width  $w_k = s_{r,\max} (\epsilon_{sm} - \epsilon_{cm})$

Where crack spacing  $s_{r,\max} = 3.4c + 0.425k_1 k_2 \phi / \rho_{p,\text{eff}}$

with  $k_1$  0.8 for high bond, 1.6 for plain bars;  
 $k_2$  1.0 for tension, 0.5 for bending.

#### SLS : deflection

Interpolated curvature

$$\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$$

where  $\zeta = 1 - \beta (\sigma_{sr} / \sigma_s)^2$

$\beta$  is 1.0 for short-term, 0.5 for sustained load,  
 $\sigma_{sr}$  is steel stress, for cracked section, but using loads which first cause cracking at the section considered.  
 $\sigma_s$  is current steel stress, calculated for cracked section.



### ULS : moment and axial force

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often  $\varepsilon_{cm} = 0.0035$ . The yield strain of steel  $\varepsilon_y$  of course depends on strength, as roughly  $f_y/E$ . Initial calculations often use uniform stress of  $0.6 f_{cd}$  on the compression zone at failure. With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d (1 - 0.5 x/d) / \gamma_s;$$

where 
$$x/d = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} b d};$$

over-reinforcement for  $x/d > 0.5$ .

For Tee beams, effective flange width  $b$  in compression is of order

$$b_w + l_o / 5 \leq b_{actual},$$

where  $l_o$  is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$$

where  $\kappa_m$  is curvature at mid-height at failure and  $l_o$  is effective length.

Eurocode multiplies by further factors  $K_r$  and  $K_\phi$ ,

where 
$$K_r = \left( \frac{n_u - n}{n_u - n_{bal}} \right) \leq 1$$

### Shear in reinforced concrete

For *unreinforced* webs at ULS, shear strength in EC2 is

$$V_{Rd,c} = \left[ \frac{0.18}{\gamma_c} k (100 \rho_l f_{ck})^{1/3} + 0.15 \sigma_{cp} \right] b_w d$$
$$\geq (v_{\min} + 0.15 \sigma_{cp}) b_w d$$

where:  $k = 1 + \sqrt{200/d} \leq 2.0$  a factor that varies with effective depth,  $d$  (with  $d$  in mm),

$\rho_l$  is the reinforcement ratio of anchored steel =  $A_s/b_w d$  but  $\rho_l \leq 0.02$ .

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

For *reinforced* webs at ULS, shear strength in EC2 is

- Concrete resistance

$$V_{Rd,max} = f_{c,max} (b_w 0.9d) / (\cot \theta + \tan \theta)$$

where:

$$f_{c,max} = 0.6 (1 - f_{ck} / 250) f_{cd}$$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d) (\cot \theta) / (s \gamma_s)$$

## Torsion at ULS

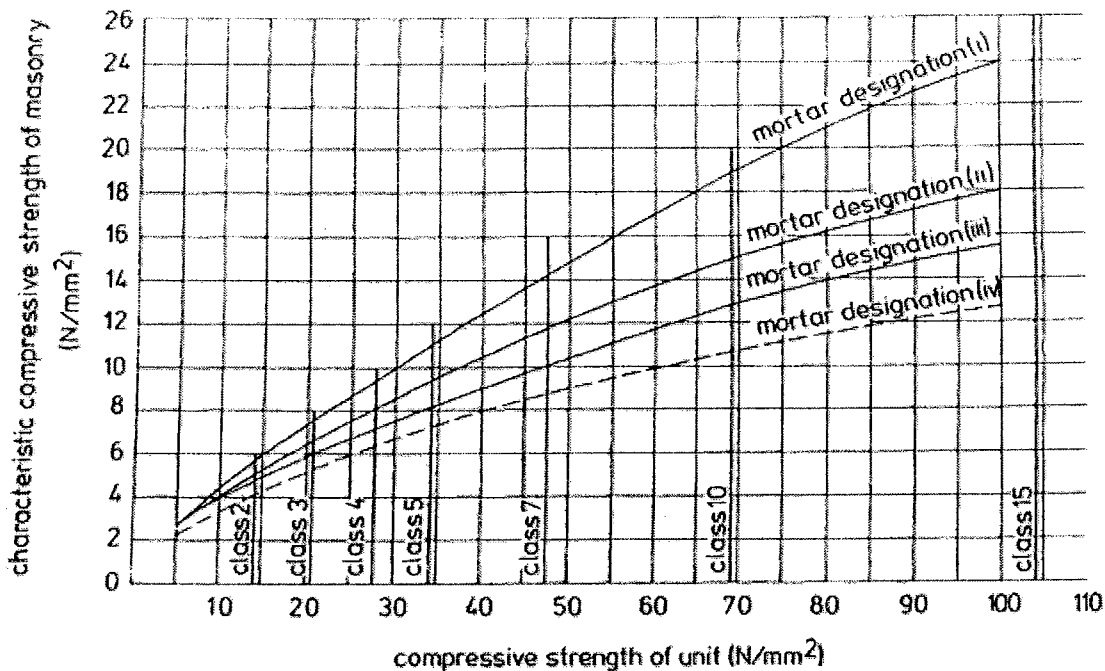
Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd} \sqrt{\frac{A_w \cdot \sum A_l}{s \cdot u}}$$

where

$$\sigma_c < v \cdot f_{cd}$$

## Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

Figure 5.6(a) Characteristic compressive strength,  $f_k$ , of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\frac{z^2}{2}} dx \quad \text{FOR } 0.00 \leq u \leq 4.99.$$

u	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.990097	.990358	.990613	.990863	.991106	.991344	.991576
2.4	.991802	.992024	.992240	.992451	.992656	.992857	.993053	.993244	.993431	.993613
2.5	.993790	.993963	.994132	.994297	.994457	.994614	.994766	.994915	.995060	.995201
2.6	.995339	.995473	.995604	.995731	.995855	.995975	.996093	.996207	.996319	.996427
2.7	.996533	.996636	.996736	.996833	.996928	.997020	.997110	.997197	.997282	.997365
2.8	.997445	.997523	.997599	.997673	.997744	.997814	.997882	.997948	.998012	.998074
2.9	.998134	.998193	.998250	.998305	.998359	.998411	.998462	.998511	.998559	.998605
3.0	.998650	.998694	.998736	.998777	.998817	.998856	.998893	.998930	.998965	.998999
3.1	.9990324	.9990646	.9990957	.9991260	.9991553	.9991836	.9992112	.9992378	.9992636	.9992886
3.2	.9993129	.9993363	.9993590	.9993810	.9994024	.9994230	.9994429	.9994623	.9994810	.9994991
3.3	.9995166	.9995335	.9995499	.9995658	.9995811	.9995959	.9996103	.9996242	.9996376	.9996505
3.4	.9996631	.9996752	.9996869	.9996982	.9997091	.9997197	.9997299	.9997398	.9997493	.9997585
3.5	.9997674	.9997759	.9997842	.9997922	.9997999	.9998074	.9998146	.9998215	.9998282	.9998347
3.6	.9998409	.9998469	.9998527	.9998583	.9998637	.9998689	.9998739	.9998787	.9998834	.9998879
3.7	.9998922	.9998964	.99990039	.99990426	.99990799	.99991158	.99991504	.99991838	.99992159	.99992468
3.8	.99992765	.99993052	.99993327	.99993593	.99993848	.99994094	.99994331	.99994558	.99994777	.99994988
3.9	.99995190	.99995385	.99995573	.99995753	.99995926	.99996092	.99996253	.99996406	.99996554	.99996696
4.0	.99996833	.99996964	.99997090	.99997211	.99997327	.99997439	.99997546	.99997649	.99997748	.99997843
4.1	.99997934	.99998022	.99998106	.99998186	.99998263	.99998338	.99998409	.99998477	.99998542	.99998605
4.2	.99998665	.99998723	.99998778	.99998832	.99998882	.99998931	.99998978	.99999022	.99999065	.99999106
4.3	.999991460	.999991837	.999992199	.999992545	.999992876	.999993193	.999993497	.999993788	.999994066	.999994332
4.4	.999994587	.999994831	.999995065	.999995288	.999995502	.999995706	.999995902	.999996089	.999996268	.999996439
4.5	.999996602	.999996759	.999996908	.999997051	.999997187	.999997318	.999997442	.999997561	.999997675	.999997784
4.6	.999997888	.999997987	.999998081	.999998172	.999998258	.999998340	.999998419	.999998494	.999998566	.999998634
4.7	.999998699	.999998761	.999998821	.999998877	.999998931	.999998983	.9999990320	.9999990789	.9999991235	.9999991661
4.8	.9999992067	.9999992453	.9999992822	.9999993173	.9999993508	.9999993827	.9999994131	.9999994420	.9999994696	.9999994958
4.9	.9999995208	.9999995446	.9999995673	.9999995889	.9999996094	.9999996289	.9999996475	.9999996652	.9999996821	.9999996981

Example:  $\Phi(3.57) = .998215 = 0.9998215.$

Answers (4D7 - 2009/2010)

- Q2 (b) (i)  $f_{yd} = 400 \text{ MPa}$ ,  $P_f = 3 \times 10^{-3}$   
(ii)  $\beta = 4.67$ ,  $P_f = 1.51 \times 10^{-6}$   
(iii) yes ( $f_{yk} = 462 \text{ MPa} > 460 \text{ MPa}$ ),  $\beta = 4.0$ ,  $P_f = 3.2 \times 10^{-5}$   
(iv)  $S_k = 75 \text{ kN}$ ,  $S_d = 105 \text{ kN}$ ,  $P_f = 2.3 \times 10^{-4}$
- Q3 (b) (i)  $M_{ult} = 237.5 \text{ kNm}$
- Q4 (a) (i)  $\Sigma A_l = 1689 \text{ mm}^2$   
(ii)  $f_{cd} = 23.6 \text{ MPa}$   
(iii) area of longitudinal reinforcement required decreases to  $430/500 \times 1689 \text{ mm}^2 = 1453 \text{ mm}^2$  but the concrete strut angle and the required design concrete strength are unchanged