ENGINEERING TRIPOS PART IIB

Monday 3 May 2010 9 to 10.30

Module 4F5

ADVANCED WIRELESS COMMUNICATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Consider the channel described in Fig. 1. This channel is a good model for optical fibre transmission; when no photon is transmitted, with probability 1, no photon will be received: when a photon is transmitted it can be lost with a certain probability. Calculate the entropies $H(Y|X)$, $H(X,Y)$, $H(Y)$, $H(X|Y)$ and the mutual information $I(X;Y)$ for equiprobable input symbols and a loss probability of $1/2$. [25%]

(b) Let Y_1 be the output of channel 1 to input X_1 and Y_2 be the output of channel 2 to input X_2 (see Fig. 2). Find the value of p that makes $I(X_1;Y_2) = I(X_1;Y_1)$ when the output of channel 1 is used as input for channel 2, i.e., $X_2 = Y_1$. Is there any value of p that makes $I(X_1;Y_2) > I(X_1;Y_1)$? Justify your answer. [25%]

(c) Find the capacity and the capacity-achieving input distribution for the channel shown in Fig. 3. Justify your answer. [25%]

(d) Let Y_1 be the output of channel 1 to input X_1 and Y_2 be the output of channel 2 to input X_2 (see Fig. 4). Consider the following transmission strategy: transmit a photon on channel 1 when a 0 is to be sent; transmit a photon on channel 2 when a 1 is to be sent. Draw the equivalent channel transition diagram and compare it to the binary erasure channel. What is its capacity? [25%]

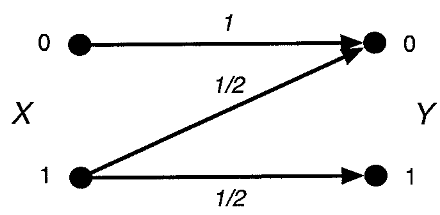

Fig. 1

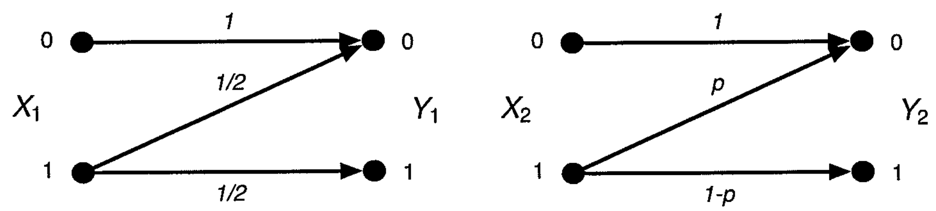

Fig. 2

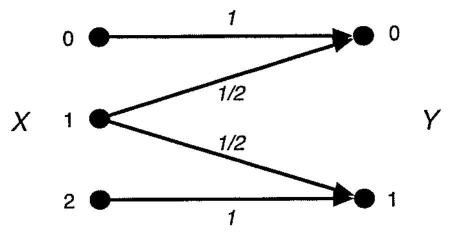

Fig. 3

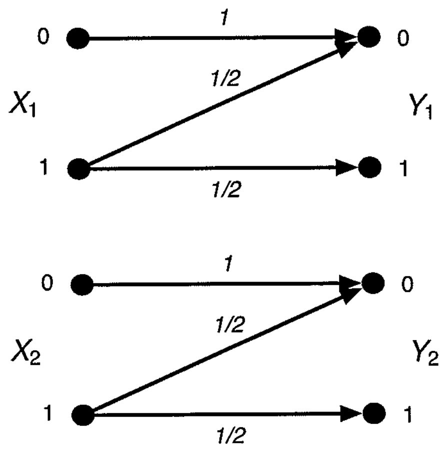

Fig. 4

2 Consider the signal constellation shown in Fig. 5.

(a) What is the dimension of the signal space? [10%]

(b) Write down the vector representation of the signal constellation as a function of $f_1(t)$ and $f_2(t)$. [20%]

(c) What is the minimum distance of the signal constellation? [10%]

(d) Find the energy of each signal vector and the average energy of the signal constellation. [15%]

(e) Given the signals $f_1(t)$ and $f_2(t)$ shown in Fig. 6, verify they are a suitable basis for the signal space. [20%]

(f) Sketch the 8 signals $x_1(t),\ldots,x_8(t)$ corresponding to the signal constellation in Fig. 5 using the basis functions $f_1(t)$ and $f_2(t)$ shown in Fig. 6. [25%]

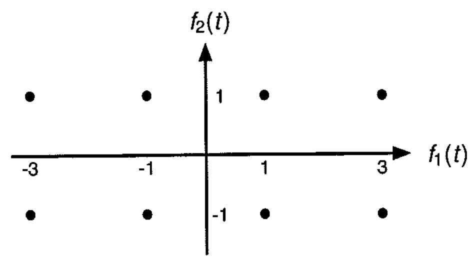

Fig. 5

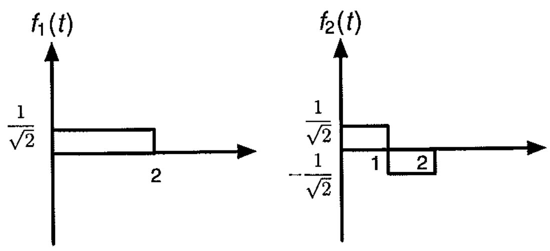

Fig. 6

3 Consider the code with factor graph representation given in Fig. 7.

(a) Write down the parity-check equations and the corresponding parity-check matrix. What is the code rate? [25%]

(b) Write down the variable and check-node degree distributions (edge perspective, i.e., λ_i and ρ_i) of the code, interpreted as a low-density parity-check code. [20%]

(c) Find a systematic generator matrix. [30%]

(d) The code is used for transmission over a binary erasure channel. The received word is $?,?,1,1,?,0,0$, where ? denotes the erasure symbol. Can the iterative decoder successfully decode? What is the decoded codeword and what are the decoded information bits? [25%]

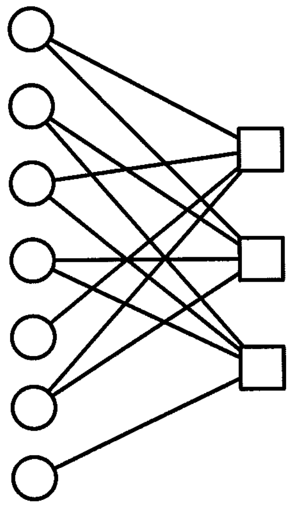

Fig. 7

4 (a) Consider transmission over a fading channel with the multipath intensity profile given in Fig. 8 and Doppler spectrum following the Jakes model given by

$$S_H(\xi) = \begin{cases} \frac{1}{\pi f_m} \frac{1}{\sqrt{1-(\xi/f_m)^2}} & |\xi| \leq f_m \\ 0 & |\xi| > f_m \end{cases}$$

where f_m is the maximum Doppler frequency.

(i) What is the coherence bandwidth of the channel? [10%]

(ii) What is the coherence time of the channel if the carrier frequency is $f_c = 10$ MHz and the mobile user is moving at a velocity $v = 10$ km h^{-1}? [10%]

(iii) Will the channel introduce frequency or time selectivity if codewords of duration $T_x = 20$ ms using signals of bandwidth $B_x = 2$ MHz are employed for transmission? [10%]

(b) Consider binary transmission over a fast Rayleigh fading channel. Show that the pairwise error probability (PEP) of an error event at Hamming distance d can be upper bounded by [25%]

$$\mathrm{PEP}(d) \leq \left(\frac{1}{1+\mathrm{SNR}}\right)^d .$$

(c) Consider the convolutional code shown in Fig. 9 followed by an interleaver and an 8-PSK modulator.

(i) What is the code diversity? [15%]

(ii) What is the overall transmission rate? [10%]

(d) The output of the modulator is now parsed across 5 transmit antennas. The receiver is equipped with 3 antennas.

(i) What is the transmission rate of the overall scheme? [10%]

(ii) Consider a maximum-likelihood detector. How many metrics does the detector need to compute? [10%]

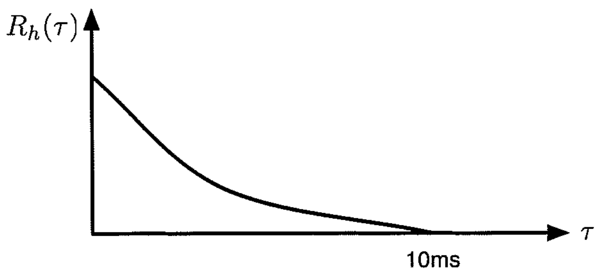

Fig. 8

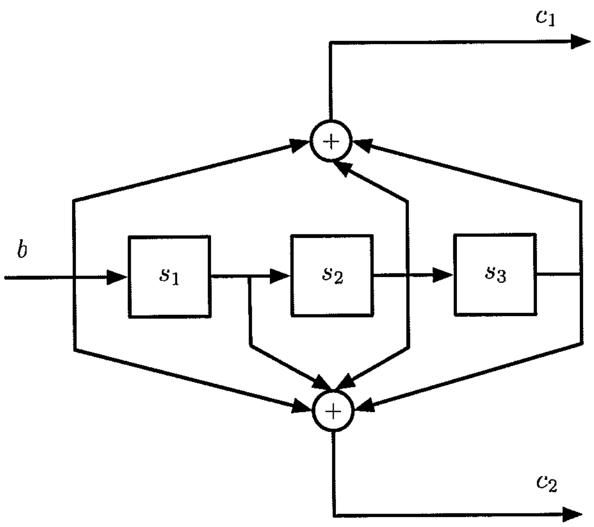

Fig. 9

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