ENGINEERING TRIPOS PART IIB

---

Wednesday 5 May 2010 9 to 10.30

---

Module 4F6

SIGNAL DETECTION AND ESTIMATION

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

wjf03

1 (a) Prove the *Neyman-Fisher Factorisation* theorem and explain why this is useful in estimation theory. [20%]

(b) The *scalar exponential family* of probability density functions for a random variable $x$ may be written as :

$$p(x|\theta) = \exp(A(\theta)B(x) + C(x) + D(\theta))$$

If data $x(n)$ for $n = 0, 1, 2, ..., N-1$ are observed which are independent and identically distributed and whose probability density function belongs to this family, show that a *sufficient statistic*, $T(x)$, for the parameter $\theta$ is given by :

$$T(x) = \sum_{n=0}^{N-1} B(x(n))$$

[40%]

(c) Show that both the Gaussian and the exponential probability density functions belong to the *scalar exponential family* and that the sufficient statistics in both cases are given by :

$$T(x) = \sum_{n=0}^{N-1} x(n)$$

[40%]

2 (a) Define the term *Cramer-Rao Lower Bound* (CRLB), explaining it's importance in estimation theory. [25%]

(b) What is meant by the term *Unbiased Estimator* ? [10%]

(c) Consider the Maximum Likelihood (ML) estimation of an unknown parameter $\theta$ using a single sample of data, $x_0$, with likelihood function given by :

$$p(x_0|\theta) = \frac{x_0}{\theta}\exp(-\frac{x_0^2}{2\theta}), \quad 0 \leq x_0$$

and zero otherwise.

(i) Find the ML estimator of $\theta$. [25%]

(ii) Determine if this estimator is biased or unbiased. [15%]

(iii) Now suppose that $\theta$ is to be estimated from N independent and identically distributed samples, $x_0, x_1, .., x_{N-1}$. Find the ML estimator of $\theta$. [25%]

3 (a) Describe, in detail, the three main decision rules used in detection theory and discuss the advantages and disadvantages of each of these rules. [20%]

(b) Multiple observations are made using a radar system and when a target is present, the received signal samples are given by :

$$y(n) = s(n) + e(n)$$

where $n = 1, 2, ..., N$, $s(n)$ is the known transmitted signal and $e(n)$ is zero mean white Gaussian noise with variance $\sigma^2$. With no target present, the received signal is just noise.

(i) Show that the likelihood ratio detector is given by :

$$\mathbf{y}^{\mathbf{T}}\mathbf{s} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2}\mathbf{s}^{\mathbf{T}}\mathbf{s} + \sigma^2 \ln k$$

where $k$ is the appropriate threshold depending on the decision rule chosen. [60%]

(ii) Give a new expression for the detector if the noise is coloured. [20%]

4 (a) Define the *Receiver Operator Characteristic* (ROC) curve of a detector. [20%]

(b) Explain how the threshold of a *Neyman-Pearson* detector is related to the slope of the ROC curve. [20%]

(c) It is required to detect a signal given by

$$s(n) = A + Bn$$

where $n = 0, 1, 2, ..., N-1$, in additive white Gaussian noise of variance $\sigma^2$ and where $A$ and $B$ are known.

Show that the data may be written in the form of a *General Linear Model* and determine the *Neyman-Pearson detector* for this problem. [60%]

**END OF PAPER**