ENGINEERING TRIPOS PART IIB

Thursday 6 May 2010 2.30 to 4

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

ep04

1 (a) If an image $g(u_1,u_2)$ is sampled on a rectangular grid (spacings Δ_1 and Δ_2 in u_1 and u_2 respectively), the sampled image $g_s(u_1,u_2)$ may be written as

$$g_s(u_1,u_2) = s(u_1,u_2)g(u_1,u_2)$$

Give an expression for $s(u_1,u_2)$ assuming an infinite extent. [10%]

(b) Write s as a Fourier series and hence find $G_s(\omega_1,\omega_2)$, the Fourier transform of the sampled image g_s, in terms of $G(\omega_1,\omega_2)$, the Fourier transform of the original image g. Use this result to explain the phenomenon of *aliasing*. [20%]

(c) Suppose that instead of a rectangular grid, we sample on a diamond grid, $s_d(u_1,u_2)$, with spacings Δ_1 and Δ_2 in u_1 and u_2, as shown in Fig. 1.

(i) By treating this grid as the sum of two rectangular grids (one centered on the origin and one shifted), show that $s_d(u_1,u_2)$ can be written as a Fourier series with Fourier coefficients, $c(p_1,p_2)$, given by [20%]

$$\frac{1}{4\Delta_1\Delta_2}[1+\mathrm{e}^{-j(p_1+p_2)\pi}]$$

(ii) Hence show that the Fourier transform of the image sampled on a diamond grid is the Fourier transform of the original image repeated at particular intervals in the frequency domain and specify those intervals. [5%]

(d) Consider the zero-phase ideal frequency response $H(\omega_1,\omega_2)$ shown in Fig. 2, where $\Omega_s < \pi/\Delta_1$ and $\Omega_s < \pi/\Delta_2$. H takes the value 1 in the shaded region and zero outside this region. The original image is sampled with spacings Δ_1 and Δ_2 in the u_1 and u_2 directions respectively.

By inverse Fourier transforming $H(\omega_1,\omega_2)$, either directly or via a change of variables, show that the ideal impulse response is given by [30%]

$$\frac{\Delta_1\Delta_2}{2\pi^2}{\Omega_s}^2 \,\mathrm{sinc}\left((n_1\Delta_1+n_2\Delta_2)\frac{\Omega_s}{2}\right)\,\mathrm{sinc}\left((n_1\Delta_1-n_2\Delta_2)\frac{\Omega_s}{2}\right)$$

(e) By considering the standard result for an ideal frequency response defined by

$$H(\omega_1,\omega_2) = \begin{cases} 1 & \text{if } |\omega_1| < \Omega_c \text{ and } |\omega_2| < \Omega_c \\ 0 & \text{otherwise} \end{cases}$$

where $\Omega_c = \Omega_s/\sqrt{2}$, show how one can obtain the result of part (d) above. [15%]

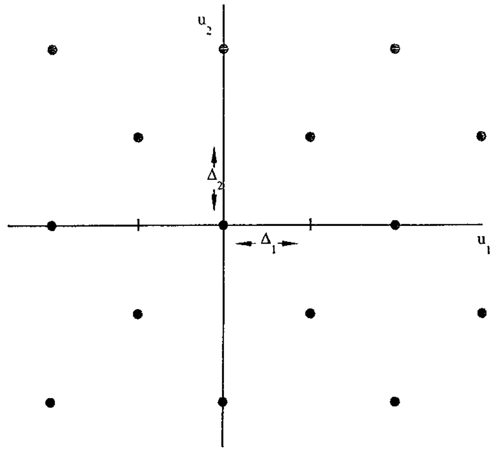
u_2
Δ_2
Δ_1
u_1

Fig. 1

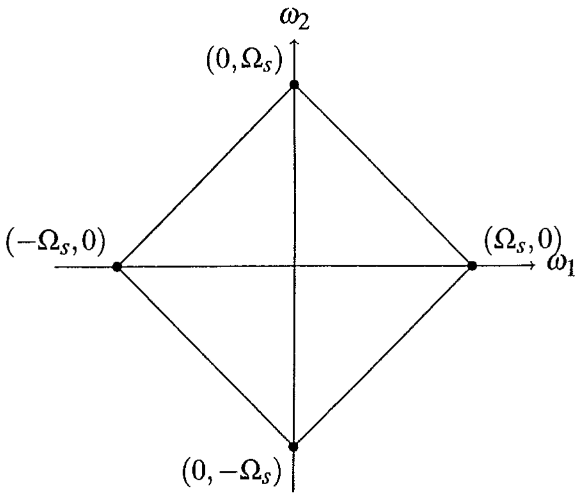
ω_2
$(0, \Omega_s)$
$(-\Omega_s, 0)$
$(\Omega_s, 0)$
ω_1
$(0, -\Omega_s)$

Fig. 2

2 (a) If an image displays poor use of available grey levels we can often improve its appearance via the process of *histogram equalisation.*

(i) Outline, qualitatively, the concept of histogram equalisation in images. [10%]

(ii) Suppose that our range of greyscale values is 1 to 18. Consider the 6×6 image given in Fig. 3.

7	8	11	9	7	8
9	6	10	10	12	8
11	8	11	9	6	7
10	8	10	10	11	9
9	12	10	10	9	8
8	7	9	10	8	9

Fig. 3

Sketch the histogram of this image and comment on the use of the available grey levels. [10%]

(iii) Perform histogram equalisation on this image by finding the set of transformed values, $\{y_k\}$, $k = 1,..,18$, onto which the original grey levels are mapped. [20%]

(iv) Sketch the new equalised image and its histogram, commenting on how well the process has worked. Indicate how the spread of grey levels might be improved. [10%]

(b) Assume that an observed image, $\mathbf{y}$, can be modelled as a linear distortion, L, of the true image, $\mathbf{x}$, plus additive noise, $\mathbf{d}$, i.e. $\mathbf{y} = L\mathbf{x} + \mathbf{d}$.

(i) If L is known and we neglect noise, explain how the simple *inverse filter* can be used to estimate the true image. Explain also why such a filter performs poorly on real images and outline how the *generalised inverse filter* can improve performance. [20%]

(ii) More sophisticated deconvolution techniques will attempt to estimate the *posterior*, $P(\mathbf{x}|\mathbf{y})$, via Bayes theorem, $P(\mathbf{x}|\mathbf{y}) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x})$.

If we assume the noise to be Gaussian, give the forms of the *likelihood*, $P(\mathbf{y}|\mathbf{x})$, and of the *prior*, $P(\mathbf{x})$, for the *Wiener filter* and *Maximum Entropy* deconvolution methods. Describe all terms you use in your expressions, and any assumptions made. [30%]

3 (a) The N-point discrete cosine transform (DCT), where N is even, is based on cosine waves with period $2N/k$ samples, where k is an integer and $0 \leq k < N$. Give expressions for the rows of the DCT matrix, $\mathbf{T}$, where the elements are either symmetric or anti-symmetric about the centre of each row and the rows are each normalised to unit magnitude. What key property of the DCT allows the inverse DCT to be calculated easily? [25%]

(b) Calculate $\mathbf{y}$, the 4-point DCT of a vector $\mathbf{x}$, where

$$\mathbf{x} = [p \;\; p \;\; q \;\; q]^T$$

expressing the result for each element y_i, $i = 1, \ldots, 4$, as simply as possible. [15%]

(c) If $\mathbf{X}$ is an $N \times N$ block of image pixels, explain how $\mathbf{Y}$, the 2-dimensional DCT of $\mathbf{X}$, may be calculated using $\mathbf{T}$ and $\mathbf{X}$, and how $\mathbf{X}$ may be recovered from $\mathbf{Y}$. [15%]

(d) A small patch of an image is represented by the matrix

$$\mathbf{X} = \begin{bmatrix} r & r & r & r \\ r & r & r & r \\ r & r & s & s \\ r & r & s & s \end{bmatrix}$$

where the pixel intensities are $r = 50$ and $s = 150$. Calculate $\mathbf{Y}$, the 2-dimensional DCT of $\mathbf{X}$, and briefly explain what subsequent processes would typically be applied to $\mathbf{Y}$ to encode the patch efficiently with binary symbols. [45%]

4 (a) Draw the block diagram of a 3-level 1-dimensional wavelet transform system, clearly showing all the highpass and lowpass filters and down-samplers that are needed. In addition, draw the equivalent block diagram of a system to perform the 3-level inverse wavelet transform. [15%]

(b) Describe the *multi-rate filtering theorem*. In particular, state how the transfer function of a filter is modified when it is moved from acting before a 2:1 upsampler to acting after the upsampler. [10%]

(c) Show how the block diagram of the inverse wavelet transform may be modified so that all up-samplers are moved to the inputs of the filter bank, and hence calculate the transfer function from each input to the output of the system. [20%]

(d) For an inverse of the Haar wavelet transform, the filters are given by

$$G_0(z) = \frac{1}{\sqrt{2}}(z+1) \quad \text{and} \quad G_1(z) = \frac{1}{\sqrt{2}}(z-1)$$

Calculate the impulse responses from the two inputs at level 3 of the inverse Haar transform to the output and sketch these functions. [20%]

(e) Explain why these responses tend to produce blocking artifacts in the reconstructed images when the Haar transform is extended to be used for 2-dimensional signals (images). [15%]

(f) Give an example of the simplest form of filter for $G_0(z)$ that overcomes the problem of blocking artifacts and explain briefly why this is achieved. [20%]

END OF PAPER