ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Monday 26 April 2010 2.30 to 4

Module 4M16

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4M16 data sheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) What are the main limitations of one-group diffusion theory in describing the distribution of neutrons in a reactor? Define *neutron flux* and *neutron current density* in this theory. [15%]

(b) A uniformly loaded unreflected nuclear reactor in the shape of a right circular cylinder of radius R and height H is critical in steady state with no sources. Derive an expression for the one-group diffusion flux distribution in the reactor, stating any assumptions made. [60%]

(c) What is the radial form factor for this reactor? [15%]

(d) How can form factors be improved in practice? [10%]

2 There is growing interest in breeding fissile uranium-233 from thorium-232. When Th-232 captures a neutron it is transmuted into Th-233, which then decays by beta emission into protactinium-233 with a half-life of 22 minutes. Protactinium-233 decays into uranium-233 by beta emission with a half-life of 27 days. This half-life is so much longer than that for the decay of Th-233 that, for the purposes of modelling the process of breeding U-233, it can be assumed that Th-232 is transmuted into Pa-233 instantaneously.

(a) An engineer is investigating the feasibility of breeding U-233 from Th-232 using a flux of fast neutrons generated by spallation using a proton accelerator. What will be the initial rate of Pa-233 production per unit volume of Th-232 if an average flux of $10^{15}\ \mathrm{m^{-2}s^{-1}}$ can be generated? Take the microscopic capture cross-section of Th-232 for fast neutrons to be 0.309 barns, its density to be $11724\ \mathrm{kg\,m^{-3}}$ and its atomic weight to be 232.04. [15%]

(b) The long-term evolution of the Th-232 and Pa-233 populations (T and P) can be modelled approximately using the following 'lumped' equations:

$$\frac{dT}{dt} = -T\sigma_{cT}\phi$$

$$\frac{dP}{dt} = T\sigma_{cT}\phi - \lambda_P P - P\sigma_{cP}\phi$$

where symbols have their usual meanings. Briefly explain the physical meaning of the terms on the right-hand sides of these equations. [10%]

(c) For the case where the initial Th-232 population is T_0 and the initial Pa-233 population is zero, show that according to these equations the Pa-233 population varies with time as

$$P = \frac{T_0\sigma_{cT}\phi}{\lambda_P + \left(\sigma_{cP} - \sigma_{cT}\right)\phi}\left[\exp(-\sigma_{cT}\phi t) - \exp\left(-\left[\lambda_P + \sigma_{cP}\phi\right]t\right)\right] \qquad [50\%]$$

(d) Using the data given above and taking the microscopic capture cross-section of Pa-233 for fast neutrons to be 0.976 barns, find the time at which the Pa-233 population reaches its peak value. [25%]

3 (a) Describe and explain the in-core and out-of-core fuel management strategies typically used with CANDU reactors and Pressurised Water Reactors. [40%]

(b) A low-enrichment reactor is fueled at start-up entirely with standard fresh fuel elements. The first cycle ends at a burnup of 18 MWd kg^{-1}. One third of the fuel is then changed at the end of each cycle on a "first in, first out" basis. Stating any assumptions made, estimate the lengths of the second and third cycles in MWd kg^{-1}. [35%]

(c) What ultimate burnup per cycle would this fuel management strategy lead to after many cycles? What would be the associated discharge burnup of the fuel? [10%]

(d) Estimate the proportion of fuel which should be replaced after the first cycle if the second cycle is to yield the same burnup as the ultimate, steady-state cycle. [15%]

4 (a) Discuss the pros and cons of the reprocessing of spent nuclear fuel. [25%]

(b) List the principal wastes arising from the reprocessing of nuclear fuel and give methods of treating each stream. [40%]

(c) A high level waste stream from the processing of spent fuel has the following initial activities:

2.5×10^6 Bq Sr-90, half-life: 29 years

2.5×10^6 Bq Cs-137, half-life: 30.2 years

10^4 Bq Am-234, half-life: 7370 years

Ignoring daughter isotope production, calculate the activity after 100 years, 1000 years and 10,000 years. [20%]

(d) Comment on the validity of the assumption concerning daughter products and on the implications of the results for the long-term storage of the wastes. [15%]

END OF PAPER

Data Sheet S/15

Revised January 2010

MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458\times10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi\times10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.626176\times10^{-32}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662\times10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892\times10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	1.6605655×10^{-27} kg (931.5016 MeV)
Electron volt	eV	1.6021892×10^{-19} J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	$10^{-28}\ \mathrm{m}^2$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

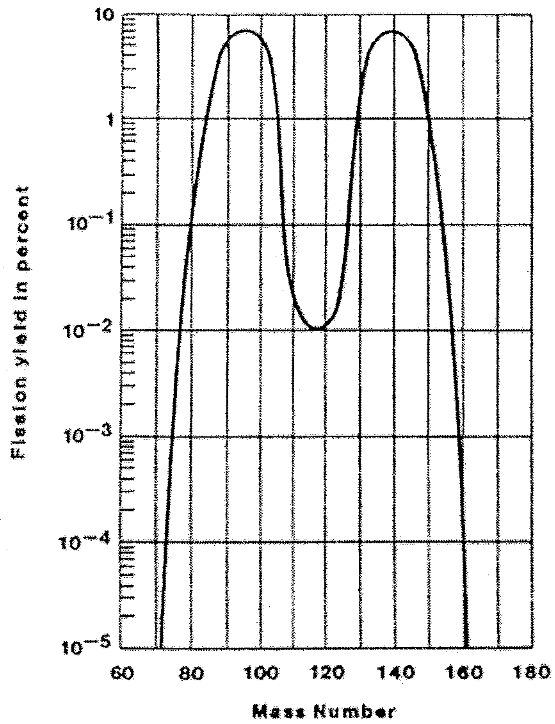
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

$$\text{Kr-90} \xrightarrow{33\text{ s}} \text{Rb-90} \xrightarrow{2.7\text{ min}} \text{Sr-90} \xrightarrow{28\text{ yr}} \text{Y-90} \xrightarrow{64\text{ hr}} \text{Zr-90}$$

Sr-90 is a serious health hazard, because it is bone-seeking.

$$\text{Sb-131} \xrightarrow{23\text{ min}} \text{Te-131} \xrightarrow{24\text{ min}} \text{I-131} \xrightarrow{8\text{ days}} \text{Xe-131}$$

I-131 is a short-lived health hazard. It is thyroid-seeking.

$$\text{I-135} \xrightarrow{6.7\text{ hr}} \text{Xe-135} \xrightarrow{9.2\text{ hr}} \text{Cs-135 (nearly stable)}$$ Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn.

$$\text{Pm-149} \xrightarrow{54\text{ hr}} \text{Sm-149}$$ Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn.

$$\text{Se-87} \xrightarrow{17\text{ s}} \text{Br-87} \xrightarrow{55\text{ s}} \text{either } \{\text{Kr-86} + n\} \text{ or } \{\text{Kr-87} \xrightarrow{78\text{ min}} \text{Rb-87}\}$$

This chain leads to a “delayed neutron”.

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor $(1/\lambda_i)$ / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons $(100\,\beta_i)$	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0^{th} Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2=D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small }x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity

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Answers

Q1 (b) $\phi(r,z)=\phi_0 J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$ with $\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2=B_m^2$

(c) 2.320

Q2 (a) $0.940\times10^{15}\ \mathrm{m^{-3}s^{-1}}$

(d) 1.72 years

Q3 (b) $\psi_2=6\ \mathrm{MWd\,kg^{-1}}$; $\psi_3=8\ \mathrm{MWd\,kg^{-1}}$

(c) $T_3=9\ \mathrm{MWd\,kg^{-1}}$; $B_3=27\ \mathrm{MWd\,kg^{-1}}$

(d) 0.5

Q4 (c) 490884.1 Bq; 9102.37 Bq; 3904.33 Bq