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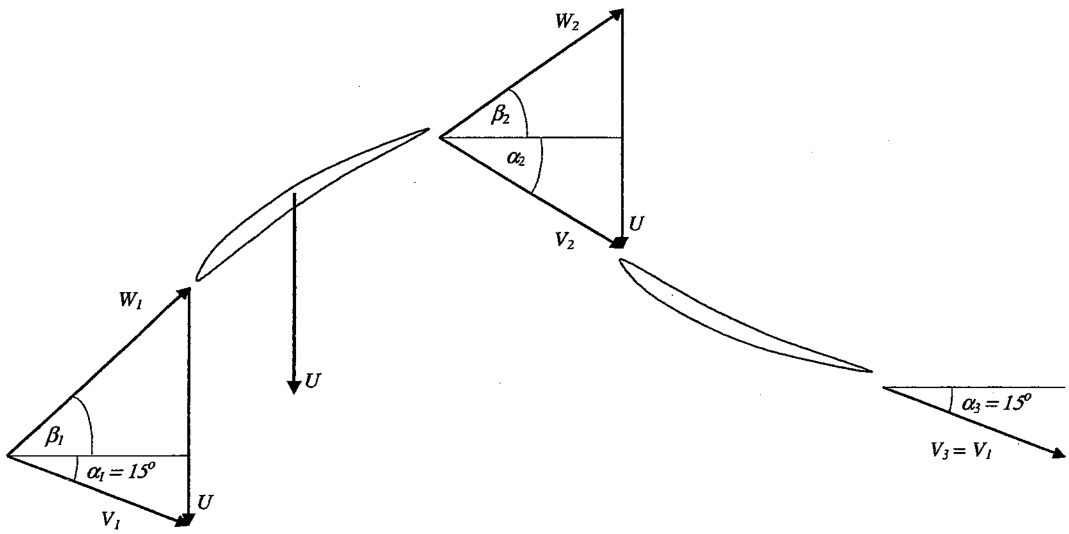
IIB Paper 4A3 Turbomachinery I Solutions 2011

1. a) Repeating stages are stages in a multi-stage machine that have identical mean velocity triangles. In a repeating stage, the axial velocity is constant and the flow angle at exit from the stage is the same as at inlet. To maintain constant axial velocity through a multi-stage compressor the blade height must reduce to allow for the change in fluid density as the flow is compressed. Repeating stages are often used as the mean design of each stage will be similar (similar blade shapes) and the performance of each stage will also be similar (similar loss, deviation).

[15%]

b)



$$\tan\beta_1 = \tan\alpha_1 - \frac{1}{\phi} = \tan 15^o - 2 \quad \Rightarrow \underline{\beta_1 = -60.0^o}$$

$$\psi = \frac{\Delta h_0}{U^2} = \frac{(V_{\theta 2} - V_{\theta 1})}{U} = \phi(\tan\alpha_2 - \tan\alpha_1)$$

$$\therefore \tan\alpha_2 = \frac{\psi}{\phi} + \tan\alpha_1 = 0.8 + \tan 15^o \quad \Rightarrow \alpha_2 = 46.88^o$$

$$\tan\beta_2 = \tan\alpha_2 - \frac{1}{\phi} = 0.8 + \tan 15^o - 2 \quad \Rightarrow \underline{\beta_2 = -42.99^o}$$

Reaction,

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{stator}}{\Delta h_0} = 1 - \frac{0.5\left(V_2^2 - V_3^2\right)}{\psi U^2} = 1 - \frac{0.5\phi^2}{\psi}\left(\tan\alpha_2^2 - \tan\alpha_1^2\right)$$

$$\Lambda = 1 - \frac{0.5 \times 0.5^2}{0.4}\left(\tan^2 46.88^o - \tan^2 15^o\right) = \underline{0.666}$$

[35%]

(c) At rotor inlet,

$$\frac{V_1}{\sqrt{c_p T_{01}}} = \frac{V_x/\cos\alpha_1}{\sqrt{c_p T_{01}}} = \frac{\phi U/\cos 15^0}{\sqrt{c_p T_{01}}} = \frac{0.5\times 220/\cos 15^0}{\sqrt{1005\times 300}} = 0.2074$$

From tables, $M_1 = 0.3315$ [approximations due to flow tables allowed for]

$$M_{1,rel} = M_1 \times \frac{\cos 15^0}{\cos 60^o} = \underline{0.6404}$$

Across the rotor,

$$T_{02} = T_{01} + \frac{\psi U^2}{c_p} = 300 + \frac{0.4\times 220^2}{1005} = 319.3K$$

$$\frac{V_2}{\sqrt{c_p T_{02}}} = \frac{V_x/\cos\alpha_2}{\sqrt{c_p T_{02}}} = \frac{\phi U/\cos 46.88^0}{\sqrt{c_p T_{02}}} = \frac{0.5\times 220/\cos 46.88^0}{\sqrt{1005\times 319.3}} = 0.2841$$

From tables, $M_2 = 0.4585$

$$M_{2,rel} = M_2 \times \frac{\cos 46.88^0}{\cos 42.99^o} = \underline{0.4284}$$

Applying continuity through the rotor passage,

$$\frac{\dot m\sqrt{c_p T_{01,rel}}}{A_{x1}\cos\beta_1\, p_{01,rel}} = Q(M_{1,rel}) = \frac{\dot m\sqrt{c_p T_{02,rel}}}{A_{x2}\cos\beta_2\, p_{02,rel}} \times \frac{\dot m/A_{x1}}{\dot m/A_{x2}} \times \frac{\cos\beta_2}{\cos\beta_1} \times \frac{p_{02,rel}}{p_{01,rel}}$$

$T_{01,rel} = T_{02,rel}$ (constant radius)

$$\Rightarrow \frac{p_{02,rel}}{p_{01,rel}} = \frac{Q(M_{1,rel})}{Q(M_{2,rel})} \times \frac{\cos\beta_1}{\cos\beta_2} \times \frac{\dot m/A_{x2}}{\dot m/A_{x1}} = \frac{1.1191}{0.8512} \times \frac{\cos 60^o}{\cos 42.99^o} \times 1.1 = 0.9884$$

$$Y_P = \frac{p_{01,rel} - p_{02,rel}}{p_{01,rel} - p_1} = \frac{1 - p_{02,rel}/p_{01,rel}}{1 - p_1/p_{01,rel}} = \frac{1 - 0.9884}{1 - 1.3177^{-1}} = \underline{0.048} \quad (\sim 5\%)$$

[35%]

c) The overall stagnation temperature ratio is found from the stage loading,

$$\frac{T_{0,ex}}{T_{01}} = 1 + \frac{N_{stage}\psi U^2}{c_p T_{01}} = 1 + \frac{10\times 0.4\times 220^2}{1005\times 300} = 1.642$$

$$OPR = \frac{p_{0,ex}}{p_{01}} = \left(\frac{T_{0,ex}}{T_{01}}\right)^{\frac{\gamma\eta_p}{\gamma-1}} = 1.642^{(1.4\times 0.92/0.4)} = \underline{4.94}$$

The isentropic efficiency is then:

$$\eta_{is} = \frac{(p_{0,ex}/p_{01})^{(\gamma-1)/\gamma} - 1}{T_{0,ex}/T_{01} - 1} = \frac{4.939^{0.4/1.4} - 1}{1.642 - 1} = \underline{0.90}$$

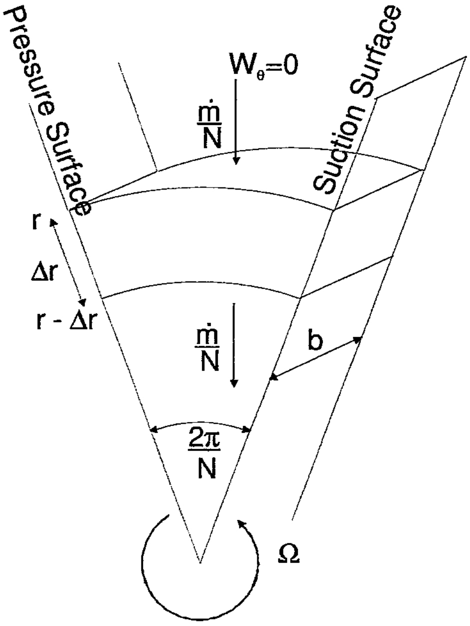
[15%]

Question 1: Multi-stage axial copressor design

This was the most popular question, despite it being difficult. For part (a) candidates generally had a clear understanding of what constitutes a repeating stage, but many struggled to articulate their use in multistage compressor design. Part (b) involved calculating flow angles and constructing velocity triangles given typical compressor stage design parameters.

Most candidates could calculate the flow angles, but the quality of many velocity triangles was surprisingly poor, especially given that their importance is stressed in 4A3 lectures and they are also covered in 3A3. Errors in the velocity triangles led to subsequent sign errors. Part (c) was a challenging problem involving compressible flow calculations and application of continuity in the relative frame. Many candidates slipped up in calculating the relative Mach numbers or accounting for a change in mass flow per unit frontal area. Part (d) also caused a surprising number of problems. It was intended as an easy finish to the question, simply applying definitions of efficiency, but many candidates thought something more complicated was needed. Several wrongly assumed a repeating stage implied the stages have equal pressure ratios, rather than equal stage loadings.

2. a)
Consider the control volume of radial thickness Δr shown below:



Let difference in pressure between suction and pressure surfaces of the N blades be: $\Delta p_\theta = p_{suction} - p_{pressure}$

The rate of change of angular momentum of the fluid <u>in the stationary frame</u> equals the torque exerted by blades, i.e.,

$$\Delta p_\theta (\Delta rb) r = \frac{\dot{m}}{N}\left(\left(rV_\theta - \frac{d(rV_\theta)}{dr}\Delta r\right) - rV_\theta\right)$$

Also, $\dot{m} = \rho 2\pi r b \overline{V_{rel}}$ and $V_\theta = \Omega r$

Substituting these in the above gives: $\Delta p_\theta = -\rho \frac{2\pi}{N}\left(\overline{V_{rel}}\, 2\Omega r\right)$ Eqn. (A)

From the Euler work equation, $h_{0,rel} - \frac{1}{2}U^2 = constant$

Assuming the flow is isentropic (and incompressible), $P_{0,rel} - \frac{1}{2}\rho U^2 = constant$

At a constant radius, the relative stagnation pressure is constant,

$$P_{0,rel} = p + \frac{1}{2}\rho V_{rel}^2 = constant \quad \Rightarrow \quad \Delta p_\theta = -\rho \overline{V_{rel}} \Delta V_{rel}$$

This can be combined with Eqn. (A) to give:

$$\left(V_{rel,suction} - V_{rel,pressure}\right) = \Delta V_{rel} = \underline{\frac{4\Omega r \pi}{N}}$$

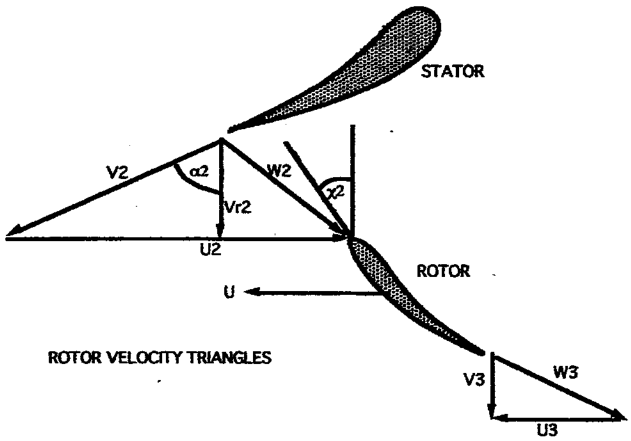
[35%]

b) Slip is the difference between the actual tangential velocity at the rotor outer radius and the ideal tangential velocity if the flow followed the blades perfectly. Slip occurs because the velocity difference ΔV_{rel} reduces from the above value at a radius *r* to zero at the outer radius of the rotor. The blade load (i.e. the pressure difference) must also drop to zero over this distance.

As the pressure force needed to provide the Coriolis acceleration is not present, the angular momentum of the flow (and thus tangential velocity) is reduced at the rotor outer radius from that needed to follow the blade direction.

[15%]

c) Radial turbine rotor velocity triangles unwrapped are shown below:



(i) In this case the blades are radial at rotor inlet, $\chi_2=0$.

Therefore, $V_{\theta 2,ideal}=U_2$

The actual tangential velocity, $V_{\theta 2}=\sigma U_2 \; = 0.9\times 260 = 234\, m/s$

The radial velocity here, $V_{r2}=V_{\theta 2}/\tan\alpha_2 \; = 234/\tan 75^o = 62.7\, m/s$

The relative inlet angle,

$$\beta_2 = \tan^{-1}(W_{\theta 2}/V_{r2}) = \tan^{-1}\left(\frac{V_{\theta 2}-U_2}{V_{r2}}\right) = \tan^{-1}\left(\frac{234-260}{62.7}\right) = \underline{-22.5^o}$$

The exit relative velocity,

$$W_3=2\times W_2 = 2\times\sqrt{W_{\theta 2}^2+V_{r2}^2} = 2\times\sqrt{26^2+62.7^2} = 135.75\, m/s$$

The exit rotor speed, $U_3 \; = 0.4\times 260 = 104\, m/s$

Thus the relative exit angle,

$$\beta_3 = -\sin^{-1}(U_3/W_3) = -\sin^{-1}(104/135.75) = \underline{-50.0^o}$$

[20%]

(ii)

$$V_3 = \sqrt{W_3^2 - U_3^2} = \sqrt{135.75^2 - 104^2} = 87.24\, m/s$$

$$\frac{Exit\ KE}{\Delta h_0} = \frac{0.5V_3^2}{\Delta h_0} = \frac{0.5V_3^2}{\sigma U_2^2} = \frac{0.5\times 87.24^2}{0.9\times 260^2} = \underline{0.0626}$$

[10%]

(iii)
The reaction,

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{h_2 - h_3}{h_1 - h_3} = \frac{h_{02} - 0.5V_2^2 - (h_{03} - 0.5V_3^2)}{h_{01} - (h_{03} - 0.5V_3^2)} = \frac{\sigma U_2^2 - 0.5V_2^2 + 0.5V_3^2}{\sigma U_2^2 + 0.5V_3^2}$$

$$\Lambda = \frac{0.9 \times 260^2 - 0.5 \times \left(234 / \sin 75^o\right)^2 + 0.5 \times 87.24^2}{0.9 \times 260^2 + 0.5 \times 87.24^2} = \underline{0.546}$$

[10%]

The efficiency of most radial inflow turbines is not as high as that of an axial flow turbine because:

i. they work at a lower specific speed (larger ratio of wetted area to flow area)

ii. they have complex 3-D flow generated by the radial to axial bend within the rotor.

iii. The tip clearance gaps represent a larger fraction of blade span

[10%]

Question 2: Radial turbine analysis with slip

This was the least popular question. Radial turbomachinery continues to be off-putting to candidates and not as well understood. Part (a) was a standard proof showing the change in relative velocities in radial turbomachinery passages is related to the Coriolis acceleration. Most candidates who attempted this completed it well. Part (b) was a related discussion of "slip", which was fairly well answered. Part (c) involved constructing velocity triangles for a radial inflow turbine with slip. Few made a good attempt at this, which was disappointing since there was nothing particularly tricky in the question and once the velocity triangles were understood, the calculations needed were straightforward. Part (d) was a discussion of losses in radial machines and this, in general, was well answered. It may be worth putting more focus in a lecture (or examples class) on the construction of radial and axial velocity triangles based on a limited, but sufficient, amount of information.

3. a) Turbine (LP) + nozzle = choked system

$$\frac{\dot{m}\sqrt{c_p T_{0IN}}}{P_{0IN}A_{IN}} = \Pi_m = constant_1 = 1.347 \qquad \text{(I)}$$

$$\frac{T_{05}}{T_{045}} = \left(\frac{P_{05}}{P_{045}}\right)^{\eta_p \frac{\gamma-1}{\gamma}} \qquad \text{(II)}$$

$\Pi_{m45} = \Pi_{m9} = 1.347$ (choked) (III)

Equating $\Pi_{m45} = \Pi_{m9}$ using Eqn (I) with $p_{09}=p_{05}$, $T_{09}=T_{05}$ (negligible losses & heat transfer)

$$\frac{\sqrt{T_{05}}}{P_{05}A_9} = \frac{\sqrt{T_{045}}}{P_{045}A_{45}} \qquad \text{(IV)}$$

Eqns (II) and (IV) very directly yield

$$\frac{P_{05}}{P_{045}} = \left(\frac{A_{45}}{A_9}\right)^{1.131} \qquad \text{(V)} \qquad \frac{T_{05}}{T_{045}} = \left(\frac{A_{45}}{A_9}\right)^{0.263} \qquad \text{(VI)} \quad \textbf{[20 \%]}$$

b) p_a = 11 kPa, T_a = 216, M = 2 – assume nozzle flow isentropic and fully expanded

$$T_{02} = T_a\left[1+\frac{(\gamma-1)}{2}M^2\right] = 390 \text{ K}, \quad p_{02} = p_a\left[1+\frac{(\gamma-1)}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} = 11 \times 1.8^{3.5} = 86 \text{ kPa}$$

Overall Pr = 12.5 and equal split between HP and LP. Therefore,

$$\frac{p_{023}}{p_{02}} = \frac{p_{03}}{p_{023}} = \sqrt{12.5} = 3.53$$ and .. $p_{023} = 3.53\, p_{02} = 3.53(86) = 304$ kPa

$$\frac{T_{023}}{T_{02}} = \left(\frac{p_{023}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma\eta_p}}, \quad T_{023} = T_{02}\,(3.53)^{1/(3.5)0.93} = 390\,(3.53)^{1/(3.5)0.93} = 575 \text{ K}$$

<u>Work balances</u>

HP $c_{pe}\Delta T_{0,HP} = c_p(T_{03}-T_{023})$, $\Delta T_{0,HP} = \frac{c_p}{c_{pe}}(T_{03}-T_{023}) = \frac{c_p}{c_{pe}}(847-575) = 238$ K

LP $c_{pe}\Delta T_{0,LP} = c_p(T_{023}-T_{02})$, $\Delta T_{0,LP} = \frac{c_p}{c_{pe}}(T_{023}-T_{02}) = \frac{c_p}{c_{pe}}(575-390) = 162$ K

The turbine entry temperature T_{04} = 1700 K and so $T_{045} = T_{04} - \Delta T_{0,HP}$ = 1700 – 238 = 1461 K, $T_{05} = T_{045} - \Delta T_{0,LP}$ = 1461 – 162 = 1300 K

For turbine

$$\frac{P_{045}}{P_{04}}=\left(\frac{T_{045}}{T_{04}}\right)^{\frac{\gamma}{(\gamma-1)\eta_p}}=\left(\frac{1462}{1700}\right)^{\frac{1.33}{0.33(0.93)}}=0.52,\quad \frac{P_{05}}{P_{045}}=\left(\frac{T_{05}}{T_{045}}\right)^{\frac{\gamma}{(\gamma-1)\eta_p}}=\left(\frac{1300}{1462}\right)^{\frac{1.33}{0.33(0.93)}}=0.6$$

Assume no pressure loss in the combustor so $p_{04} = p_{03}$

$$\frac{p_{05}}{p_a}=\frac{p_{03}}{p_{02}}\frac{p_{02}}{p_a}\frac{p_{05}}{p_{045}}\frac{p_{045}}{p_{04}}=31$$

$$V_j=\sqrt{2c_{pe}(T_{05}-T_a)}=\sqrt{2c_{pe}T_{05}(1-T_a/T_{05})}=\sqrt{2c_{pe}T_{05}\left(1-(p_a/p_{05})^{\gamma-1/\gamma}\right)}=$$

$$\sqrt{2(1149)1300\left(1-\left(1/31\right)^{0.33/1.33}\right)}=\mathbf{1311\ m/s}$$ **[30 %]**

c) As shown in Part (a), the turbine temperature and pressure ratios are fixed by geometry. From Part (a) $p_{05}/p_{045} = 0.6$. This gives from Eqn. (V) $A_{45}/A_9 = 0.64$. Increasing A_9 by 10% gives $A_{45}/A_9 = 0.529/1.1 = 0.58$. From Eqn. (V)

$$\frac{P_{05}}{P_{045}}=(0.58)^{1.131}=0.54 \text{ and from Eqn. (VI) } \frac{T_{05}}{T_{045}}=\left(\frac{A_{45}}{A_9}\right)^{0.263}=(0.58)^{0.263}=0.87$$

For the LP turbine the temperature drop $T_{045} - T_{05} = T_{045}(1 - T_{05}/T_{045}) = 194$ K

LP energy balance $c_p\,\Delta T_{LPC} = c_{pe}\,(T_{045} - T_{03}) \rightarrow c_{pe}(T_{045} - T_{05})/c_p = 222$ K

$T_{023} = T_{02} + \Delta T_{LPC} = 390 + 222 = 612$ K.

$$\frac{p_{023}}{p_{02}}=\left(\frac{T_{02}+\Delta T_{LPC}}{T_{02}}\right)^{\gamma\eta_p/\gamma-1}=\left(1+\frac{222}{T_{02}}\right)^{\gamma\eta_p/\gamma-1}=4.32$$

HP turbine work unchanged from datum build values hence $W_{HPC} = c_p\,(T_{03} - T_{023}) = c_p(847 - 575)$ – datum build values - c_p (272)

$$\frac{p_{03}}{p_{023}}=\left(\frac{T_{023}+\Delta T_{HPC}}{T_{023}}\right)^{\gamma\eta_p/\gamma-1}=\left(1+\frac{272}{612}\right)^{\gamma\eta_p/\gamma-1}=3.317$$

$$\frac{p_{05}}{p_a}=\frac{p_{05}}{p_{045}}\frac{p_{045}}{p_{04}}\frac{p_{023}}{p_{02}}\frac{p_{03}}{p_{023}}\frac{p_{02}}{p_a}=31.78$$ (p_{045}/p_{04} datum build value, $T_{05} = (T_{05}/T_{045})T_{045}$)

$$V_j=\sqrt{2c_{pe}T_{05}\left(1-(p_a/p_{05})^{\gamma-1/\gamma}\right)}=1298.5\ \text{m/s}$$

$$\eta_{prop}=\frac{2V}{V+V_j}=\frac{2M\sqrt{\gamma RT_a}}{M\sqrt{\gamma RT_a}+V_j}=\frac{(2)2.0\sqrt{(1.4)(287.3)(216)}}{2.0\sqrt{(1.4)(287.3)(216)}+1311}=0.620$$

For 10% larger nozzle

$$\eta_{prop} = \frac{2V}{V+V_j} = \frac{(2)2.0\sqrt{(1.4)(287.3)(216)}}{2.0\sqrt{(1.4)(287.3)(216)}+1298.5} = 0.624$$ i.e. about a 0.4% η_{prop} increase relative to the datum build.

[30 %]

d) Higher η_{prop} (as shown above), lower take off noise, moving working line away from the surge line. **[10 %]**

e) The blade design could be poor and prone to separation hence a redesign could be needed. The end wall flow might also be a zone of high separation and hence the design in this area could be improved. The blades could be too highly loaded and hence more stages or greater solidity could be required. However, this would all add weight and extra cost. The higher pressure ratio would need for a greater need for bleed valves (computer controlled), variable stators and re-staggering of the blades. These things would prevent stall at part speed and potential surge. Variable stators present sealing problems and are mechanically complex. Bleed ports are inefficient, mechanically unreliable and noisy. **[10 %]**

Question 3: Two-spool turbojet with variable area nozzle

Those who answered this seemed to know the topic well, but it was a complex question that was perhaps slightly too long. Part (a) was a variation on a fairly standard proof for choked gas turbine operation. This was generally well done and was useful for the subsequent parts of the question. Part (b) was a cycle calculation at the design condition, applying polytropic efficiencies and a shaft work balance to find the jet velocity. Many candidates correctly spotted how to simplify this to consider the two-shaft engine as a thermodynamically equivalent single- shaft. Part (c), however, was very tough and no candidates got this completely right. It involved computing the off-design operation of the two-shaft turbojet due to a variable area exhaust. This required recalculating the operating point of the low-pressure shaft to find a new exhaust jet velocity. Most candidates couldn't see how to proceed, but some made a simplifying assumption to find the total pressure in the exhaust nozzle which led to a result that wasn't far wrong. Part (d) asked for benefits of a variable exhaust and generated some good answers. Many candidates were not too clear what was expected in Part (e) on increasing the compressor pressure ratio, although some described good approaches that could be used to modify the design.

IIB Paper 4A3 2011 **Turbomachinery I** **Numerical answers**

1.
(b) -60.0°, -43.0°, 0.666
(c) 0.640, 0.428, 0.048 (~5% loss)
(d) 4.94, 0.90

2.
(c) (i) -22.5°, -50.0°
(ii) 6.3%
(iii) 0.546

3.
(b) 1310 m/s
(c) 1300 m/s, 0.4% increase