DR LXU

Engineering Tripos Part 2B 4A11 Turbomachinery II, 2011 Girbs

1.(a) [20%] Meridional Streamline Curvature Equation:

$$V_m \sin\phi \frac{\partial V_m}{\partial m} + \frac{V_m^2}{R_m}\cos\phi - \frac{V_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{F_r}{\rho}$$

Where $V_m \sin\varphi \frac{\partial V_m}{\partial m}$: the radial component of the meridional acceleration

$\frac{V_m^2}{R_m}\cos\varphi$: the radial component of the centripetal acceleration due to the streamline curvature

$\frac{V_\theta^2}{r}$: the centripetal acceleration due to tangential velocity

$\frac{1}{\rho}\frac{\partial p}{\partial r}$: the radial pressure gradient

$\frac{F_r}{\rho}$: the body force term modeling the blade force in radial direction

The equation is derived from Euler equations assuming axisymmetric and steady flow. As it is based on Euler Equations the flow is assumed to be inviscid but the effects of viscosity can be included in the system in terms of entropy generation at the calculation stations as a source term, so can the work input/output. The axisymmetric flow assumption is a good one as it is compatible to the steady flow assumption, and helps to reduce the equation to a manageable form: by assuming flow being steady and axisymmetric, there is no need to solve the equation of moment of momentum. Only all important momentum equation in the radial direction which controls the radial equilibrium needs to ne solved. If compared to Vx, Vr is small, or, the flow accelerations in meridional direction is small, then the radial component of the meridional acceleration term $V_m \sin\varphi \frac{\partial V_m}{\partial m}$ can be neglected. Further, if the meridional flow is straight and parallel, the radius of the streamline curvature is large, the radial component of the centripetal acceleration due to the streamline curvature term $\frac{V_m^2}{R_m}\cos\varphi$ is small and can be neglected. Finally, them the blades are stacked radially or the computational stations are located outside of the blade rows the body force term modeling the blade force in radial direction $\frac{F_r}{\rho}$ can be neglected. Thus the equation only has two term left: the radial pressure gradient term and the centripetal acceleration due to tangential velocity: $\frac{V_\theta^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{1}{\rho}\frac{dp}{dr}$.

Since $dh - Tds = \frac{1}{\rho}dp$, $\frac{\partial h}{\partial r} - T\frac{\partial s}{\partial r} = \frac{1}{\rho}\frac{\partial p}{\partial r}$; $h_o = h + \frac{1}{2}(V_m^2 + V_\theta^2)$

$$\frac{\partial h}{\partial r} = \frac{\partial h_o}{\partial r} - V_m \frac{\partial V_m}{\partial r} - V_\theta \frac{\partial V_\theta}{\partial r} \Rightarrow \frac{\partial h_o}{\partial r} - T\frac{\partial s}{\partial r} = V_m \frac{\partial V_m}{\partial r} - V_\theta \frac{\partial V_\theta}{\partial r} + \frac{V_\theta^2}{r} = V_m \frac{\partial V_m}{\partial r} + \frac{V_\theta}{r}\frac{\partial (rV_\theta)}{\partial r}$$

As required.

(b) (i) [40%] $r_h = 0.3m;\ \ r_t = 0.5m;\ \ V_{x1} = 50.0m/s;\ \ \alpha_2 = 60.0°;\ \ \rho = 1.2kg/m^3$

Simple radial equilibrium: uniform inlet: $\frac{\partial h_o}{\partial r} = 0;\ \frac{\partial s}{\partial r} = 0$ at inlet. Along blade span $\frac{\partial s}{\partial r} = 0$ in the axial gap, so downstream of the stator in the axial gap:

$$V_m \frac{\partial V_m}{\partial r} + \frac{V_\theta}{r}\frac{\partial r V_\theta}{\partial r} = 0. \quad \because\ V_\theta = V_m tg\alpha_2 = \sqrt{3}V_m;\ \because\ V_m \frac{\partial V_m}{\partial r} + \frac{\sqrt{3}V_m}{r}\frac{\partial r\sqrt{3}V_m}{\partial r} = 0$$

$$V_m \frac{\partial V_m}{\partial r} + 3\frac{V_m}{r} + 3V_m \frac{\partial V_m}{\partial r} = 0 \ \Rightarrow\ 3\frac{V_m^2}{r} + 4V_m \frac{\partial V_m}{\partial r} = 0;\ \Rightarrow\ -\frac{3}{4}\frac{dr}{r} = \frac{dV_m}{V_m}$$

Integrate: $\ln V_m = -\frac{3}{4}\ln r + C;\ \ V_m = Ar^{-3/4};\ \ V_{mc}/V_{mh} = (1/0.6)^{-3/4} = 0.6^{3/4} = 0.682$

Use continuity to find constant A:

At inlet: $\dot m = \rho \int_{0.3}^{0.5} 2\pi r 50 dr$; at the axial gap: $\dot m = \rho \int_{0.3}^{0.5} 2\pi r A r^{-3/4} dr$

$$50\cdot(0.25-0.09) = \int_{0.3}^{0.5} 2Ar^{1/4}dr; \qquad A = \frac{50\cdot 0.16}{2\cdot 4/5\cdot r\big|_{0.3}^{0.5}} = \frac{50\cdot 0.16}{8/5\cdot 0.198} = 28.25$$

$$\therefore\ V_m = 25.25r^{-3/4}; \qquad V_{mc} = 42.46m/s \qquad V_{mh} = 62.29m/s \quad V_h = 2\cdot V_{mh} = 124.58m/s$$

$$V_c = 2\cdot V_{mc} = 84.92/m/s \qquad p_c - p_h = \frac{1}{2}\rho(V_h^2 - V_c^2) = \frac{1}{2}\cdot 1.2\cdot(124.58^2 - 84.92^2) = 4985pa$$

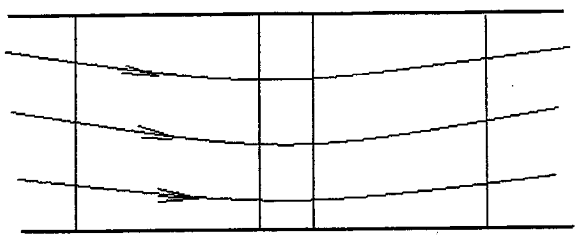
(ii) [20%] $V_{mid2} = 2.0\cdot 25.25\cdot 0.4^{-3/4} = 100.4m/s$

$$\Lambda = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}};\ \ \Lambda_{hub} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}}\bigg|_{hub} = 1 - \frac{124.58^2 - 50^2}{2\cdot(100.4^2 - 50^2)} = 1 - \frac{13020}{15160} = 0.14 = 14\%$$

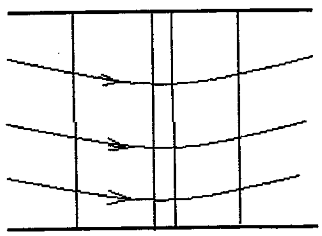
$$\Lambda_{casing} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}}\bigg|_{casing} = 1 - \frac{84.92^2 - 50^2}{2\cdot(100.4^2 - 50^2)} = 1 - \frac{4711}{15160} = 0.69 = 69\%$$

(iii) [10%] The variation of V_m along the span compared to the mid span value is not extreme (~20%), due to the use of constant angle vortex distribution. So there is small streamline curvature effect which is neglected in the simple radial equilibrium. In a sense, the SRE will underpredict the spanwise flow variation as the treamline curvature term has the same sign of that of the centripetal acceleration due to V_θ^2 term. Also the hub reaction becaomes very low, thus the loss is likely to increase which leads to the assumption of $\frac{\partial s}{\partial r} = 0$ less likely good near the hub section.

(c) [10%] Refer to the sketch below. Build (a) with lower aspect ratio will be closer to the simple radial equilibrium than build (b) as the streamline curvature will be smaller thus its effect weaker.



(a)



(b)

Examiner's comment

Most candidates could deal with the radial equilibrium equation well and understood the effects of streamline curvature term on the radial equilibrium. In general the conceptual parts of the question were well answered. However the handling of algebra was less satisfactory that many tried to integrate the pressure variation by using the mean value theorem, instead of working it out through difference in specific kinetic energy.

2. (a) [30%] Loss of efficiency in compressor:

$$\Delta\eta = \frac{T\Delta s}{h_2 - h_1} \quad \Rightarrow \quad h_2 - h_1 \approx \frac{p_2 - p_1}{\bar{\rho}} \approx \frac{p_2 - p_1}{\rho_1} \text{ for small pressure change}$$

$$T\Delta s \approx \frac{\gamma+1}{12\gamma^2} TR\left(\frac{\Delta p}{p_1}\right)^3 \quad \Rightarrow \quad \Delta\eta \cong \frac{\gamma+1}{12\gamma^2} RT\rho\left(\frac{\Delta p}{p_1}\right)^3 \frac{1}{\Delta p} \approx \frac{\gamma+1}{12\gamma^2}\left(\frac{\Delta p}{p_1}\right)^2 \quad \text{as required.}$$

This is only valid for very weak shockwaves where Δp across the shock is very small. For finite strength the shockwave the expression is likely to over estimate the loss of efficiency.

(b)(i) [30%] For M=1.3, π_{rotor}=2.125, 0.85 of $\pi_{rotor} \Rightarrow \pi_{shock} = 1.806$

$$\Delta\eta_s = \frac{\gamma+1}{12\gamma^2}(\pi_s - 1)^2 = 0.0663; \quad \Delta\eta_r = \Delta\eta_s \cdot 1.5 = 0.0994$$

$$\underline{\eta = 1 - \Delta\eta = 0.9006 = 90.06\%}$$

This shows a reasonable value of the rotor efficiency. However as the pressure difference across the shock is substantial, the expression obtained in (a) is likely to be inaccurate, it overestimates the shock loss, thus an overestimate of the diffusion loss downstream of the shockwave is also expected as the consequence.

(ii) [20%] The opposite trend of the test result against the estimates and expectations in (b)(i) are likely due to several factors: a). the existence of tip clearance loss which could be substantial in transonic flow where the leakage flow is driven by high pressure difference due to the shock wave; b). the blockage due to the shock/leakage flow/endwall flow interactions reduces the inlet mass flow rate thus leads to higher tip incidence, resulting in stronger shockwave and higher loss. Tp have a very tight control of the tip clearance and reduce the forepart of the blade loading should be able to improve the flow and reduce the loss of efficiency.

(c) [20%] The clearance leakage flow in an unshrouded rotor is driven by chordwise pressure difference so very much depends on the chordwise loading distribution, while as for the shrouded rotor the leakage flow is driven by the overall pressure rise across the rotor thus a function of the overall rotor loading. Although for the shrouded rotor it is possible to have more seals to control the leakage mass flow rate the main concern in transonic rotors, on one hand is the blade stress at the hub is likely to be the limiting factor for the structural integrity, a shroud will put too much strains onto it; on the other hand, the so called offset loss for the shrouded transonic rotor is likely to be higher due to high windage loss. It is likely, unless new engineering materials/processing are used, the shrouded transonic compressor rotor will remain impractical. While as for an unshrouded rotor if the tip clearance can be controlled the damage due to the leakage flow can be limited.

Examiner's comment:

Most straight forward question and answered well by candidates who attempted it. The derivation and numerics were handled well. The difference between the shrouded and unshrouded rotor tips was well understood. The weaknesses in candidates were not able to assess and to appreciate the errors involved in the assumptions while deriving the expression for the loss of efficiency.

3 (a) [25%]

Subscripts 1 = stator inlet; 2 = stator exit; 3 = rotor exit.

Repeating stage:

$$\psi = 2(1 - R - \phi \tan\alpha_1)$$

$$2 = 2(1 - 0.5 - 0.5\tan\alpha_1)$$

$$\alpha_1 = \alpha_3 = -45°$$

Euler (no radius change):

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 3})$$

$$\frac{\Delta h_0}{U^2} = \frac{V_x}{U}(\tan\alpha_2 - \tan\alpha_3)$$

$$2 = 0.5(\tan\alpha_2 + 1)$$

$$\alpha_2 = 71.6°$$

Relative angles:

$$V_\theta^{rel} = V_\theta - U$$

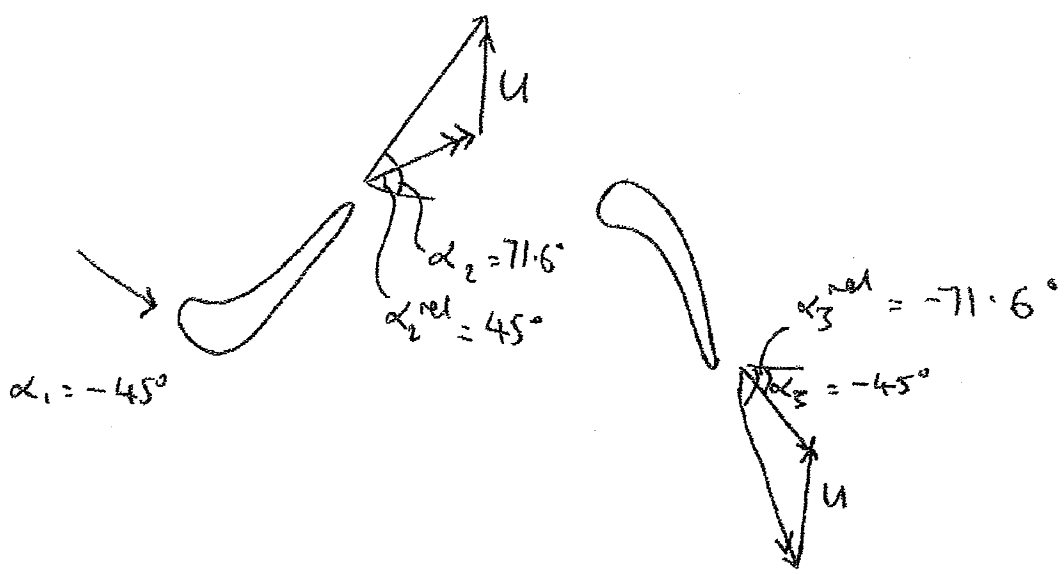
$$\tan\alpha^{rel} = \tan\alpha - \frac{1}{\phi}$$

so,

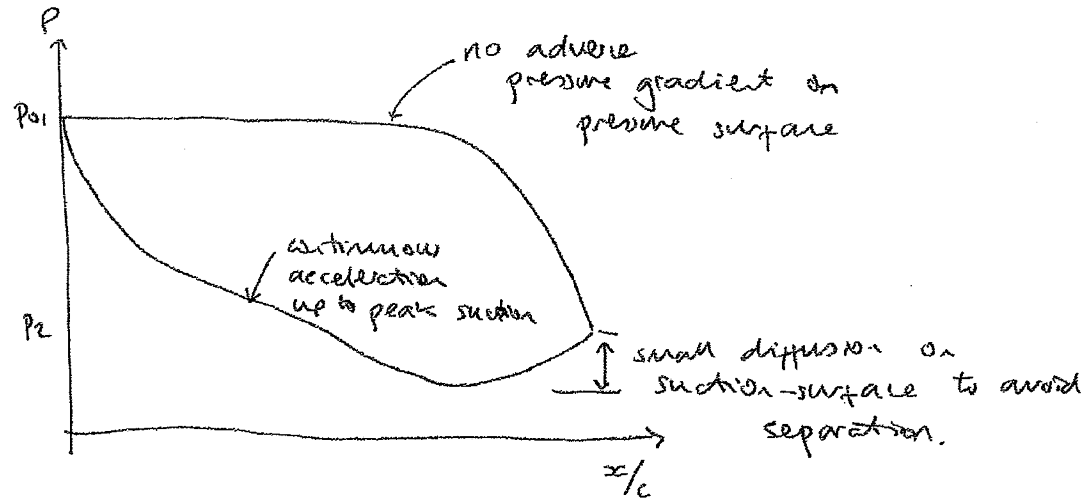
$$\alpha_2^{rel} = 45°$$

$$\alpha_3^{rel} = -71.6°$$

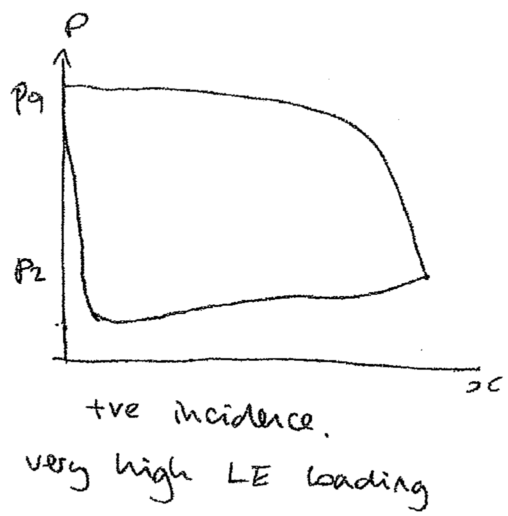
So the velocity triangles are symmetric, as expected for a 50% reaction machine.

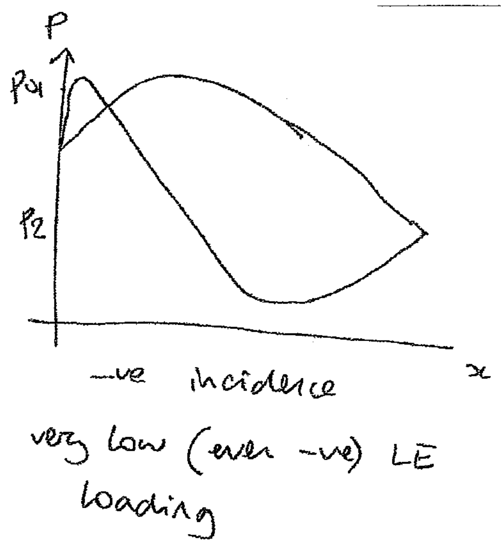


3(b) [15%] A good pressure distribution will minimise areas of deceleration ("diffusion"). So there should be a continuously falling pressure on the PS (acceleration from LE to TE), and a falling pressure for most of the SS (until 75% chord, say) and then a small region of increasing pressure from peak suction to the TE. Too much deceleration here will result in a separation on the rear SS.



3(c)(i) [10%] At postive incidence, a lot of turning is done right at the LE, resulting in high loading here. At negative incidence, reduced turning is done, hence reduced or even negative loading.





3(c)(ii) [10%] Skew is caused by a change of reference frame. The boundary layer from the upstream rotor endwall can be assumed to be at the same relative flow angle as the mainstream flow, but with a smaller velocity magnitude:

$$V_{\theta 1,BL}^{rel} + U = V_{\theta 1,BL}$$

$$V_{x,BL} \tan \alpha_1^{rel} + U = V_{x,BL} \tan \alpha_{1,BL}$$

$$\frac{V_{x,BL}}{V_x} \tan \alpha_1^{rel} + \frac{1}{\phi} = \frac{V_{x,BL}}{V_x} \tan \alpha_{1,BL}$$

$$0.5 \tan(-71.6°) + 2 = 0.5 \tan \alpha_{1,BL}$$

$$\alpha_{1,BL} = 45°$$

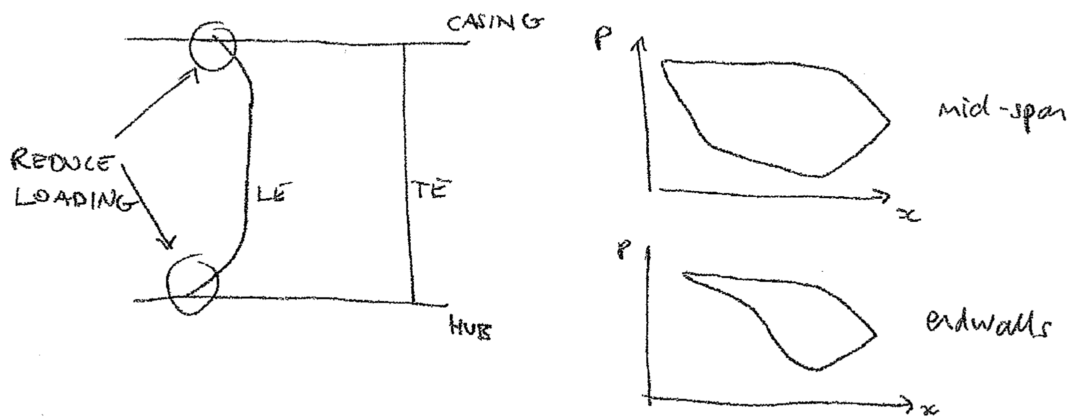
i.e. 90 degrees of negative incidence (angled toward the SS of the stator).

3(c)(iii) [10%] Assume that the flow is not turned as it flows over the shroud (in reality, it will turn a little due to the skin friction on the surface of the shroud). Then the stator inlet flow angle will be the same as the absolute rotor inlet flow angle:

$$\alpha_{1,Shroud\_leak} = \alpha_2 = 71.6°$$

i.e. 116.6 degrees of negative incidence.

3(c)(iv) [10%] The sensitive of the blade to incidence can be reduced by reducing the leading edge loading. The leading edge loading at the endwalls can be reduced by extended the LE forward at the endwalls. Since the pressure gradient perpendicular to the endwalls is very small (compared to the cross-passage pressure gradient), the loading at the LE at the endwalls will be reduced because there is no blade inboard (toward mid-span) of the endwall sections.



3(d) [20%] No blade geometry is required to run a throughflow code, only the exit angles. Also, a throughflow code can be run at conditions far away from the design operating point where a NS code may not converge properly.
A throughflow method will not predict the endwall flows described above, so models would have to be used. The correlation would have to specify what the average boundary layer skew or shroud leakage flow angle was, and then impose this on the flow as a deviation from the intended flow angle.

Examiner's comments:

Similar to Q1, the conceptual parts of the question have been answered well, apart from the numerical method part where the candidates seem to be rather biased towards in favour of the streamline curvature method and very few commented on the N-S Solver. The concept of velocity triangle is soundly established and only a couple of candidates got that wrong. Again the numerical parts were less well handled, for example some had wrong flow angles estimated which affected the later calculations.