Engineering Tripos Part IIB: Module 4C2
Designing with Composites
CRIB - 2010/11

Michael Sutcliffe

1. (a) Should mention tensile-shear interactions – tensile strains from applied shear stresses & vice versa lead to distortions. These can be minimised if the stacking sequence is chosen carefully, since the distortions from individual laminae will largely cancel each other out. A *balanced* laminate is one in which the laminate as a whole exhibits no tensile-shear interactions for any loading angle. Also, mention through-thickness stresses, which can cause out-of-plane distortions. These can be minimised by choice of stacking sequence. In general, a symmetric laminate (ie one possessing a mirror plane lying in the plane of the laminae) will exhibit low distortion from through-thickness stresses, because the various interactions will tend to cancel out.

(b)

From Datasheet: $\overline{S}_{16} = (2\,S_{11} - 2S_{12} - S_{66})c^3 s - (2\,S_{22} - 2\,S_{12} - S_{66})c\,s^3$

(i) For $\left|\overline{S}_{16}\right|$ to be zero (no tensile-shear interaction):

$A = 2\,S_{11} - 2S_{12} - S_{66}$ and $B = 2\,S_{22} - 2\,S_{12} - S_{66}$

$$S_{11} = \frac{1}{E_1} = \frac{1}{30}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{6}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.25}{30}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{2}$$

$\Rightarrow A = -0.42$ GPa^{-1}

$B = -0.15$ GPa^{-1}

Now $\overline{S}_{16} = Ac^3 s - Bcs^3 = cs(Ac^2 - Bs^2)$

$\Rightarrow \overline{S}_{16} = 0$ when

(1) $\cos\theta = 0 \Rightarrow \theta = 90°$

(2) $\sin\theta = 0 \Rightarrow \theta = 0°$

(3) $Ac^2 = Bs^2 \Rightarrow \theta = \tan^{-1}\left(\dfrac{A}{B}\right)^{1/2}$

$\therefore \theta = 59°$

(ii)

$\overline{S}_{16} = Ac^3 s - Bcs^3$

$\dfrac{\partial \overline{S}_{16}}{\partial \theta}$ =A(-3c$^2s^2 + c^4) - B(-s^4 + 3c^2s^2)$

$= Ac^4 + Bs^4 - 3Ac^2s^2 - 3Bc^2s^2$

$= A(1-s^2)^2 + Bs^4 - (3A+3B)s^2(1-s^2)$

$= s^4(A+B+3A+3B) + s^2(-2A-3A-3B) + A$

$= s^4(4A+4B) + s^2(-5A-3B) + A$

$\Rightarrow \dfrac{\partial \overline{S}_{16}}{\partial \phi}$ =0 when $\left(4+\dfrac{4B}{A}\right)s^4 - \left(5+\dfrac{3B}{A}\right)s^2 + 1 = 0$

$5.44s^4 - 6s^2 + 1 = 0$

$s^2 = 0.91$ & 0.2

for $0 \le \theta \le \pi/2$, θ □ 73° or 27°

By substituting these values into the equation for $\left|\overline{S}_{16}\right|$

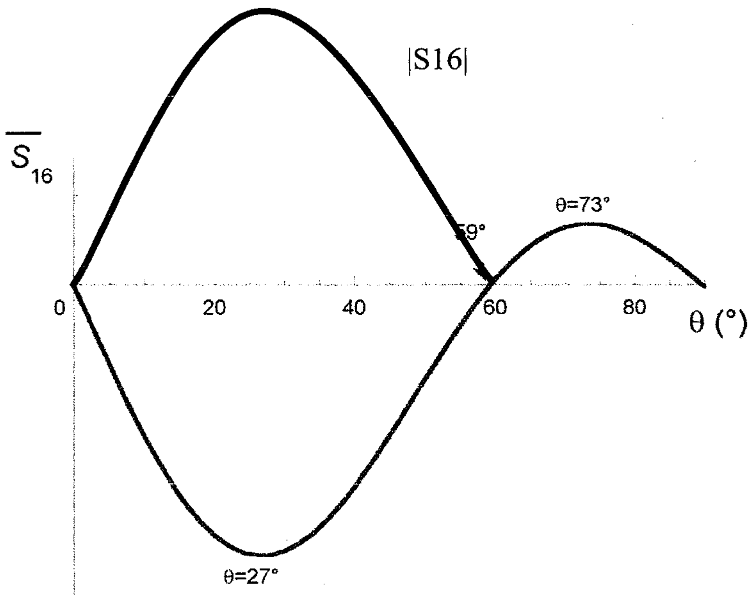
We find that $\overline{S}_{16}$ is maximum at

$\therefore$ (i) $\overline{S}_{16} = 0.028$ GPa$^{-1}(\theta = 73°)$

(ii) $\overline{S}_{16} = -0.12$ GPa$^{-1}(\theta = 27°)$

$\left|\overline{S}_{16}\right|$ is a tensile-shear interaction term, an important feature of off-axis loading of laminates.

As a result of a tensile stress σ_x, a shear strain $\gamma_{xy} = \overline{S}_{16}\sigma_x$ is produced.



A relatively straightforward question applying laminate plate theory. Hence very popular and well answered. Most of the problems arose in the algebra, rather than the composites understanding.

2. **Materials**: performance indicates CFRP with excellent specific stiffness and strength. High value component so cost not so critical.

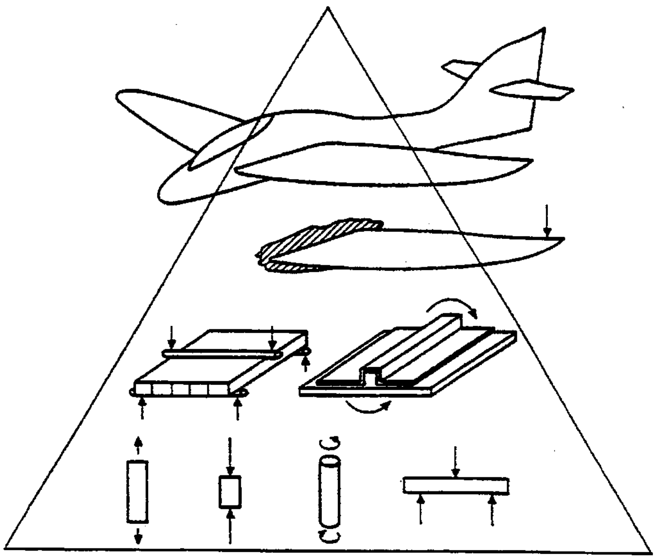
Process: High performance and relatively large number of components indicates autoclave use. Size doesn't preclude this. Hand lay-up or (more likely) tape lay-up of pregpreg components will give the required quality. Perhaps some form of moulding (RTM) for attachment structure.

Structural design. Need to consider loading and constraints. Here the wind pressure loading gives a maximum bending moment at the attachment point. Could manufacture using a plate with decreasing thickness from root to tip. Or perhaps include a network structure (c.f. stiffeners or spars) to carry the main loads, with a thinner skin to spread the load and provide the correct surface.

Laminate lay-up. Loading predominantly in bending away from the hinge, but need some material in other directions to prevent splitting and provide buckling resistance. Also impact resistance will be very important (run-way stones) so perhaps include 45s or a woven protective layer.

Joints: this aspect will be critical, with the hinge joint and lugs to get the actuation load in. Need to consider how to spread the load, perhaps via metal inserts or thicker sections. Where stiffeners are used, should consider how to bond them (adhesives would be preferred).

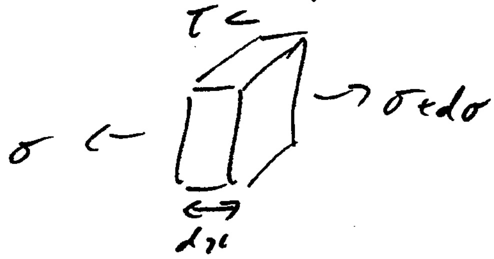
Testing and certification: Again critical. Air worthiness certification requirements force a very expensive testing pyramid as illustrated, from resin to coupon to sub part to full-scale prototype. Late in the design process much of this testing will already have been done, and the delays and cost involved in starting again are prohibitive. Need to include design of attachments, fatigue, compression after impact. Non-destructive testing at the manufacturing stage will be required, for example using ultrasonic c-scanning. Effect of environment (hot-wet, cold) should be included, as well as corrosion with metal inserts/connections.



Quite a few good answers, showing a good depth of knowledge about the issues affecting design and manufacture of composite parts. Marks tended to be lost for a scatter-gun approach to answers, listing information rather than focussing on the particular application.

3(a) Perhaps crimp (waviness) in tows leads to a stress concentration.
Or local tow damage, impact damage, manufacturing flaw.
Or due to inherent variability in tow strength.

(b) Assume slip occurs everywhere



$$d\sigma \cdot b^2 = 4\tau b\, dx$$

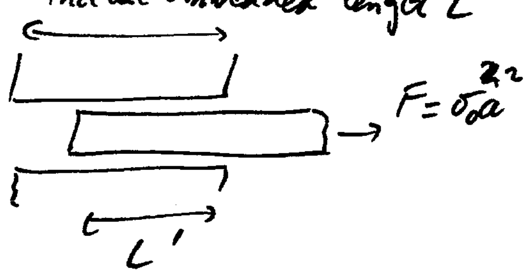
$$\Rightarrow \frac{d\sigma}{dx} = \frac{4\tau}{b}$$

At $x = 0$, $\sigma = 0$ $\Rightarrow$ $\sigma = \frac{4\tau}{b} x$

Remote σ: $\sigma_0 \cdot a^2 = \sigma_{x=L} \cdot b^2$

$$\Rightarrow \sigma_0 = \frac{4\tau}{b} L \cdot \frac{b^2}{a^2} = \frac{4\tau L b}{a^2}$$

(c) Pull out energy / tow



$$\frac{E}{a^2} = -\frac{1}{a^2} \int_L^0 F\, dL'$$

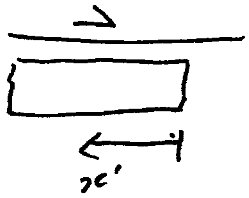
$$= -\frac{1}{a^2} \int_L^0 4\tau L' b\, dL' = \frac{1}{a^2} 4\tau \frac{L^2}{2} b = \frac{2\tau L^2 b}{a^2}$$

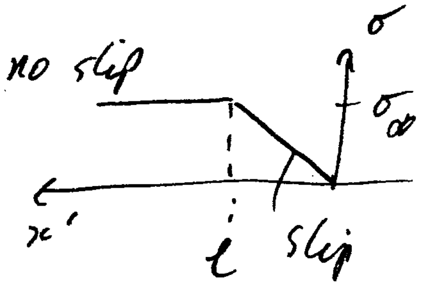
Here $\tau = 50 \times 10^6$ Pa, $L = 20$ mm, $a = 5$ mm, $b = 1$ mm

$$\frac{E}{a^2} = 2 \times 50 \times 10^6 \left(\frac{20}{5}\right)^2 \cdot 1 \times 10^{-3} = 1600 \text{ kJ/m}^2$$

This is large compared with typical values of G_c (~ 30 kJ/m^2) so would be useful.

3(d)





To left of break tow stress rises from zero to remote value. Shear lag theory applies as before. At distance l σ rises to remote value.

σ_t - stress in tow

σ_0 - stress in laminate far from break

In no slip region ε is same in laminate & tow

$$\Rightarrow \frac{\sigma_t}{E_t} = \frac{\sigma_0}{E_L} \Rightarrow \sigma_t = \sigma_0 \frac{E_T}{E_L}$$

Also $\dfrac{d\sigma_t}{dx'} = \dfrac{4\tau}{b}$ as before

$$\Rightarrow \frac{4\tau}{b} l = \sigma_t = \sigma_0 \frac{E_T}{E_L}$$

$$\Rightarrow l = \frac{b^2 L}{a^2} \frac{E_T}{E_L} \quad \text{using } \sigma_0 = \frac{4\tau L b}{a^2}$$

$$\sigma = \frac{4\tau}{b} x' \quad \text{for } x' \leq \frac{b^2 L}{a^2} \frac{E_T}{E_L}$$

$$\sigma = \sigma_0 \frac{E_T}{E_L} \quad \text{for } x' > \frac{b^2 L}{a^2} \frac{E_T}{E_L}$$

This looked like a difficult question on pull-out of tows and shear lag theory, hence the unpopularity. In fact the analysis was quite straightforward and mirrored material learnt in lectures so that those students who attempted it (perhaps the braver ones) tended to do well.

4. (a) From databook, with $c = \frac{1}{\sqrt{2}}$, $s = \frac{1}{\sqrt{2}}$, $c^2 s^2$ etc $= \frac{1}{4}$
[$c = \cos\theta$, $s = \sin\theta$]

$\bar{Q}_{11} = (Q_{11} + Q_{22} + 2Q_{12} + 4Q_{66})/4 = E.\frac{3}{4}$

$\bar{Q}_{12} = \frac{1.4}{4} E$, $\bar{Q}_{22} = \bar{Q}_{11}$ by symmetry, $\bar{Q}_{16} = \bar{Q}_{26} = 0$ as balanced symmetric.

$\bar{Q}_{66} = \frac{1.8}{4} E$

$$\Rightarrow (\bar{Q})_{\pm 45} = E \begin{bmatrix} 0.75 & 0.35 & 0 \\ 0.35 & 0.75 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}$$

(b) The woven plies are each behaving as though an intimate mixture of plies at right angles, so that internally the plies don't behave as ±45, say. Instead the central ply is effectively symmetric, as is the overall arrangement. Balanced as equal numbers of + and − θ plies.

(c) $A = 2t[Q]_{0/90} + t[\bar{Q}]_{\pm 45} = Et \begin{bmatrix} 2.75 & 0.55 & 0 \\ 0.55 & 2.75 & 0 \\ 0 & 0 & 0.85 \end{bmatrix}$

(d) Need σ in 0/90 ply.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = A^{-1} \begin{bmatrix} N \\ 0 \\ 0 \end{bmatrix}$$ – shear is zero by symmetry

$$\Rightarrow \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \frac{1}{Et} \begin{pmatrix} 2.75 & 0.55 \\ 0.55 & 2.75 \end{pmatrix}^{-1} \begin{pmatrix} N \\ 0 \end{pmatrix}$$

4e (d) cont

$$\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = [Q]\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix} = \frac{E}{E\,t}\begin{bmatrix}1 & 0.1\\ 0.1 & 1\end{bmatrix}\begin{bmatrix}2.75 & 0.55\\ 0.55 & 2.75\end{bmatrix}^{-1}\begin{pmatrix}N\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0.1\\ 0.1 & 1\end{pmatrix}\begin{pmatrix}2.75 & -0.5\\ -0.55 & 2.75\end{pmatrix}\begin{pmatrix}N\\ 0\end{pmatrix}\frac{1}{7.26}\frac{1}{t}$$

$$= \begin{pmatrix}0.371\\ -0.0379\end{pmatrix}\frac{N}{t}$$

Tensile failure - look at $\sigma_1 = 0.371\,N/t$

Tsai-Hill $\left(\frac{\sigma_{11}}{s^+}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{s^{+2}} + \frac{\sigma_{22}}{s^{-2}} = 1$

Putting $\sigma^+ = s_L^-$, $s^- = s_T^-$

$$\Rightarrow \frac{N^2}{t^2} = \left[\frac{0.371}{s^{+2}} + \frac{0.371.0.0379}{s^{+2}} + \frac{0.0379^2}{s^{-2}}\right]^{-1}$$

In many years questions about failure can be rather tricky, involving substantial algebraic manipulation, but this year the question proved no match for the students, with many essentially correct answers. Clearly the students had learnt the principles well (in fact a substantial part of this question was about laminate plate theory) and could apply it.