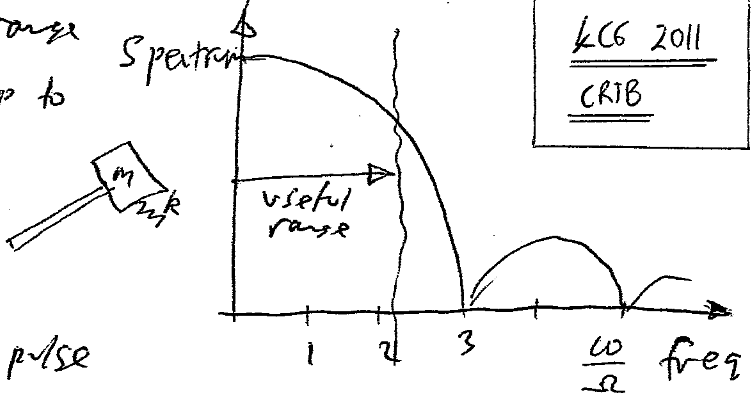
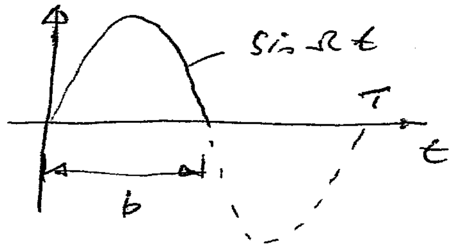
4C6 2011
CRIB

1(a) The form of the spectrum of the impulse is as shown here so the useful range of frequencies excited is up to about $\frac{\omega}{\Omega} = 2$



For a hammer $\Omega = \sqrt{\frac{k}{M}}$

and the duration of the pulse is $\frac{T}{2} = \frac{2\pi}{2\Omega} = \frac{\pi}{\Omega} = b$



So $\omega = 2\Omega = \frac{2\pi}{b}$

is the maximum frequency excited usefully by an impulse of duration b, and in Hz we have $f_{max} = \frac{\omega}{2\pi} = \frac{1}{b}$

[This is a useful rule of thumb that candidates might just quote, and they should be given full marks for remembering it.]

From the modes given, $f_{max} = 175\,Hz$ $\therefore$ $b = \frac{1}{175}\,s = 6\,ms$

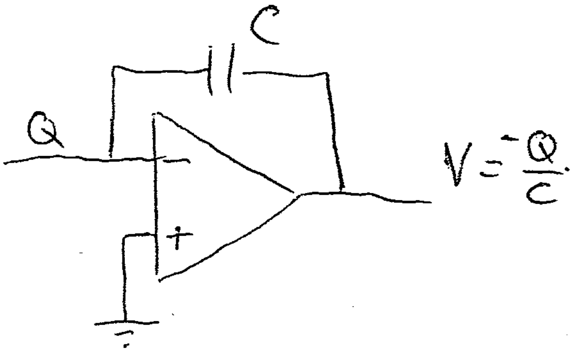
So an impulse of duration 6 ms will suffice.

(b) $\Omega = \frac{\omega}{2} = \frac{2\pi f_{max}}{2} = \pi f_{max} = 550\ rad\,s^{-1} = \sqrt{\frac{k}{M}}$

$\therefore$ $k = M\Omega^2 = 0.005 \times (550)^2 = 1500\ N/m$

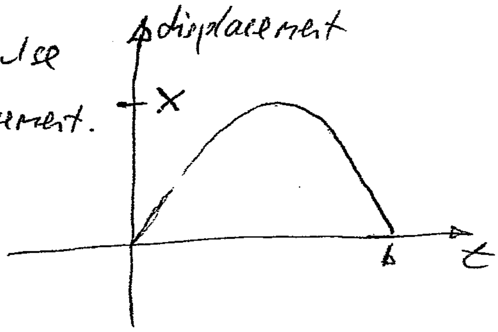
= stiffness of the hammer tip

(c)



The gain of the charge amplifier is $-\frac{1}{C}$ where C is the feedback capacitance

(c) cont. The peak force during the impulse is kX where X is the peak displacement. The approach velocity V is ΩX



because $x = X \sin \Omega t$

$\dot{x} = X\Omega \cos \Omega t$

$\therefore V = X\Omega$ at $t = 0$

So the peak force $F = kX = \frac{kV}{\Omega} = m\Omega V$

$= 0.005 \times 550 \times 1$

$= 2.75\ N$

The force transducer produces $Q = 1\ pC/N \times 2.75\ N$

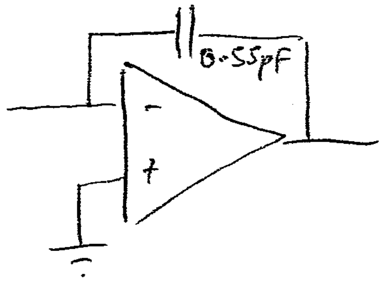
$= 2.75\ pC$

We want a peak ~~velocity~~ output voltage of 5V

so with $V = \frac{Q}{C}$ $\therefore$ $C = \frac{Q}{V} = \frac{2.75}{5}\ pF$

$= 0.55\ pF$

So the circuit is

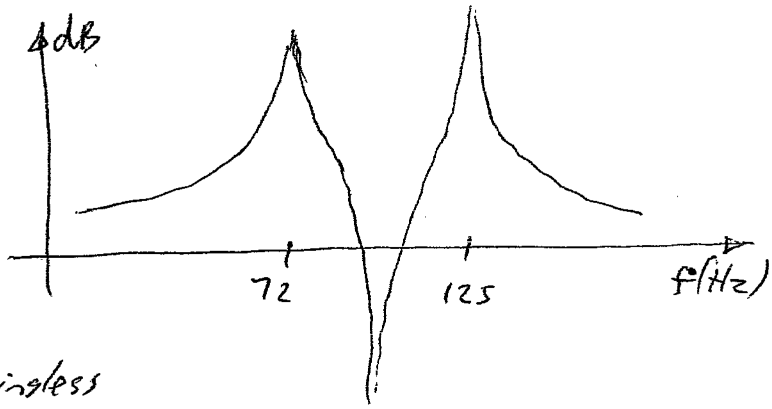


(d) The Nyquist frequency is set to, say, twice the maximum frequency of interest and $f_{nyq} = \frac{f_{sample}}{2}$

$\therefore f_{sample} = 4 f_{max} = 4 \times 175 = 700\ Hz$

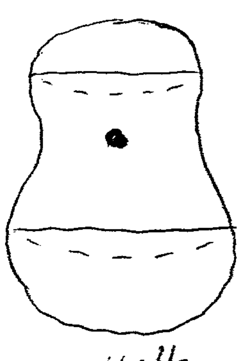
Say 1 kHz to be safe.

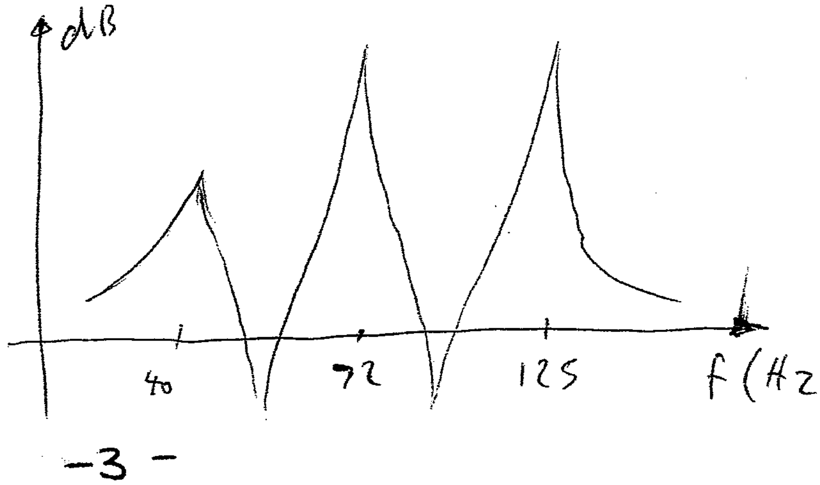
(e) If either A or B are on nodal lines then they do not appear in the transfer function. This means that the only modes to be excited are modes 2 & 5. The charge amp will need a high-pass filter so the transfer function will ~~drop to zero at~~ be meaningless at low frequencies



There is one nodal line between A & B for both modes so expect an antinode between the two peaks

(f) If we ignore rotary inertia (ie the mass is really tiny) then the only mode affected by the mass is mode 1. All other modes are unchanged because the mass is on a nodal line. Mode 1 will be shifted to a lower frequency. The mass (0.024g) is quite heavy compared with the guitar so expect a significant change in frequency - say from 60 Hz to 40 Hz. The nodal lines for mode 1 will shift perhaps as shown here. The new transfer function might be as below





Q1 Examiner's comment:

This question was answered well by the majority of candidates. One recurring error was in the calculation of the required sampling frequency; only five students obtained the correct answer for this.

2 (a) The transient decay rate (log dec) of individual resonant modes can be measured provided the decay is slow enough that a large number of cycles can be detected. That makes the method good for materials with low damping, and increasingly accurate as damping gets lower. It would not be accurate for material with higher damping, giving insufficient cycles before decay below the noise floor of the measurement. The geometry of the test specimen needs to be designed to have well-separated modes: low modal overlap is needed so that decay rates of different modes can be separated, by some kind of band-pass filtering approach. Care is always needed when measuring very low damping, because extra damping is likely to be contributed by whatever is used to support the test specimen, any attached sensors, and losses by viscosity or sound radiation into the surrounding air.

Frequency domain methods based on transfer functions, e.g. by circle fitting, are the second possibility. This method is difficult for very low damping, because you need to collect data for a very long time to get enough points around the modal circle to obtain a good fit. The same is true of a half-power bandwidth measurement: it is easier to get an accurate bandwidth of a moderately wide peak. But if damping is too high then a clear peak may not be seen, and there is also more danger of modal overlap with neighbouring modes, which complicates the analysis. So the approach is best for material of moderate damping. With loss factors around 1% the method overlaps with the transient decay method, and both can be used as a cross-check.

For material with very high damping no method based on resonance will work well. A forced vibration test is then the only option. Some kind of cyclic strain cycle is imposed, at the chosen frequency. The energy dissipation per cycle, e.g. via the loop area in the stress-strain plot, gives the damping value. The method relies on relatively elaborate and expensive equipment, and it is not easy to push to very high frequencies because of the difficulties of moving mass and possible resonances of the test machine. This method is less good for material with low damping because of additional damping coming from the grips used to hold the test specimen.

(i) Glass has high stiffness and very low damping, so transient decay will be the best approach. The material is also brittle, so excitation amplitude needs to be kept low. Support structures need to carefully chosen to minimise added damping. Non-contact sensing, e.g. by laser or microphone, would be a good choice. A specimen cut as a rectangular beam exciting in bending would be a common choice.

(ii) PMMA, from the data sheet, has damping η around 0.01, so either transient decay or circle fitting would be possible. Probably circle fitting would be slightly preferable, because an approach based on transfer functions allows averaging to be used when collecting the data, to improve the signal-to-noise ratio. Again, a rectangular beam specimen would be a good choice.

(iii) Butyl rubber, from the data sheet, has damping η >1. A forced vibration method is needed. A sample could be cycled in tension or in shear, with appropriate fittings in a tensile test machine or Dynamic Mechanical Analyser.

(b) From data sheet, the effective bending rigidity of a two-layer beam is

$$EI = E_1 I_1 \left[1 + eh^3 + 3(1+h)^2 \frac{eh}{1+eh}\right]$$

where

$$e = \frac{E_2}{E_1}, \quad h = \frac{h_2}{h_1}.$$

If $h \ll 1$, ignore high powers of h to get

$$EI \approx E_1 I_1 [1 + 3eh] = E_1 I_1 + 3E_2 I_1 h$$

Now let $E_2 \rightarrow E_2(1+i\eta_2)$ while E_1 remains real. Then the required effective η is given by

$$EI(1+i\eta) \approx E_1 I_1 + 3E_2(1+i\eta_2)I_1 h$$

so

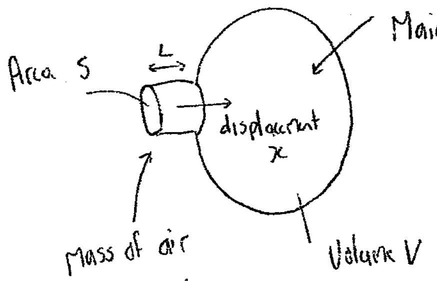
$$\eta = \frac{\mathrm{Im}(RHS)}{\mathrm{Re}(RHS)} \approx \frac{3E_2\eta_2 h}{E_1}$$

(c) For given values of E_1 and *h*, the best coating material will have the maximum value of $E_2\eta_2$. Since the chart in the Data Sheet has logarithmic scales on both axes, materials with equal values of $E_2\eta_2$ lie along straight lines with slope -1. One such line is included on the chart. To maximise $E_2\eta_2$ the line must be pushed as high as possible. The highest values correspond to high-loss metals, and the best named materials from this chart are in the region labelled "lead alloys". All polymers perform significantly worse than the best metals.

Examiner's comment:

The descriptive part of this question was answered well, but in the calculation part very few students made proper use of the fact that the thickness ratio *h* was very small. This meant that most expressions obtained for the loss factor were massively over complicated, and this in turn caused problems with the application of the Ashby chart.

3 a)



Main volume acts like a spring

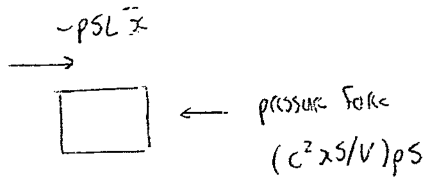
Change in volume due to displacement x

$= xS$

Change in density $= [(xS)/V]\rho$

Change in pressure $= (c^2 xS/V)\rho$

Mass of air
in neck acts
as a lumped mass
Mass $= \rho SL$



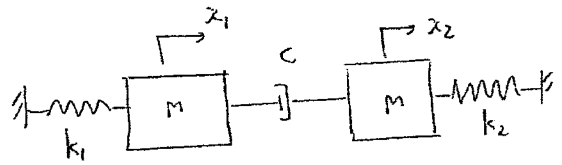
$$\rho SL\ddot{x} + (c^2 S^2 \rho / V)x = 0$$

$$\Rightarrow \quad \omega_n^2 = \frac{c^2 S^2 \rho}{V\rho SL} \quad \Rightarrow \underline{\omega_n = c\sqrt{\frac{S}{VL}}}$$

[25%]

S must be modified for "added mass" effects

b)



By inspection
$$\underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{M}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \underbrace{\begin{pmatrix} c & -c \\ -c & c \end{pmatrix}}_{C}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \underbrace{\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}}_{k}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

From data sheet
$$A = \begin{bmatrix} 0 & I \\ -M^{-1}k & -M^{-1}C \end{bmatrix} = \left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline -k_1/m & 0 & -c/m & c/m \\ 0 & -k_2/m & c/m & -c/m \end{array}\right)$$

$$\frac{d\underset{\sim}{y}}{dt} = A\underset{\sim}{y} \qquad \underset{\sim}{y} = \begin{pmatrix} \underset{\sim}{x} \\ \dot{\underset{\sim}{x}} \end{pmatrix}$$

[35%]

c) $\lambda \underset{\sim}{y} = A \underset{\sim}{y}$

$$\begin{pmatrix} \lambda \\ \lambda\alpha \\ \lambda^2 \\ \lambda^2\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m & 0 & -c/m & c/m \\ 0 & -k_2/m & c/m & -c/m \end{pmatrix} \begin{pmatrix} 1 \\ \alpha \\ \lambda \\ \lambda\alpha \end{pmatrix}$$

Row 3 $\Rightarrow$ $\lambda^2 = -k_1/m - (c/m)\lambda + \underbrace{(c/m)\lambda\alpha}_{\uparrow \text{ small}}$

$\Rightarrow$ $\underline{m\lambda^2 \approx -k_1 - \lambda c}$ — ①

Row 4 $\Rightarrow$ $\lambda^2\alpha = -k_2\alpha/m + (c/m)\lambda - \underbrace{(c/m)\lambda\alpha}_{\uparrow \text{ small}}$

$\Rightarrow$ $\underline{m\alpha\lambda^2 \simeq -\alpha k_2 + \lambda c}$ — ②

① $\Rightarrow$ $m\lambda^2 + \lambda c + k_1 = 0$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk_1}}{2m} \approx \underline{-\frac{c}{2m} \pm i\sqrt{\frac{k_1}{m}}}$$

② $\Rightarrow$ $\alpha = \dfrac{\lambda c}{m\lambda^2 + k_2}$ $\qquad \lambda c \approx \pm ic\sqrt{\dfrac{k_1}{m}}$

$m\lambda^2 + k_2 \approx -k_1 + k_2$

$$\underline{\alpha \approx \frac{\pm ic\sqrt{k_1/m}}{k_2 - k_1}}$$

$\uparrow$

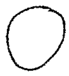
imaginary $\Rightarrow$ 90° phase difference between x_1 and x_2

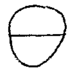
[40%]

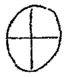
Most students showed an excellent understanding of the use of the first order formulation to obtain damping and natural frequencies; most errors were algebraic mistakes towards the end of the question.

4) a) From data sheet

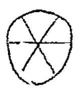
$$\omega_n = k_n\sqrt{\frac{T}{M}} = \underline{\frac{x_n}{a}\sqrt{\frac{T}{M}}}$$ with x_n given in table (ka) in data sheet

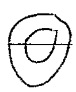
ω_1 : $x_1 = 2.404$  $n=0$

ω_2 : $x_2 = 3.832$  $n=1$

ω_3 : $x_3 = 5.135$  $n=2$

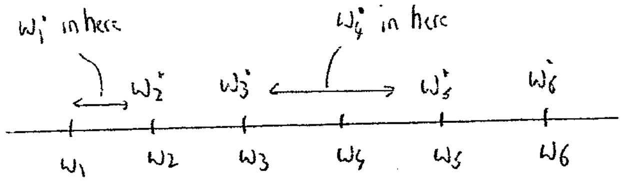
ω_4 : $x_4 = 5.520$  $n=0$

ω_5 : $x_5 = 6.379$  $n=3$

ω_6 $x_6 = 7.016$  $n=1$

[25%]

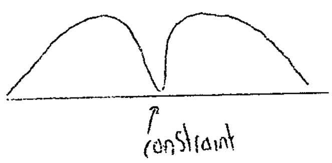
b) Only ω_1 and ω_4 will be affected - see mode shape plots above



← constrained frequencies : interlace with original frequencies

← original frequencies

Guess of first mode (cross-section) :



Axi-symmetric shape

[35%]

c) Put $w(r) = A_1 J_0(kr) + A_2 Y_0(kr)$

Must have $w(a) = w(b) = 0$

$$\Rightarrow \left.\begin{aligned} A_1 J_0(ka) + A_2 Y_0(ka) &= 0 \\ A_1 J_0(kb) + A_2 Y_0(kb) &= 0 \end{aligned}\right\} \Rightarrow J_0(ka)\underbrace{Y_0(kb)}_{\text{tends to } \infty \text{ for } b\to 0} - Y_0(ka)\underbrace{J_0(kb)}_{\text{tends to } 1 \text{ as } b\to 0} = 0$$

$b \to 0 \Rightarrow J_0(ka) = 0 \Rightarrow$ <u>natural frequency is the same as the unconstrained membrane</u>

[40%]

Examiner's comment:

The final part of this question required the students to apply boundary conditions to a membrane beyond those considered in the course. This was very well done, and overall the students displayed a good understanding of membrane vibrations, constraints, and the interlacing theorem.