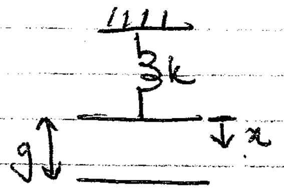
PART IIB 2011 4C15 : MEMS DESIGN

DR AA SESHIA

Q1. (a)

$$F_{NET} = kx - \frac{1}{2}\frac{\epsilon_0 A V^2}{g^2}$$



$$x = g_0 - g$$

$$\therefore\ k(g_0 - g) = \frac{1}{2}\frac{\epsilon_0 A V^2}{g^2}$$

(b) Stability criterion

$$\frac{\partial F_{NET}}{\partial g} < 0$$

$$\frac{\epsilon_0 A V^2}{g^3} - k < 0$$

At pull-in:

$$k = \frac{\epsilon_0 A V_{PI}^2}{g_{PI}^3} \qquad (1)$$

$$k(g_0 - g_{PI}) = \frac{1}{2}\frac{\epsilon_0 A V_{PI}^2}{g_{PI}^2} \qquad (2)$$

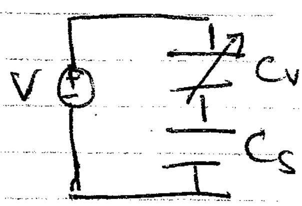
(1) + (2) gives:

$$g_0 - g_{PI} = \frac{g_{PI}}{2}$$

$$\text{or}\quad g_{PI} = \tfrac{2}{3} g_0$$

$$V_{PI} = \sqrt{\frac{8}{27}\frac{k}{\epsilon_0}\frac{g_0^3}{A}}$$

(c)



1

(c) Voltage dropped across capacitor actuator:

$$V_A = \frac{V}{1+\frac{C}{C_S}} = \frac{V}{1+K\frac{g_0}{g}}$$

$$F_c = \frac{\epsilon_0 A V_A^2}{2g^2} = \frac{\epsilon_0 A V^2}{2(g+Kg_0)^2}$$

Equilibrium gives:-

$$K(g_0-g) = \frac{\epsilon_0 A V^2}{2(g+Kg_0)^2} \quad (3)$$

Stability criterion:-

$$-K + \frac{\epsilon_0 A V^2}{(g+Kg_0)^3} < 0 \quad (4)$$

Substituting (3) into (4):

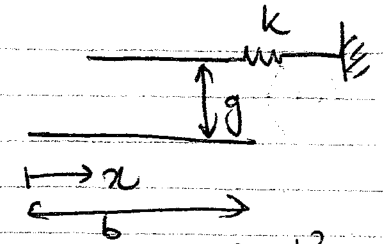
$$\frac{\epsilon_0 A V^2}{(g+Kg_0)^3}(g_0-g) < \frac{\epsilon_0 A V^2}{2(g+Kg_0)^2}$$

$$g_0 - g < \frac{g}{2} + \frac{Kg_0}{2}$$

or $$g > \frac{(2-K)g_0}{3}$$

$\therefore$ For $K > 2$, stable behaviour is possible.

The first two parts of the question was generally well done. Most students had difficulty with part (c) in deriving the condition for actuator stability over the entire operating gap.



Q2

(a)

$$E^* = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \frac{\epsilon_0 A}{g} V^2 = \frac{1}{2} \frac{\epsilon_0 t (b-x)}{g} V^2$$

$$\therefore F = \frac{-\partial E^*}{\partial x} = \frac{1}{2} \frac{\epsilon_0 t}{g} V^2$$

For N electrode gaps

$$F = \frac{N}{2} \frac{\epsilon_0 t}{g} V^2$$

(b) Dynamic actuation :-

$$F = \frac{N \epsilon_0 t}{g} V_{DC} \cdot V_{ac} = \frac{5\times10^{-9} \times (2\pi\times10^{4})^2 \times 5\times10^{-}}{100}$$

Set $V_{DC} = 10$ V as minimum V_{ac} desired.

$$\therefore V_{ac} = 0.558 \text{ V}$$

(c) $F_{coriolis} = 2 m \Omega_z \cdot v_x$

$$= 2 m \Omega_z \cdot (1 \mu m) \times (2\pi \times 1000) \cos(2\pi \times 1000 t) \text{ say}$$

Part I databook

$$m \ddot{y} + \frac{k}{m} y = 2 m \Omega_z (1 \mu m) \cos(2\pi \times 1000 t) \times (2\pi \times 1000)$$

$$\therefore \frac{y \cdot \omega_y^2}{(2\pi \times 1000)\, 2 (1 \mu m) \times \left(100 \times \frac{2\pi}{360}\right)} = \frac{1}{1 - \left(\frac{1000}{11000}\right)^2}$$

$$y = 4.63 \text{ pm}$$

(d)

$$\Delta C = \frac{\epsilon_0 A}{-x + x_0} - \frac{\epsilon_0 A}{x_0}$$

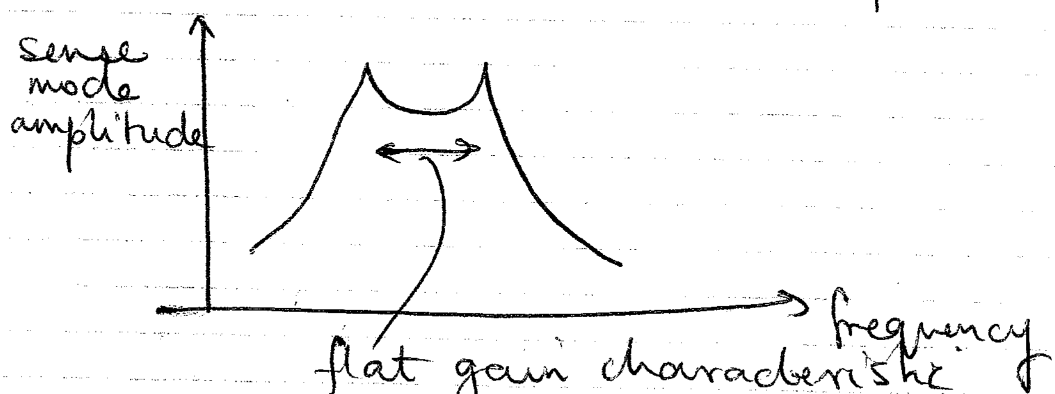
$$= \frac{\epsilon_0 A}{x_0}\left[\frac{x_0 + x - x_0}{x_0}\right]$$

$$= C_{nom}\left(\frac{x}{x_0}\right)$$

$$= 100fF\left(4.63 \times 10^{-6}\right)$$

$$= 0.463 \text{ aF}$$

(e) Mode-matching in a micromechanical vibratory rate gyroscope involves matching the natural frequencies in the drive and sense mode to "gain" up the response of the device in response to an applied rotation rate. While sensitivity is increased, bandwidth of operation is consequently reduced. One solution to address this trade-off is to increase the degrees of freedom along the sense mode to create a "band-pass" characteristic



. Most parts of this question were generally well done but some students had difficulties explaining the bandwidth-sensitivity trade-off in part (c).

Q3

(a)

$$\frac{d^2u}{dz^2} = \frac{\sigma_w E_x}{\eta L_D} e^{-z/L_D}$$

$$u = u_0\left(1 - e^{-z/L_D}\right)$$

$$\frac{d^2u}{dz^2} = -\left(\frac{1}{L_D}\right)^2 u_0 \cdot e^{-z/L_D}$$

$$\therefore u_0 = -\frac{\sigma_w E_x L_D}{\eta}$$ and the solution for plug flow is satisfied

The sign of u_0 indicates flow opposite to the direction of the applied field if $\sigma_w > 0$. and positive otherwise.

(b)

$$\sigma_w = -0.1\ C/m^2$$

$$E_x = \frac{100\ V}{10\ mm} = 10^4\ V/m$$

$$u_0 = \frac{(0.1)(10^4)(10^{-9})}{1.5\times10^{-3}} = 6.66\times10^{-4}\ m/s$$

$$Q = A u_0 = 100\times100\times10^{-12}\times6.66\times10^{-4}$$

$$= 6.66\times10^{-12}\ m^3/s$$

To travel from Port 1 to Port 2 :→

$$t = \frac{10\ mm}{6.66\times10^{-4}\ m/s} = 15.02\ secs$$

(c) $$v_x = \mu_{ep} \cdot E_{3\to4}$$
$$= 10^{-9} \times 2000$$
$$= 2 \times 10^{-6} \ \text{m/s}.$$

$$t_{3\to4} = \frac{20 \ \text{mm}}{1.332 \times 10^{-4}} = 150 \ \text{secs}$$

$\therefore$ bands are separated by a distance Δx

$$\Delta x = 2 \times 10^{-6} \times 150$$
$$= 300 \ \mu\text{m}.$$

(d) width of the band after separation

$$\approx \sqrt{D \cdot t_{3\to4}}$$
$$\approx \sqrt{50 \ \mu\text{m}^2/\text{s} \times 150 \ \text{secs}}$$
$$\approx 86.6 \ \mu\text{m}.$$

This is comparable to the band separation by electrophoresis

$$\text{band size} = \sqrt{D \cdot t}$$
$$= \sqrt{D \cdot L/u_0}$$
$$= \sqrt{D \cdot L\eta/(\epsilon_w \cdot E_x L_0)}$$

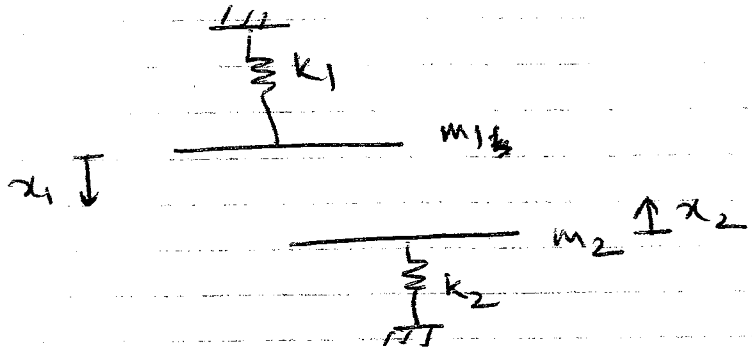
$\therefore$ use short columns, large electric fields and low ionic strength buffers to achieve large L_D and good separation.

need to ensure plug-flow is satisfied as well

It was also generally well done. Some students had difficulties with formulating the design criteria for good electrophoretic separation in (d).

Q4

(a)



$$m_1\ddot{x}_1 + k_1 x_1 - f_e = 0$$

$$m_2\ddot{x}_2 + k_2 x_2 - f_e = 0$$

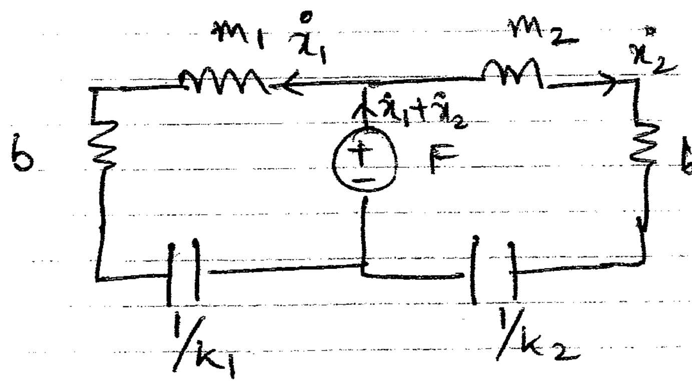
$$f_e = \frac{\epsilon_0 A V^2}{2(g - x_1 - x_2)^2}$$

Assuming $\frac{x_1}{g}, \frac{x_2}{g} << 1$ we have:

$$m_1\ddot{x}_1 + k_1 x_1 - \frac{\epsilon_0 A V^2}{2g^2} - \frac{\epsilon_0 A V^2}{g^3}(x_1 + x_2) = 0$$

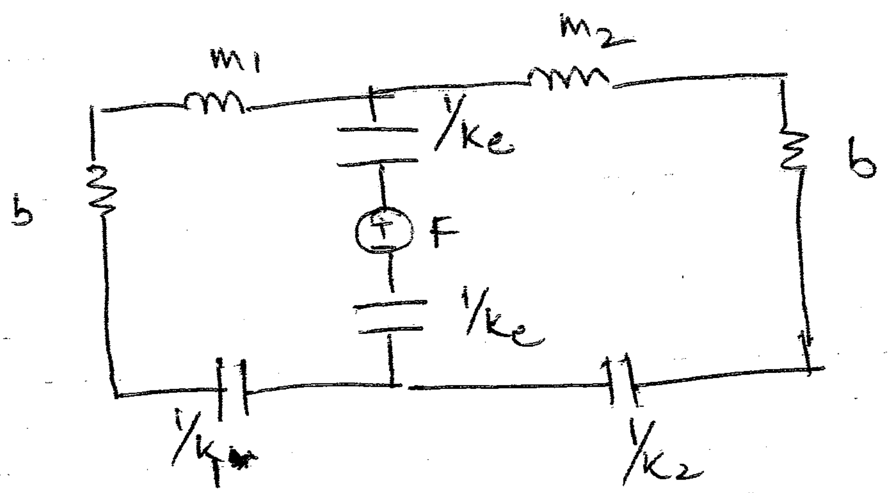
$$m_2\ddot{x}_2 + k_2 x_2 - \frac{\epsilon_0 A V^2}{2g^2} - \frac{\epsilon_0 A V^2}{g^3}(x_1 + x_2) = 0.$$

(b)



$\Downarrow$

adding electrical springs to the circuit



(c) (i) $k_1 >> k_2$, the pull-in behaviour is expected to be similar to the case of collapse to a fixed electrode

(ii) $k_1 \approx k_2$, the pull-in behaviour will involve approximate simultaneous collap of the two beams (triggered by motion of the more compliant beam)

(d) Device application include micromecha switches and band-pass filters. Pull-in modelling will apply to mechanical switches while the dynamic modelling of the coupled behaviour can be used construct transfer function for microw band-pass filters.

Most students who attempted this question had difficulties with parts (a) and (d). Some students appeared to have attempted this question last and run out of time resulting in a few incomplete solutions.