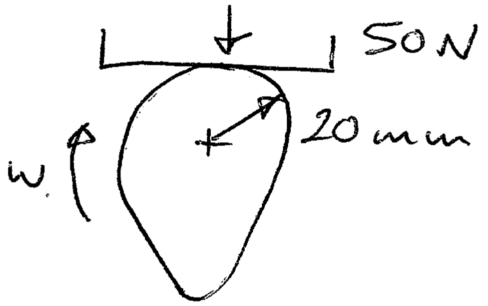
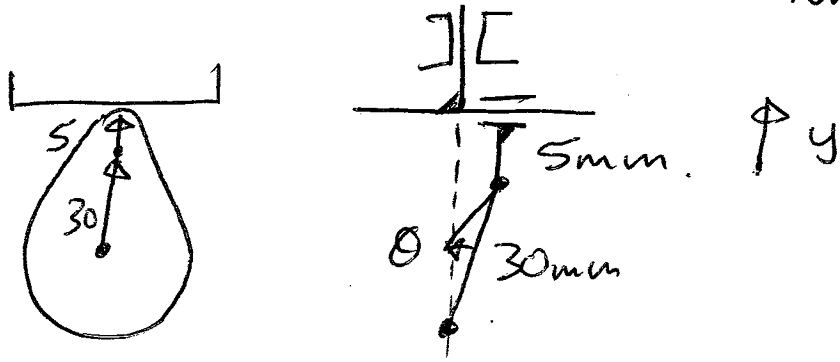
MODULE 4C16 ADVANCED MACHINE DESIGN
2011 D J COLE.

1. a)



10 mm width.

contact force per unit length $= \frac{50N}{10mm} = \underline{\underline{5\,kN/m}}$



$y = 0.03.\cos\theta \qquad \theta = \omega t$

$\dot{y} = -\omega\, 0.03 \sin \omega t$

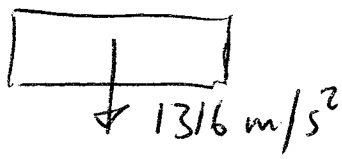
$\ddot{y} = \omega^2\, 0.03 \cos \omega t$

at max lift $\ddot{y} = \omega^2\, 0.03$

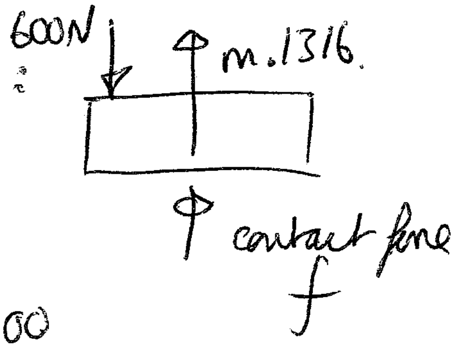
$= \left(2000.\frac{2\pi}{60}\right)^2 0.03 = 1316\ m/s^2 \downarrow$

follower mass

accns:



forces:



sum forces to zero $\quad f + m1316 = 600$

$m = 0.1 \qquad f = 600 - 131.6$

$= 468.4N.$

contact force per unit length $\frac{468.4N}{10mm} = \underline{\underline{46.84\,kN/m}}$

b). find Hertz pressure

radii of curvature equal to cam radii

line contacts.

base circle $p_0 = \left(\frac{P' E^*}{\pi R}\right)^{\frac{1}{2}} = \left(\frac{5000 . 115 . 10^9}{\pi \; 0.005}\right)^{\frac{1}{2}} = 957 \text{ MPa}$ <u>safe</u>

tip circle $p_0 = \left(\frac{46.84 . 10^3 . 115 . 10^9}{\pi \; 0.005}\right)^{\frac{1}{2}} = 5856 \text{ MPa}$

comparable to yield stress of steel, so <u>may be problematic</u>

<u>Lubrication conditions</u>

entraining velocities

base circle $\bar{u} = \frac{u}{2} = \frac{\omega r}{2} = \frac{2000 . 2\pi}{2 . 60} . 0.02 = 2.09 \text{ m/s}$

tip circle $\bar{u} = \frac{2000 . 2\pi}{2 . 60} . 0.035 = 3.67 \text{ m/s}$

calculate g_1 and g_3 for base and tip circles

$$g_1 = \left(\frac{\alpha^2 P'^3}{\eta_0 R^2 \bar{u}}\right)^{\frac{1}{2}} = \left(\frac{(2.10^{-8})^2 (5000)^3}{0.1 . 0.02^2 \; 2.09}\right)^{\frac{1}{2}}, \left(\frac{(2.10^{-8})^2 (46.84 . 10^3)^3}{0.1 . 0.005^2 . 3.67}\right)^{\frac{1}{2}}$$

$$= 0.77 \qquad = 66.9$$

$$g_3 = \left(\frac{P'^2}{2 \eta_0 R \bar{u} E^*}\right)^{\frac{1}{2}} = \left(\frac{5000^2}{2.0.1.0.02.2.09 \; 115.10^9}\right)^{\frac{1}{2}}, \left(\frac{(46.84 . 10^3)^2}{2.0.1.0.005.3.67 \; 115.10^9}\right)^{\frac{1}{2}}$$

$$= 0.16 \qquad = 2.28$$

plotting g_1 and g_3 on lubrication map gives $h^* \sim 3$ for base circle

$$h^* = \frac{W h_{min}}{\eta_0 R L \bar{u}}$$

so $h_{min} = \dfrac{3 \cdot 0.1 \cdot 0.02 \cdot 2.09}{5000} = 2.5\,\mu m$

$h^* \sim 30$ for tip circle.

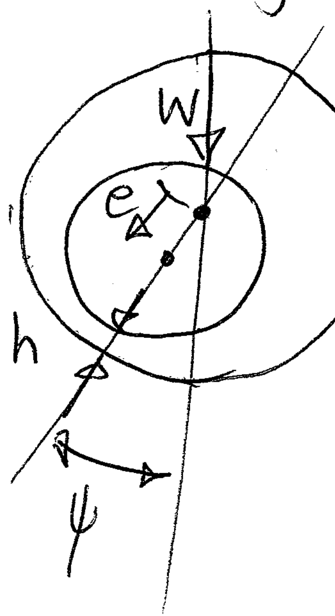
$$h_{min} = \frac{30 \cdot 0.1 \cdot 0.005 \cdot 3.67}{46.84 \cdot 10^3} = 1.18\,\mu m$$

$1.18\,\mu m$ is $\sim R_q$ ($1.0\,\mu m$) so poor lubrication conditions, leading to wear or polishing of the tip.

Base circle has $h_{min} > R_q$ so lubrication is better.

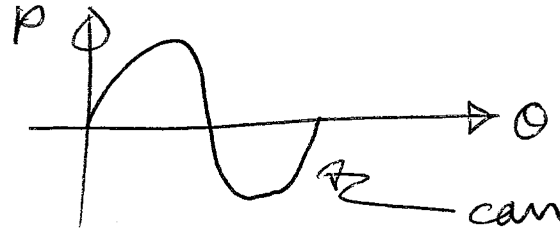
2. a) attitude angle ψ

eccentricity ratio $\varepsilon = \frac{e}{c}$

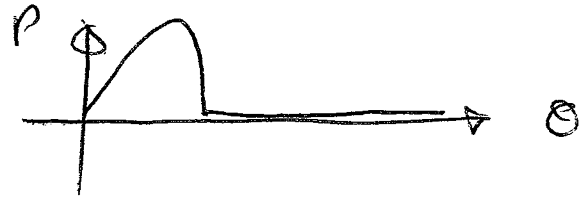


Use Reynolds equation in 1D or 2D to find pressure, hence load W

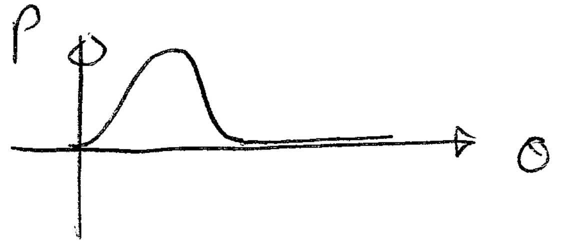
exit conditions:



full Somerfeld
okay for gas, not liquid.



half Somerfeld



Reynolds

$\frac{dp}{d\theta} = p = 0$ at exit

b) $W = 500\,kN$
$L = 0.4\,m$
$R = 0.05\,m$
$C = 50\,\mu m$
$\eta = 0.05\,Pa\,s$
$\omega = 200\,rad/s$

$$W^* = \frac{500.10^3/0.4}{0.05 . 0.1 . 200}\left(\frac{50.10^{-6}}{0.05}\right)^2$$

$$\underline{\underline{W^* = 1.25}}$$

interpolate table to find ε

$$\varepsilon = 0.2 + (0.3 - 0.2)\,\frac{1.25 - 0.882}{1.304 - 0.882}$$

$$\varepsilon = 0.287$$

$$h_{min} = c - e = c\left(1 - \frac{e}{c}\right) = c(1 - \varepsilon)$$

$$= 50.10^{-6}(1 - 0.287)$$

$$\underline{\underline{= 35.6\,\mu m}}$$

c) find c to maximise the minimum film thickness.

$$h_{min} = c - e \quad \therefore \quad \frac{h_{min}}{c} = 1 - \frac{e}{c} = 1 - \varepsilon$$

define W^* in terms of h_{min}, not c

$$W^* = \frac{W/L}{\eta D \omega}\left(\frac{h_{min}^2}{(1-\varepsilon)^2 R^2}\right)$$

$$W^*(1-\varepsilon)^2 = \frac{W/L}{\eta D \omega}\left(\frac{h_{min}^2}{R^2}\right)$$

maximise this

ε	0.2	0.3	0.4	0.5
W^*	0.882	1.304	1.767	2.318
$\sqrt{W^*}(1-\varepsilon)$	0.751	0.799	0.798	0.761

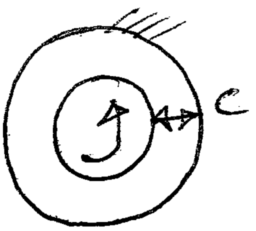
hence max value around $\varepsilon = 0.35$

$\therefore W^* = 1.536$

$c = 50\,\mu m \times \sqrt{\frac{1.536}{1.25}}$ (Scaling)

$c = 55.4\,\mu m$

d) Petrov.



shear stress $\tau = \eta \cdot \frac{\omega R}{c}$ (vel. gradient)

$\therefore M = 2\pi R L \tau R$

power $= M\omega$

$= 2\pi R L \eta \frac{\omega R}{c} . R . \omega$

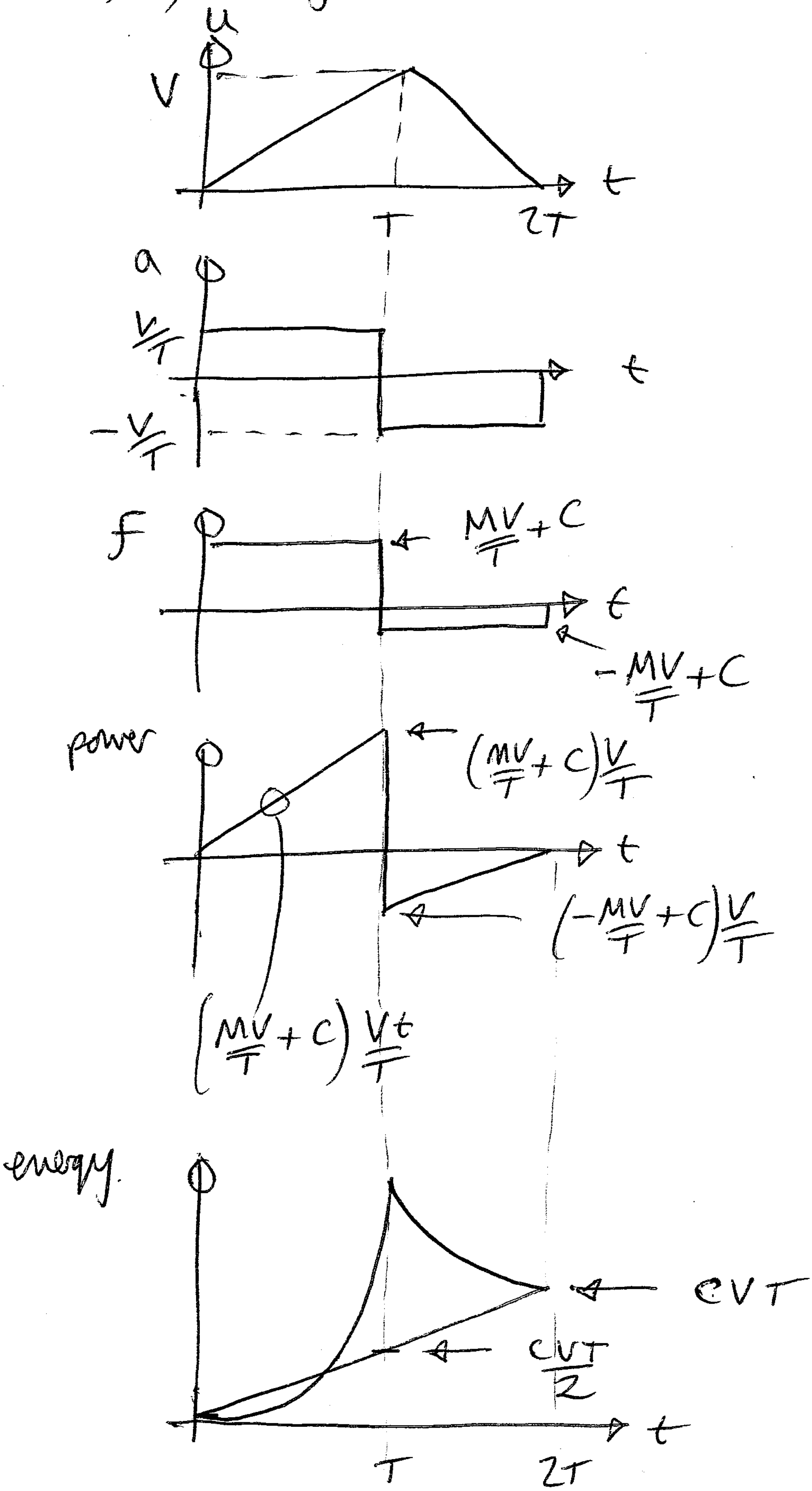
$= 2\pi \frac{L}{c} R^3 \omega^2 \eta$

$= 2\pi \frac{0.4}{50 . 10^{-6}} . 0.05^3 \, 200^2 \, 0.05$

power $= 12.6$ kW

power → rise in oil temperature
→ reduction in viscosity
→ reduction in h_{min}.

3. a) i) $F = Ma + C$



u
V
t
T
2T
a
V/T
t
-V/T
f
MV/T + C
t
-MV/T + C
power
(MV/T + C)V/T
t
(-MV/T + C)V/T
(MV/T + C)Vt/T
energy
CVT
CVT/2
t
T
2T

max energy is at $t = T$

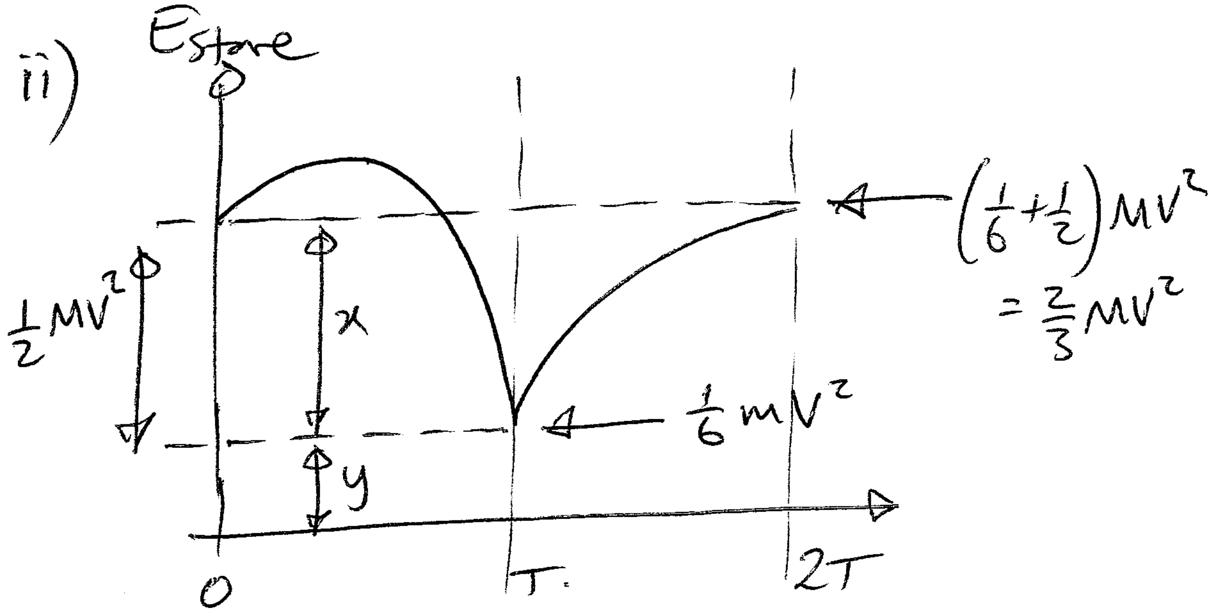
$$= \int_0^T \left(\frac{MV}{T} + C\right)\frac{Vt}{T}\,dt$$

$$= \left[\left(\frac{MV}{T} + C\right)\frac{Vt^2}{2T}\right]_0^T$$

$$= \frac{MV^2}{2} + \frac{CVT}{2}$$

$$= (MV + CT)\frac{V}{2}$$

ii)



if speed at $t=0$ is twice speed at $t=T$

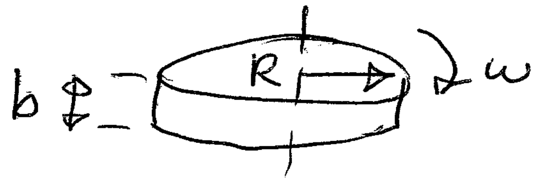
then $\frac{x+y}{4} = y$

$$x = 3y$$

but $x = \frac{1}{2}mV^2$ so $y = \frac{1}{6}mV^2$

thus $x + y = \left(\frac{1}{6} + \frac{1}{2}\right)mV^2 = \frac{2}{3}MV^2$

b) i) mass m.



energy stored $U = \frac{1}{2} J \omega^2 = \frac{\pi}{4} \rho R^4 b \omega^2$

mass $m = \pi R^2 b \rho$

Stress $\sigma = \frac{1}{2} \rho R^2 \omega^2$

Thus specific energy $\frac{U}{m} \sim \frac{1}{2} \frac{\sigma}{\rho}$

Hence for given U, minimise mass by maximising $\frac{\sigma}{\rho}$ for material of flywheel.

Examiner's comments

Q1 Cam mechanism

Part (a) was generally answered well, although there were several examples of incorrect equivalent mechanisms. Shortcomings in the answers to part (b) included: incorrect expression for entraining velocity; no discussion or interpretation of numerical results; no contact pressure calculation; numerical errors.

Q2 Hydrodynamic bearing

Parts (a) and (b) were generally answered well. In part (c) some answers were inaccurate because the tabulated data was not interpolated, or because a trial and error procedure was used to find the solution. The numerical answers to part (d) covered a wide numerical range, unreasonable values were often stated without comment, and units were often wrong or absent.

Q3 Hybrid drive

In part (a)(i) the energy required to overcome the constant force term was omitted by some candidates. In part (a)(ii) the constant force term was sometimes incorrectly included in the energy storage analysis. Apart from these problems the question was answered well.