Module 4D6
Dynamics in Civil Engineering

May 2nd 2011 9am

Crib

① (a) $K_{eq} = \dfrac{3EI}{L^3} = \dfrac{3(8\times10^{13})}{(100)^3} = 2.4\times10^8\ N/m$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.4\times10^8\ N/m}{4\times10^6\ kg}} = \sqrt{60}\ rad/s$$

$\boxed{f_n = 1.23\ Hz}$; $T_n = 0.81\ s.$

$$\frac{t_d}{T_n} = \frac{0.4}{0.81} = 0.49$$

$\rightarrow$ From Data Sheets $\rightarrow$ DAF = 1.3

$$u_{st} = \frac{F_0}{K_{eq}} = \frac{m(0.5\,g)(9.81\ m/s^2)}{2.4\times10^8} = 0.082\ m$$

$$u_{dyn} = DAF(u_{st}) = 0.082(1.3) = 0.106\ m$$

$$V_{max} = \omega_n^2\, u_{dyn}\, m = (\sqrt{60})^2(0.106)(4\times10^6) = \boxed{25.5\times10^6\ N}$$

(b) $\omega_n = \dfrac{\sqrt{60}}{5} = \sqrt{\dfrac{k_b}{m+\frac{2}{3}m}}$

$$k_b = \frac{60}{5^2}\left(\frac{5}{3}\right)(4\times10^6\ kg) = \boxed{1.6\times10^7\ N/m}$$

Estimate shear in superstructure:

$$T_n = 5(0.81) \approx 4.0$$

$$\frac{t_d}{T_n} = \frac{0.4}{4} = 0.1 \longrightarrow DAF \approx 0.3$$

$$V_{max} = DAF(\ddot{u}_{g\,max})\,M$$
$$= 0.3(0.5g)(9.81)(4.0\times10^6) = \boxed{5.9\times10^6\ N}$$

(c)

$$\frac{k_b}{k_{eq}} = \frac{1.6\times10^7}{2.4\times10^8} = \frac{1}{15} \rightarrow K_{eq} = 15\,k_b = k$$

EOM:
$$\begin{bmatrix} m_b & \\ & m \end{bmatrix}\begin{bmatrix} \ddot{u}_b \\ \ddot{u}_t \end{bmatrix} + \begin{bmatrix} k+k_b & -k \\ -k & k \end{bmatrix}\begin{bmatrix} u_b \\ u_t \end{bmatrix} = 0$$

$\det\left[\underline{K} - \omega^2 \underline{M}\right] = 0 \rightarrow$ Find natural frequencies

$$\underline{K} - \omega^2 \underline{M} = k\begin{bmatrix} 16/15 & -1 \\ 1 & 1 \end{bmatrix} - m\begin{bmatrix} 2/3 & \\ & 1 \end{bmatrix}$$

$$\lambda = \omega^2 \frac{m}{k} \rightarrow \det\begin{bmatrix} \frac{16}{15} - \frac{2}{3}\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)\left(\frac{16}{15} - \frac{2\lambda}{3}\right) - 1 = 0$$

$$\frac{2}{3}\lambda^2 - \frac{26}{15}\lambda + \frac{1}{15} = 0$$

$$10\lambda^2 - 26\lambda + 1 = 0 \rightarrow \lambda = 2.56,\ 0.04$$

$$\omega_1 = \sqrt{\frac{\lambda k}{m}} = \sqrt{0.04(60)} = 1.55 \text{ rad/s}$$

$$\boxed{f_1 = 0.25 \text{ Hz}} \rightarrow T_1 = 4\text{s.}$$

$$\omega_2 = \sqrt{2.56(60)} = 12.4 \text{ rad/s} \rightarrow \boxed{f_2 = 1.97 \text{ Hz}} \rightarrow T_2 = 0.5 \text{ s.}$$

Mode shapes:
$$\begin{bmatrix} \frac{16}{15} - \frac{2}{3}\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix}\begin{bmatrix} \phi_b \\ \phi_t \end{bmatrix} = 0$$

$$1.04\,\phi_b - \phi_t = 0 \rightarrow \phi_1 = \begin{bmatrix} 0.96 \\ 1 \end{bmatrix}$$

$$-0.64\,\phi_b - \phi_t = 0 \rightarrow \phi_2 = \begin{bmatrix} -1.56 \\ 1 \end{bmatrix}$$

(d) Mode 1

$$M_{eq_1} = \frac{2}{3}m\,(0.96)^2 + m\,(1)^2 = 1.61\,m$$

$$K_{eq_1} = \frac{k}{15}\,(0.96)^2 + k\left(1-0.96\right)^2 = 0.063\,k$$

$$\Gamma_1 = \frac{\frac{2}{3}m\,(0.96) + m\,(1)}{M_{eq}} = 1.02$$

$$F_{eq_1} = -\Gamma_1 M_{eq_1}\ddot{u} = -(1.02)(1.61\,m)\,\ddot{u}_{max} = -1.64\,m\,\ddot{u}_{max}$$

$$u_{dyn} = DAF\,\frac{F_{eq}}{K_{eq}} \approx (0.3)\,\frac{1.64\,m\,\ddot{u}_{max}}{0.063\,k} = 7.83\,\frac{m}{k}\,\ddot{u}_{max}$$

Storey Force $= V = k\delta$

$$= \cancel{k}\left[7.83\,\frac{m}{\cancel{k}}\,0.5g\,(9.81)\,(1-0.96)\right]$$

$$\boxed{V_{mode_1} = 6.1\times10^{6}\ N}$$

2nd Mode:

$$M_{eq} = \frac{2}{3}m\,(-1.56)^2 + m\,(1)^2 = 2.62\,m$$

$$K_{eq} = \frac{k}{15}\,(-1.56)^2 + k\,(1-(-1.56))^2 = 6.72\,k$$

$$\Gamma_2 = \frac{\frac{2}{3}m(-1.56) + m\,(1)}{M_{eq}} = -0.015$$

$$\frac{t_d}{T_n} = \frac{0.4}{0.5} \rightarrow DAF = 1.52$$

Alternate Method: $|V_2| = |\Gamma\, m\, \phi_t\,(DAF)\,\ddot{u}_g|$

$$= 0.015\,(m)(1)(1.52)(0.5)(9.81)$$

Mode 2: $\boxed{V_2 = 4.5\times10^{5}\ N}$

→ No damping → Add peaks → $V_{total} = 6.55\times10^{6}\ N$

2 a) The distributed mass system must be converted into an equivalent SDOF system.

~~Thus~~ A mode shape must be assumed.

$$K.E = \tfrac{1}{2} M_{eq} \dot{v}^2$$

$$P.E = \tfrac{1}{2} K_{eq} v$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$$

True natural frequency $\leqslant$ answer calculated.

b) $$M_{eq} = \int_0^L m\,\bar{u}^2\,dx = m\int_0^{15}\left(1 - \cos\frac{n\pi x}{30}\right)^2 dx$$

$$= \left[1.5x - \frac{60}{n\pi}\sin\frac{n\pi x}{30} + \frac{15}{2n\pi}\sin\frac{2n\pi x}{30}\right]_0^{15}$$

$n = 1$ $\quad M_{eq} = 170{,}070$ kg

$n = 2$ $\quad M_{eq} = 1{,}125{,}000$ kg

$$K_{eq} = \int_0^L EI\left(\frac{\partial^2 u}{\partial x^2}\right)^2 dx = \frac{EI n^4 \pi^4}{30^4}\int_0^L \cos^2\frac{n\pi x}{30}\,dx$$

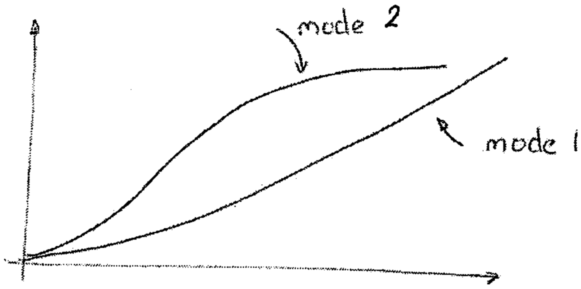
$$= \frac{EI n^4\pi^4}{810000}\left[\frac{x}{2} + \frac{15}{n\pi}\sin\frac{n\pi x}{15}\right]_0^{15}$$

$n = 1$ $\quad K_{eq} = 54 \times 10^3$ N/m

2 $\quad K_{eq} = 866 \times 10^3$ N/m

$f_1 = 0.09$ Hz $\qquad f_2 = 0.14$ Hz

c)



Boundary conditions

			Mode 1	Mode 2
	$x = 0$	$v = 0$	✓	✓
	$x = 0$	$\frac{dv}{dx} = 0$	✓	✓
$M = 0$	$x = L$	$\frac{d^2v}{dx^2} = 0$	✓	×
$S = 0$	$x = L$	$\frac{d^3v}{dx^3} = 0$	×	✓
			unsuitable	unsuitable.

d) Direct solution of equations of motion <u>OR</u>
Superposition of time histories only for <u>LINEAR</u> behaviour

- computationally expensive
- correct answer

Addition of peak responses

- overly conservative
- very quick

SRSS $\quad S = \sqrt{\delta_1^2 + \delta_2^2 + \dots}$

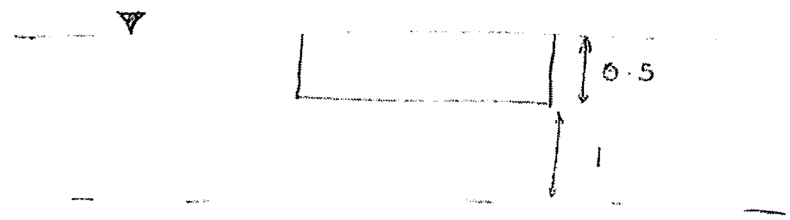
- good approximation for well separated n.f.s.
- response often dominated by a single mode.

This was a very popular question, being answered by all candidates. The question was in general very well answered. Part c caused problems to some candidates in identifying the errors with shear and bending moment causing mode shapes to be inadmissible. In part d many candidates offered superposition of time histories in different modes, rather than solution of the equation of motion. This is an exact solution only if the system is linear.

3 a) Soil must be

			Remediation
i)	Loose	$\longrightarrow$	Densification
ii)	Sandy (not too permeable)	$\longrightarrow$	Drainage
iii)	Saturated	$\longrightarrow$	Air-sparging (de-saturation)

b)



$$G_{max} = 100 \frac{(3-e)^2}{1+e} \sqrt{p'} \qquad (MPa)$$

$e = 0.8$ $\qquad K \sim 0.5 \quad \left[\text{elastic } K = \frac{\nu}{1-\nu} = \frac{0.3}{0.7}\right]$

$\sigma_v' = 0.5 \times 14 + 1 \times 9 = 16 \text{ kPa} \qquad p' \sim \frac{16 + 2\times 8}{3} = 10.7 \text{ kPa}$

$G_{max} = \cancel{87.8 \text{ MPa}} \ \underline{27.8} \text{ MPa}$

Databook p.2 $\quad l = 1m \quad b = 1m \quad e = 0.5m$

$\frac{l}{b} = 1 \qquad \frac{e}{b} = \frac{1}{2}$

$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8 + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{2}\right)\frac{1}{2}^{0.8}\right]$$

$$= \frac{27.8}{1.7} \times 9.2 \times 1.574 = 236.8 \text{ MN/m}$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73 + 0.27\right]\left[1 + \frac{1}{2} + \frac{1.6}{1.35} \times \frac{1}{2}^2\right]$$

$$= \frac{27.8}{0.7} \times 4 \times 1.796$$

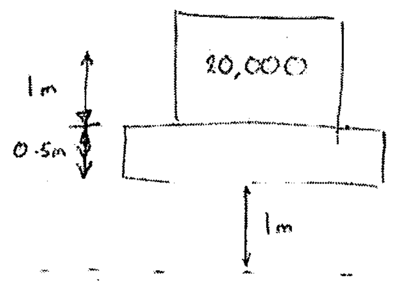
$$= 285 \text{ MNm/rad}$$

3 c) Mass of concrete block $= 2 \times 2 \times \frac{1}{2} \times 2400$

$= 4800$ kg

Mass of machine $= 20,000$ kg

Rocking



$$I \simeq 4800 \times 1.25^2 + 20000 \times 2.5^2$$

$$= \cancel{87500}\ \text{kg m}^2$$

$$132000$$

Assuming no change in foundation stiffness due to machine mass:

$$f_{rock} = \frac{1}{2\pi}\sqrt{\frac{285 \times 10^6}{87.5 \times 10^3}} = 7.4\ \text{Hz}$$

$$f_{horiz} = \frac{1}{2\pi}\sqrt{\frac{236.8 \times 10^6}{24800}} = 15.5\ \text{Hz}$$

d) 750 rpm $= \frac{750}{60} = 12.5$ Hz

Close to resonance in both modes

Resonance in rocking during spin-up.

e) Increase stiffness in both rocking & sliding - changes n.f. and reduces response at all frequencies.

Increase foundation dimensions, especially L

Increase damping to reduce resonance.

Improve soil by compaction, grouting etc to increase G.

Lower machine CG.

This was a very popular question, being answered by all candidates. This question was well answered by many candidates. The units for rotational stiffness were sometimes wrong (MN/m rather than MNm). Answers to part e often focussed on dampers for controlling resonance, rather than other methods of changing stiffness and damping such as foundation size and soil improvement which would have the added benefits of reducing response away from resonance.

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a) Full marks will be obtained for a clear explanation of how the paradox arises in irrotational potential flow theory, and how the boundary layer and boundary layer separation do not satisfy potential flow theory.

b) Full marks will be obtained for a clear explanation of "added mass" – perhaps with reference to Morison's equation – explaining how the inertial terms associated with accelerating the beer around the bubble lead to "m×a" terms comparable to the uplift forces.

c). ~~Full~~ Marks will be obtained for a clear description of classical flutter ~~as a 2 d.o.f.~~ coupling via the fluid force terms of lift and pitching moment of the ~~2 d.o.f.~~ vertical and rotational terms of a 2 d.o.f. structural system such as a plate. Further marks may be awarded for discussion of Theodorsen theory, Scanlan theory, section model tests, etc. It should also be stated that it is a catastrophic (oscillatory divergent) phenomenon that occurs above a critical wind speed, and that structures are particularly susceptible to it ~~for~~ if the vertical and rotational frequencies are close.

d) Full marks will be obtained for a clear description of how rivulets of rain on the underside of cables lead to a change in the aerodynamic cross-section, making the cables thereby susceptible to galloping.

e) Full marks will be obtained for a clear description of the long slow positive pressure ~~wave~~ profile from gas explosions compared with the very-high-positive followed by longer slower negative pressure profile of HE blasts.

f) Full marks will be obtained for a description of how the aerodynamic admittance function takes account of decorrelation of local pressure fluctuations via the areal averaging over the frontal area of a structure, and how it includes the frequency (but not the phase) information of the resultant net forces.

This question required written descriptions of a number of concepts in wind and blast engineering. It was generally answered well by those few who tackled it who also scored highly on their other questions .

Q1 Examiner's comment

The majority of students did well on parts (a) and (b), although a surprising number had trouble calculating the maximum shear. In part (c), most students followed the correct procedure to find natural frequencies and modes, but calculation errors were common. In part (d), very few students calculated the shear in the superstructure correctly.