PART IIB 2011

DR C J BURGOYNE

Paper 4D7 Concrete and Masonry Structures 2011

Solutions

1 (a) Bookwork.

Candidates had to provide examples of 3 failures. The commonest reported were:-

Ronan Point tower block collapse, where a relatively minor gas explosion led to progressive collapse of parts of a tower block caused by inadequate tying-together of wall and floor precast units. It led directly to requirements to prevent collapses of structures that were out of proportion to the original failure (disproportionate collapse). Changing the factors of safety on the codes would not have made much difference.

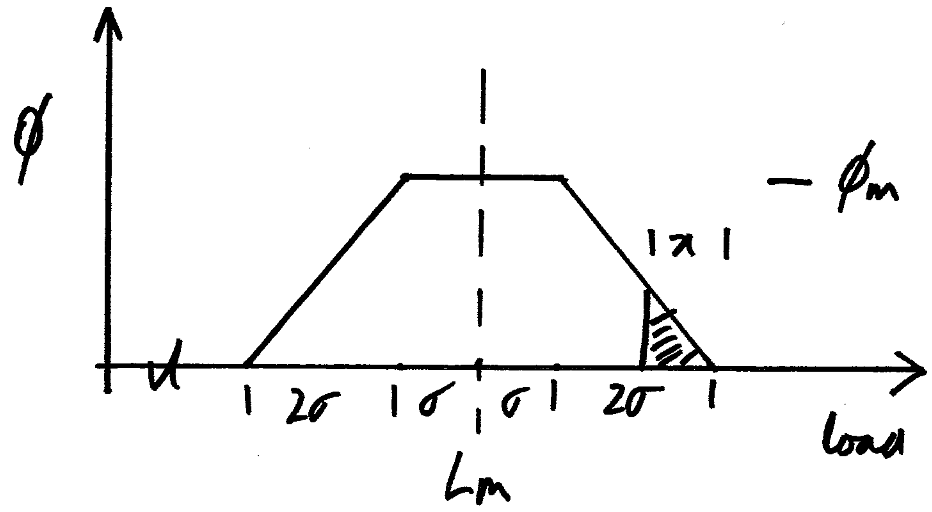
Ynys-y-Gwas Bridge, which failed with no load applied to it due to corrosion of prestressing tendons. There were several aspects of the design that allowed the corrosion to take place. Transverse joints between precast elements were filled with mortar, and the deck had no continuous slab. In addition, the ducting for the tendons provided no barrier to water. One notable fact was that the oxygen supply was limited so the steel corroded to a product that did not stain the concrete, so it was not possible to see it from external inspection. Changing factors of safety in codes would have made no difference to the likelihood of failure, although having more or bigger tendons might have delayed it. The principal cause of failure was poor detailing.

Ferrybridge Cooling Towers, several of which collapsed under high, but not excessive winds. The designers had used a wind speed lower than the BS, and had not made any allowance for gusts, or for disturbances to the flow caused by the grouping of the towers. There was also inconsistent application of load factors (factoring the resulting stress, which was the difference of two components, rather than applying factors in the worst sense to the individual load elements).

Concorde Overpass Bridge in Montreal which failed when a brittle shear failure propagated from a half joint through a cantilever. There were inadequacies in the original design (especially in the absence of shear steel), the construction quality management, and in the poor inspection and maintenance regime. Higher safety factors would have made a difference provided shear steel had been included.

Various other failures were discussed by candidates (Buildings in Chinese Earthquakes, Tasman Bridge Collapse. Tacoma Narrows Bridge was not allowed as it was not a concrete structure.

(b) (i)



$$\text{Area under curve} = 2\sigma \cdot \phi_m + 2 \cdot \frac{\phi_m}{2} \cdot 2\sigma$$
$$= 4\sigma \phi_m$$

But area $= 1$

$$\therefore \quad \phi_m = 1/4\sigma$$

Characteristic value at 5% fraction

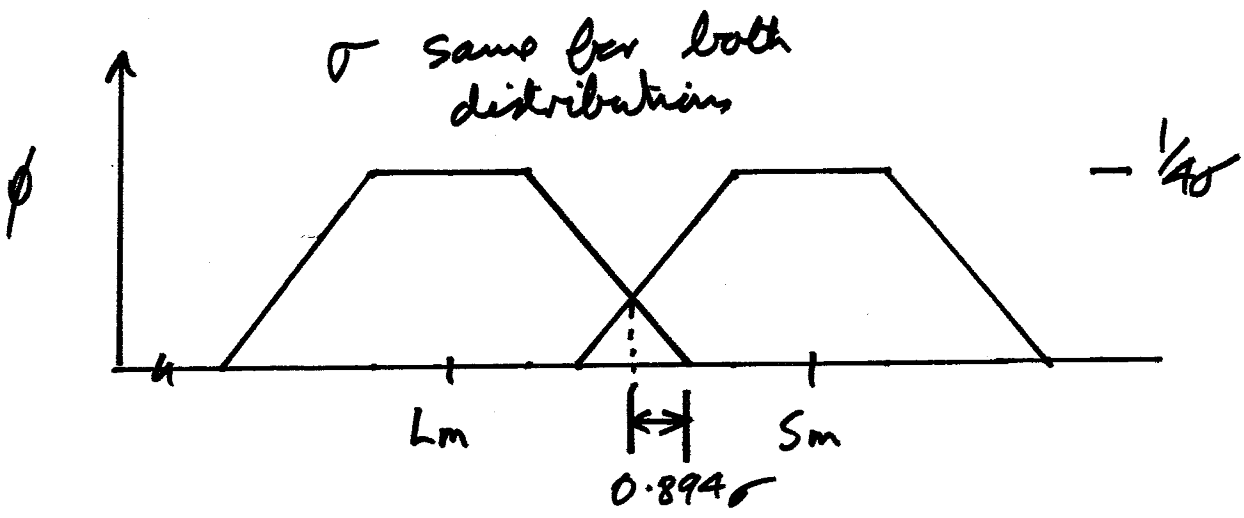
Shaded area $= 5\% \equiv 0{\cdot}2\sigma\phi_m$

$$\frac{\phi_m}{2\sigma} \cdot \frac{x}{2} \cdot x = 0{\cdot}2\sigma\phi_m$$

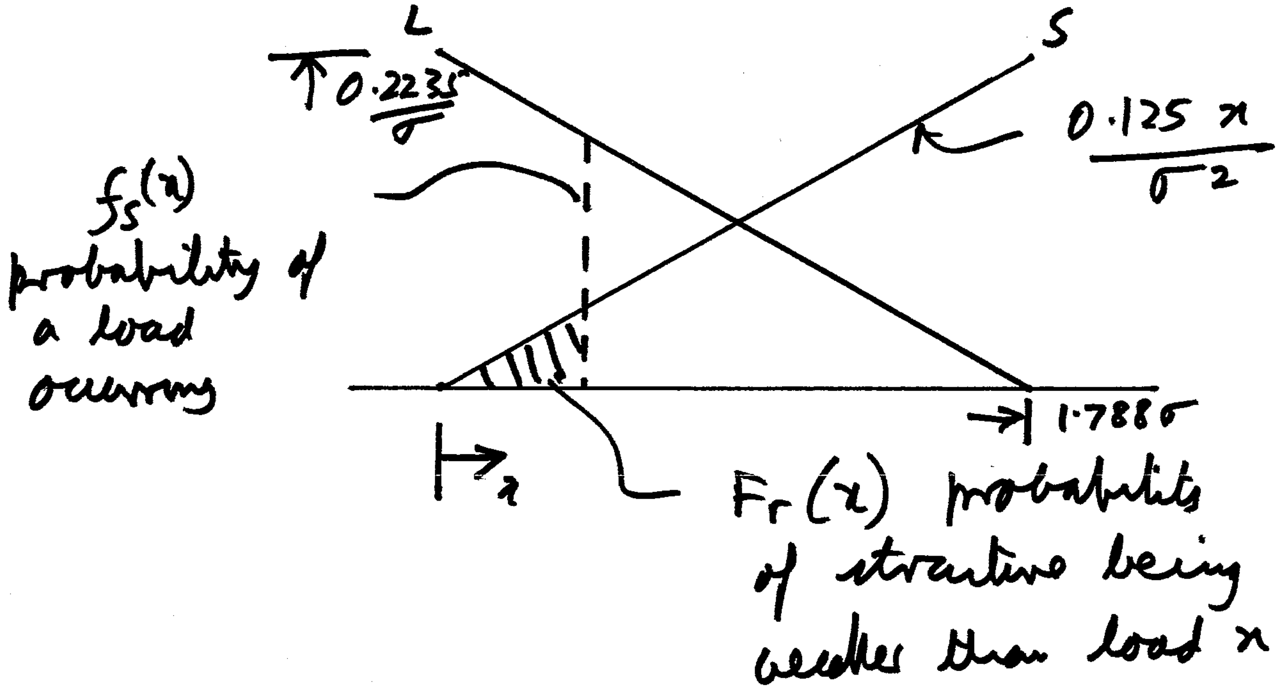
$$\Rightarrow \quad x^2 = 0{\cdot}8\sigma^2$$

$$x = 0{\cdot}894\sigma$$

(b)(ii) Load and strength curves overlap



Only need to consider overlap zone



$$f_S(x) = \frac{0.2235}{\sigma} - \frac{0.125x}{\sigma^2}$$

$$F_R(x) = \frac{x}{2} \cdot \frac{0.125x}{\sigma^2} = \frac{x^2}{16\sigma^2}$$

Convolution $\int_{-\infty}^{\infty} f_s(x) . F_R(x)\, dx$

$$= \int_0^{1.788\sigma} \left(\frac{0.2235}{\sigma} - \frac{x}{8\sigma^2} \right)\left(\frac{x^2}{16\sigma^2} \right) dx$$

Outside this range either f_s or F_R are zero

$$= \left[\frac{0.2235\, x^3}{48\sigma^3} - \frac{x^4}{4.128\,\sigma^4} \right]_0^{1.788\sigma}$$

$$= 0.0216616 - 0.019962$$

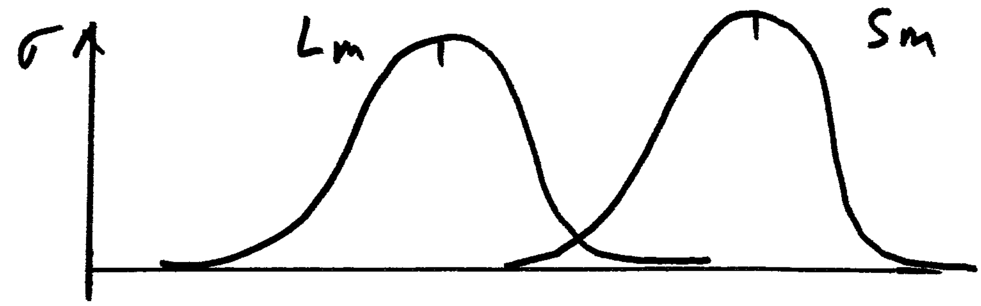
$$= \underline{\underline{0.00665}}$$

Probability can be reduced to zero by making sure the two curves don't overlap

i.e. $S_m > L_m + 6\sigma$

1. Probability of failure. The descriptive parts were done well, and generally showed a level of understanding over and above the bare minimum from the lectured material (and many also covered examples of failure not covered in lectures). However, the main calculation part, (b)(ii), was done very badly. The question had been set so that the calculation was pretty straightforward for anyone who thought clearly, but very few did. There was little attempt to sketch what they were trying to do, and very few made use of the calculation they had performed, in most cases correctly, in b(i). Instead, they put apparently randomly chosen functions into the formula given on the data sheet, took a couple of pages of algebra, and got nowhere.

(iii) When normally distributed



When curves normally distributed probability of failure is never zero because the tails go to infinity

Characteristic value = 1.64σ from mean

$\therefore S_m - L_m = 3.28\sigma$

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{L_m - S_m}{\sqrt{\sigma^2 + \sigma^2}} = \frac{3.28\sigma}{\sqrt{2}\sigma}$$

$= 2.319$

From cumulative distribution function

$P(\text{tail}) = 1 - 0.98983 = \underline{\underline{0.01017}}$

i.e. higher than in (ii)

2 (a) Bookwork

<u>During casting</u> water is required for full hydration. C_3A hydrates first followed by hydration of C_2S & C_3S to give tobomorite gel. Water/cement ratio will influence workability & ultimate strength

<u>Curing</u> Prevent water loss and maintain hydration. If water evaporates plastic shrinkage can occur.

<u>In service</u> Water can lead to deterioration — corrosion of rebar and freeze thaw action on concrete. Water involved in creep & shrinkage mechanisms.

(b) Chloride penetration governed by

$$C = C_0 (1 + \text{erf}(z)) \quad \text{where } z \propto \frac{x}{2\sqrt{Dt}}$$

At $t = 4$ years $Cl^- = 0.2\%$ at 12 mm

How long will it take to reach 0.2% at 30 mm

$$z \propto \frac{x}{2\sqrt{Dt}} \Rightarrow \frac{x}{\sqrt{t}} \text{ is a constant}$$

$$\therefore \frac{30}{\sqrt{t}} = \frac{12}{\sqrt{4}} \Rightarrow t = 25 \text{ years.}$$

Corrosion <u>will not</u> take place at this time.

At $t = 8$ years $Cl^- = 0.6\%$ at 12 mm

$$\therefore \frac{30}{\sqrt{t}} = \frac{12}{\sqrt{8}} \Rightarrow t = 50 \text{ years.}$$

$\therefore$ Corrosion <u>will</u> take place at this time

$$\therefore 25 \leq t_{corr} \leq 50 \text{ years.}$$

Carbonation reached 10 mm after 7 years.

How long will it take to reach 30 mm.

Depth of carbonation $\propto \sqrt{t}$

$$\therefore \frac{10}{\sqrt{7}} = \frac{30}{\sqrt{t}} \Rightarrow t = 63 \text{ years.}$$

$$\therefore \underline{\underline{t_{corr} \leq 63 \text{ years.}}}$$

(ii) $C = C_0 (1 + \text{erf}(Z))$

$Z \propto \frac{x}{2\sqrt{Dt}}$

if x, D the same

$$Z \propto \frac{1}{\sqrt{t}} = \frac{K}{\sqrt{t}}$$

Condition 1. $Z_1 = \frac{K}{\sqrt{4}}$

4 years $Cl^- = 0.2\%$

Condition 2. $Z_2 = \frac{K}{\sqrt{8}}$

8 years $Cl^- = 0.6\%$

$$\therefore \frac{Z_2}{Z_1} = \frac{\sqrt{4}}{\sqrt{8}} = \frac{1}{\sqrt{2}}$$

We know $\frac{0.2}{0.6} = \frac{\cancel{C_0}\,(1 - \mathrm{erf}\, z_1)}{\cancel{C_0}\,(1 - \mathrm{erf}\, z_2)}$

$\therefore\ 0.2(1 - \mathrm{erf}(z_2)) - 0.6(1 - \mathrm{erf}(z_1)) = 0 = F(z)$

Can't solve this directly. Guess values of z_1

Look up $\mathrm{erf}(z_1)$ & $\mathrm{erf}(z_2)$ and hence $F(z)$

z_1	z_2	$\mathrm{erf}(z_1)$	$\mathrm{erf}(z_2)$	$F(z)$
0.5	0.35	0.52	0.38	−0.164
1.0	0.707	0.84	0.68	−0.032
1.5	1.06	0.97	0.87	0.008
1.4	0.99	0.95	0.84	0.002

$\therefore$ solution just less than 1.4

Exact solution calculated accurately

$z_1 = 1.316 \qquad z_2 = 0.9307$

Want $Cl^- = 0.4\%$ at $x = 30$ (z_3)

$\therefore\ \frac{0.2}{0.4} = \frac{C_0\,(1 - \mathrm{erf}(z_1))}{C_0\,(1 - \mathrm{erf}(z_3))}$

$\therefore\ 1 - \mathrm{erf}(z_3) = (1 - 0.937)\cdot\frac{0.4}{0.2} = 0.126$

$\Rightarrow z_3 = 1.09$

$$Z_1 = \frac{K x_1}{\sqrt{t_1}} \qquad Z_3 = \frac{K x_3}{\sqrt{t_3}}$$

$$K = \frac{Z_1 \sqrt{t_1}}{x_1} = \frac{Z_3 \sqrt{t_3}}{x_3}$$

$$\Rightarrow \frac{1.316 \,.\, \sqrt{4}}{12} = \frac{1.09 \sqrt{t_3}}{30}$$

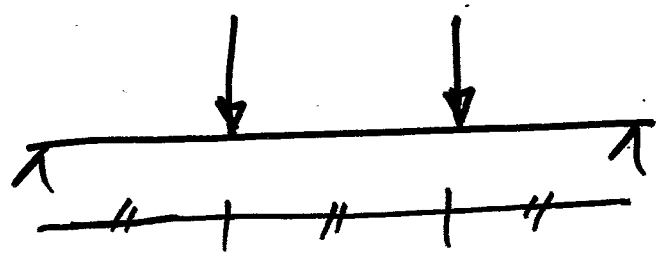
$$\Rightarrow \sqrt{t_3} = 6.04 \qquad \underline{\underline{t_3 = 36 \text{ years.}}}$$

N.B. very difficult for candidates to interpolate precisely using tabulated erf values. Marks were awarded for logic of method rather than precision of solution.

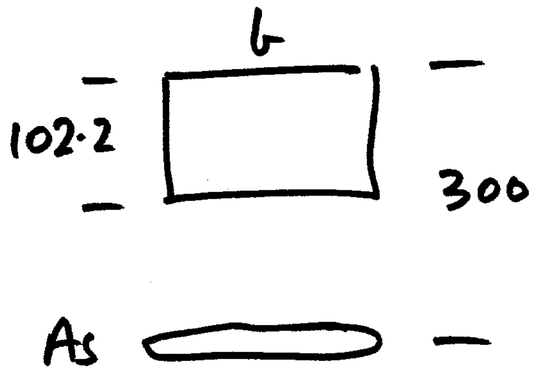
(iii) Corrosion affected by porosity, presence of water, oxygen, chloride, cover, temperature.

2. Diffusion into concrete. Done reasonably well by most, although most marks were earned during the descriptive part (a) rather than the calculation part (b).

3.



(a) Area of steel can be found by considering 1st moment of area in cracked state since steel and concrete are both still elastic



$$m = \frac{E_s}{E_c} = \frac{200}{20} = 10$$

$$102\cdot2 \times 200 \times \frac{102\cdot2}{2} = (300 - 102\cdot2) \cdot 10 \cdot A_s$$

$$\Rightarrow A_s = 528 \text{ mm}^2$$

$$\rho = \frac{A_s}{bd} = \frac{528}{200 \cdot 300} = \underline{0.88\%}$$

(ii) Uncracked stiffness (ignoring steel)

$$= \frac{bh^3}{12} = \frac{200 \cdot 330^3}{12} = \underline{599 \cdot 10^6 \text{ mm}^4}$$

$$\zeta = 1 - \beta \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \quad \text{(data sheet)}$$

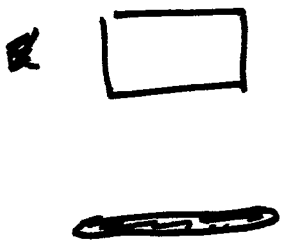
σ_{sr} & σ_s will be proportional to the moments. Take $\beta = 1.0$ (short term)

$$\therefore \zeta = 1 - \left(\frac{17.4}{48.6}\right)^2 = 0.872$$

$$\alpha = \underbrace{\zeta \alpha_{II}}_{\text{cracked}} + \underbrace{(1-\zeta)\,\alpha_{I}}_{\text{uncracked.}}$$

$$\alpha_I = \frac{M}{E_c I_{un}} = \frac{48.6 \cdot 10^6}{20000 \cdot 599 \cdot 10^6} = 4.06 \cdot 10^{-6} \ (\text{mm}^{-1})$$

200



$\overline{102.2}$

10×528

$\uparrow$ m $\quad$ A_s

$$\Rightarrow \quad I_{cr} = \frac{102.2^3 \cdot 200}{12} + \left(\frac{102.2}{2}\right)^2 \cdot 102.2 \cdot 200 + 10 \cdot 528 \cdot (300 - 102.2)^2$$

$$= \underline{\underline{277.8 \cdot 10^6 \text{ mm}^4}}$$

$$\alpha_{II} = \frac{M}{E_c I_{cr}} = \frac{48.6 \cdot 10^6}{20000 \cdot 277.8 \cdot 10^6}$$

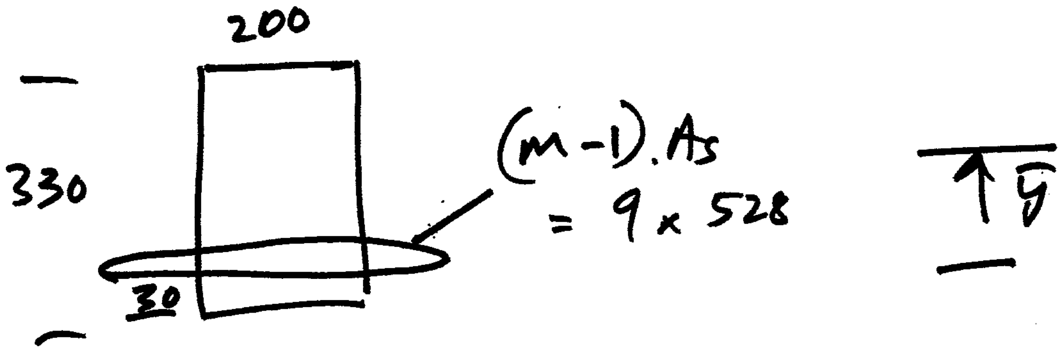
$$= 8.747 \cdot 10^{-6}$$

$$\therefore \text{Curvature} = \zeta \alpha_{II} + (1-\zeta)\alpha_I$$

$$= 0.872 \cdot 8.747 \cdot 10^{-6} + (1 - 0.872) \cdot 4.06 \cdot 10^{-6}$$

$$\underline{\underline{= 8.147 \cdot 10^{-6}}} \quad mm^{-1}$$

(iii) If steel is taken into account in the uncracked section, n.a not at centre



$$\bar{y} = \frac{200 \cdot \frac{330^2}{2} + 9 \cdot 528 \cdot 30}{200 \cdot 330 + 9 \cdot 528} = 156 \text{ mm.}$$

$$I_{un} = \frac{200 \cdot 330^3}{12} + (165 - 156)^2 \cdot 200 \cdot 330$$

$$+ (156 - 30)^2 * 9 \cdot 528$$

$$= 680 \cdot 10^6 \text{ mm}^4$$

[Check. f_t for concrete = 4 MPa

Expected cracking load $\frac{\sigma}{y} = \frac{M}{I}$

$$M_{cr} = \frac{4 \cdot 680 \cdot 10^6}{156} = 17.4 \text{ kNm} \checkmark$$]

I_{un} is 13% greater than before but this will only change expected curvature by ≈ 1% since the effect of the uncracked stiffness is reduced by the $(1-\zeta)$ term.

3. Calculation of Stiffnesses. The least popular question, possibly because of the length of the actual question caused by the amount of data provided. There was no descriptive part. Clearly the last question attempted; many of the solutions were incomplete rather than wrong.

(b) Shear force = P = 150 kN

$$A_{sw} = \frac{\pi 8^2}{4} \times \underset{\substack{\uparrow \\ \text{legs}}}{2} = 100.5\ mm^2$$

$f_{c,max}$ = 8 MPa (specified)

steel $$V_{rd,s} \leq \frac{A_{sw} f_y . 0.9d . \cot\theta}{s \gamma_s}$$

$$\leq \frac{100.5 . 475 . 0.9 . 300 . \cot\theta}{100}$$

$$\Rightarrow \cot\theta \geq 1.16$$

$$\therefore \tan\theta \leq \frac{1}{1.16} \leq 40.7^\circ$$

Concrete

$$V_{rd,max} \leq \frac{f_{c,max} . t_w . 0.9d}{\cot\theta + \tan\theta}$$

$$\leq \frac{8 . 200 . 0.9d}{\cot\theta + \tan\theta}$$

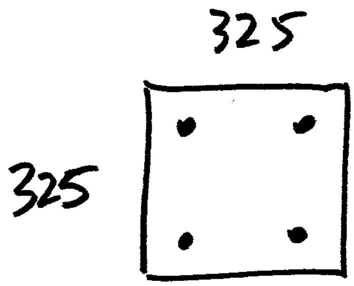
$$\cot\theta + \tan\theta \leq 2.88$$

$$\Rightarrow \tan^2\theta - 2.88\tan\theta + 1 \leq 0$$

$$\therefore \tan\theta \geq 22^\circ$$

$$\therefore 22^\circ \leq \tan\theta \leq 40.7^\circ$$

4 (a)



32 mm bar area = 804 mm^2 per bar

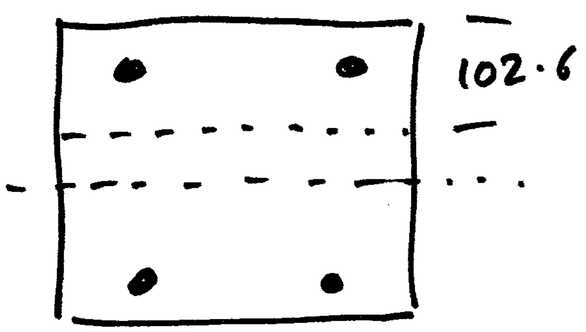
A_{st} total = 3217 mm^2

f_{yd} = 410 MPa
f_{cd} = 30 MPa — facts at $0 \cdot 6 f_{cd}$

(i) N = 600 kN. When in loading, at ult load, two bars yield in tension & two in compression.

$\therefore$ depth of compression zone (x) not affected by steel.

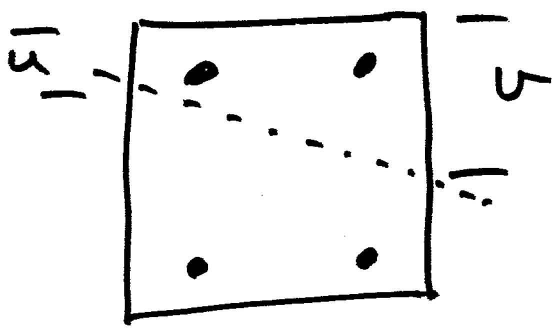
$$\therefore \quad x = \frac{600000}{0 \cdot 6 \cdot 30 \cdot 325} = 102 \cdot 6 \text{ mm}$$



$$M_{xx} = 2 . A_s f_y (d - d') + 0 \cdot 6 f_{cd}\, b x \left(\frac{h}{2} - \frac{x}{2}\right)$$

$$= 228 \text{ kNm}$$

(ii)



Assume n.a. such that 2 bars in tension & 2 in compression.

$$\therefore \left(\frac{u+v}{2}\right).0{\cdot}6.30.325 = 600000$$

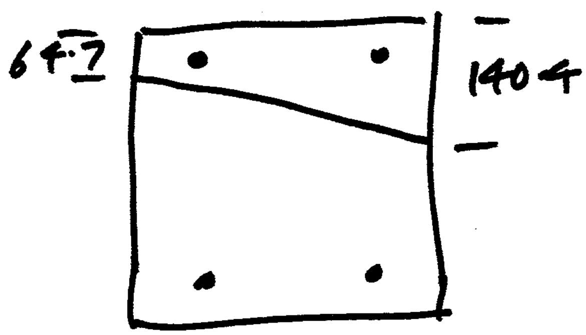
$$\Rightarrow u+v = 205 \text{ mm} \quad \left(= 2 \times 102{\cdot}6 \text{ as expected}\right)$$

$$M_y = \frac{(v-u).325}{2}.0{\cdot}6.30\left(\frac{325}{2} - \frac{325}{3}\right)$$

$$= 158.10^3(v-u) = 12.10^6 \text{ Nmm}$$

$\therefore$ Solve for u & v $\quad u = 64.7$ mm, $v = 140.4$ mm

(check - assumption above is valid).



$$M_x = 2.804.410(285-40)$$

$$+ 0{\cdot}6.30\left[325.64{\cdot}7\left(\frac{325}{2} - \frac{64{\cdot}7}{2}\right)\right.$$

$$+ 0{\cdot}6.30 \times \frac{1}{2} \times 325 \times (140{\cdot}4 - 64{\cdot}7)$$

$$\left.\times\left[\frac{325}{2} - 64{\cdot}7 - \frac{1}{3}(140{\cdot}4 - 64{\cdot}7)\right]\right]$$

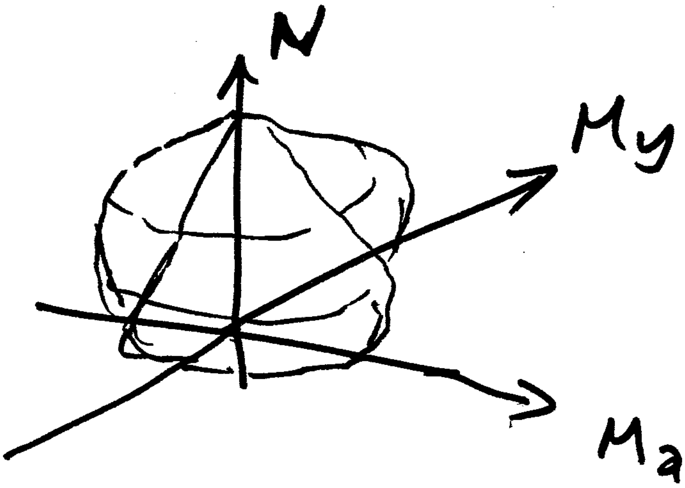
$$= 226{\cdot}8.10^6 \text{ Nmm}$$

$\therefore$ Just less than before.

Biaxial approximation takes the form

$$\left(\frac{M_x}{M_{ux}}\right)^n + \left(\frac{M_y}{M_{uy}}\right)^n \leq 1$$

The coefficient n depends on axial load and leads to a 3D surface



(b) Bookwork.

4. Column stress analysis. Generally done well. The commonest mistake was taking moments about the wrong axis. When an axial load is present, as here, it matters!