IIB 2011

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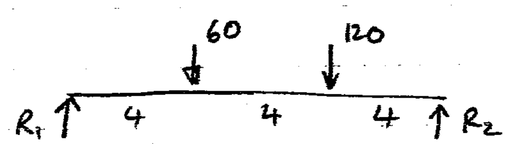
4D10 Structural Steelwork ~~2010~~.

Q1. a) When plotting $\bar{\lambda}$ versus $\chi$ where $\chi$ = Critical / Plastic
then obviously when critical = plastic, then $\chi = 1$
but also, when critical = elastic,
then $y = \frac{\text{elastic}}{\text{plastic}}$ and $x = \sqrt{\frac{\text{plastic}}{\text{elastic}}}$ so $y \sim 1/x^2$

i.e. $\chi$ falls away as $1/\bar{\lambda}^2$, even though LTB eqn does not fall away as $1/L^2$ $\left( M_{LT} = \frac{\pi}{L}\sqrt{GJEI}\left(1 + \frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{1/2} \sim \frac{A}{L}\left(1 + \frac{B}{L^2}\right)^{1/2} \right)$

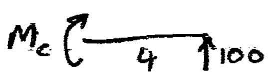
This has the advantage that the same format graph can be used for beams as is traditionally used for columns, (even though beam LTB does not go as $1/L^2$). EC3 exploits this and actually uses Perry-Robertson curves for plastic-elastic interaction that were developed for columns, and uses them for beam design.

b)



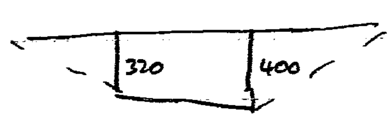

$12R_2 = 8(120) + 4(60) = 1200$
$\Rightarrow R_2 = 100$ kN
$R_1 = 180$ kN $- 100$ kN $= 80$ kN


$M_B = 4(80) = 320$ kNm.




$M_c = 4(100) = 400$ kNm.

Critical segment is BC




$\psi = \frac{320}{400} = 0.8$

$C_{unequal} = 0.6 + 0.4\psi = 0.6 + 0.4(0.8) = \underline{\underline{0.92}}$

533 x 210 x 92 UB, S275 MPa

$M_{pl} = (275 \times 10^6 \text{ N/m}^2) \times (2360 \times 10^{-6} \text{ m}^4)$ ← $Z_{pl,major}$
$= \underline{649 \text{ kNm}}$

4D10, Q1 cont'd.

$$M_{LT} = \frac{\pi}{L}\sqrt{GJEI}\left(1+\frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{1/2} \qquad G = \frac{E}{2(1+\nu)} = \frac{E}{2\cdot6} \text{ for steel } (\nu = 0.3)$$

$J = 75.7 \times 10^{-8}\ m^4$ (SDB)

$I = 2389 \times 10^{-8}\ m^4$ (SDB, minor axis)

$\Gamma = \frac{I_{minor} D^2}{4}$ , $D = 533.1 - 10.1 = 523\ mm = 0.523\ m$

$$\Gamma = \frac{(2389 \times 10^{-8})(0.523)^2}{4} = 163.4 \times 10^{-8}\ m^4$$

$$\frac{\pi^2 E\Gamma}{L^2 GJ} = \frac{\pi^2 (2.6)}{L^2}\frac{\Gamma}{J} = \frac{\pi^2}{4^2}(2.6)\left(\frac{163.4}{75.7}\right) = 3.461$$

$$\sqrt{1+\frac{\pi^2 E\Gamma}{L^2 GJ}} = \sqrt{1+3.461} = \underline{\underline{2.112}}.$$

$$\frac{\pi}{L}\sqrt{GJEI} = \frac{\pi}{L}E(10^{-8})\sqrt{\frac{(75.7)(2389)}{2.6}} = \frac{\pi}{4}(210 \times 10)(263.7)$$

$$= \underline{\underline{435\ kNm}}$$

$$M_{LT} = 435\ kNm \times 2.112 = \underline{\underline{918.8\ kNm}}$$

$$M_{cr} = M_{LT}/C_{unequal} = 918.8/0.92 = \underline{\underline{999\ kNm}}$$

$M_{pl} = 649\ kNm.$ $\quad \bar{\lambda} = \sqrt{\frac{M_{plastic}}{M_{elastic}}} = \sqrt{\frac{649}{999}} = 0.806$

Rolled I-sections: $h/b = \frac{533.1}{209.3} = 2.547$ so $h/b > 2$

$\rightarrow$ so use curve (b), where $\bar{\lambda} = 0.806 \rightarrow \chi = 0.72$

so $M_{\substack{max\\allowable}} = \chi M_{pl} = 0.72(649) = \underline{\underline{467\ kNm}}$

cf. 400 kNm max applied $\therefore$ <u>Beam is adequate</u>

(need to check shear, and to check all local elements at least Class 2 compact).

Q1 examiner's comment:

A popular question in which candidates were asked to explain the underlying definition of slenderness for beams and columns according to the Eurocode practice, making reference to its simplicity and ubiquity in design, before determining whether a certain loaded beam would be safe. The main errors were: not giving a full explanation of the slenderness methodology; stating the right choice of buckling curve but reading from the wrong one; and specifying "*D*" incorrectly in the calculation of the critical moment.

Q2. 4D10, 2011.

a) 356 × 406 × 235 UC. S275.

$Z_{major} = \cancel{2383}\ 4687 \times 10^{-6}\ m^3$ $\quad \sigma_y = 275$ MPa.

$M_{pl} = 275 \times 10^6\ N/m^2 \times \cancel{2383}\ 4687 \times 10^{-6}\ m^3$

$= \cancel{658\ kNm}\ .\ \underline{\underline{1289\ kNm}}$

$N_{pl} = A\sigma_y = 299 \times 10^{-4} m^2 \times 275 \times 10^6\ N/m^2$

$= \underline{\underline{8222\ kN}}$

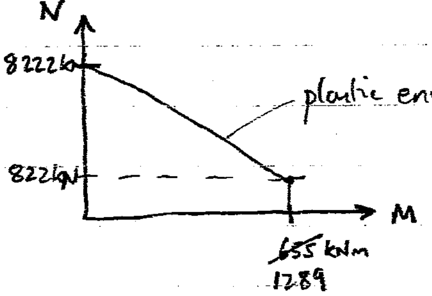
Web fraction : Area $A = 299\ cm^2$

Flange area $= 2 \times \underset{T}{3.02\ cm} \times \underset{B}{39.48\ cm} = 238.5\ cm^2$

$a = \text{Web fraction} = \frac{299-238}{299} = \frac{61}{299} = \underline{\underline{0.2040}}$

$\underline{\underline{a/2 = 0.102}}$ (say 0.1)

(so y-coord $= a/2 \times N_{pl} = 0.1 \times 8222\ kN = \underline{\underline{822\ kN}}$




b). Axial effects.

$N_{pl} = 8222\ kN.$

$I_{minor} = 30990 \times 10^{-8}\ m^4$

$N_{Euler} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (210 \times 10^9)(30990 \times 10^{-8})}{(3)^2} \left[\frac{N/m^2 \cdot m^4}{m^2}\right] = [N]$

$= \underline{\underline{71,367\ kN}}$

slenderness $\bar{\lambda} = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{8222}{71\,367}} = \underline{\underline{0.3394}}$

DS2 : → $h/b = \frac{381}{394.8} = 0.965 \leq 1.2.$

$t_f = 30.2\ mm \leq 100\ mm$

Buckling about z-z :→ Curve c. → $\chi = 0.94$ for $\bar{\lambda} = 0.3394$

→ $N_{cr} = \chi N_{pl} = 0.94\,(8222) = \underline{7729\ kN}$

Moment effects.

$M_{pl\,(major)} = 1289$ kNm.

Need $M_{LT} = \frac{\pi}{L}\sqrt{GJEI}\left(1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right)^{1/2}$

$J = 812\ cm^4 = 812 \times 10^{-8}\ m^4$

$I_{minor} = 30990\ cm^4 = 30990 \times 10^{-8}\ m^4$

$G = \frac{E}{2(1+\nu)} = \frac{E}{2.6}$ (for $\nu = 0.3$).

$\frac{\pi}{L}\sqrt{GJEI} = \frac{\pi}{L}\frac{E}{2.6}\sqrt{JI} = \frac{\pi}{3}\frac{210 \times 10^9\ N/m^2}{(2.6)}\sqrt{(812 \times 10^{-8})(30990 \times 10^{-8})}\ \frac{m^4}{m}$ $\times 10^{-8}$

$= \underline{4243\ kNm}$ ~~Nm~~

$\Gamma = \frac{ID^2}{4}$ with $D = 381 - 30.2$ mm $= 350.8$ m $= 0.3508$ m

$\Gamma = \frac{30990 \times 10^{-8} \times (0.3508)^2}{4} = 2422 \times 10^{-8}\ m^6$

$\frac{\pi^2 E\Gamma}{L^2 GJ} = \frac{\pi^2(2.6)}{(3)^2}\left[\frac{2422}{812}\right] = \underline{8.50}$

$\left(1 + \frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{1/2} = \sqrt{9.50} = \underline{3.083}$

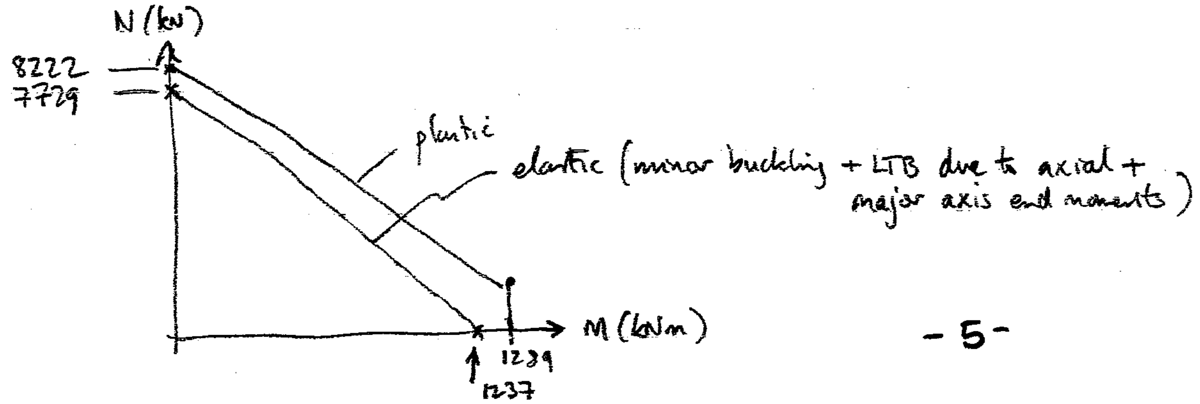
$\rightarrow M_{LT} = 4243\ kNm \times 3.083 = \underline{13\,080\ kNm}$

$\bar{\lambda} = \sqrt{\frac{Plastic}{Elastic}} = \sqrt{\frac{1289}{13080}} = \underline{0.314}$

Which curve? $h/b = 0.965$ still, so $h/b \le 2$ so DS3 $\rightarrow$ Curve (a).

$\rightarrow \chi = 0.96$ (DS1)

so $M_{Design} = \chi M_{pl} = 0.96\,(1289) = \underline{1237\ kNm}$.

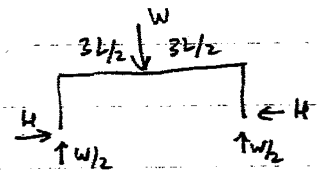


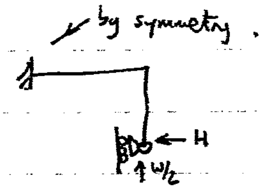
Q2 Examiner's comment:

Another popular question in which candidates were asked to construct the interaction diagram between axial force and major axes bending moment for a beam under plastic and elastic considerations. The main errors were: numerical miscalculations; improper sketching of the interaction graphs.

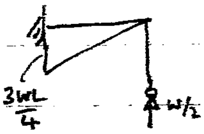
Q3

a) Solution by ~~work~~ virtual work:

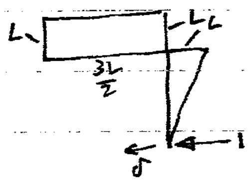



Pick H as redundancy: Statically determinate system. ⓪




①




Real system = ⓪ + H①  Dot with ①. real = ⓪ + H① compat.

$$F.\Delta = \int \frac{mM}{EI}\,dx$$

virt. equilib ①

$$\cancel{1.\delta = \int_0^L \frac{\times 0}{EI}\,dx + \int_0^{3L/2} \frac{\frac{3WL}{4} \times L}{EI}\,dL}$$

$$= \cancel{\frac{3WL^2}{4} \times \frac{1}{2} \times \frac{3L}{2}} = \cancel{\frac{9WL^3}{16}}$$

$$1.0 = \int \left( \quad + H \times \quad \right) \times \left( \quad \right) \frac{1}{EI}\,dx$$

So

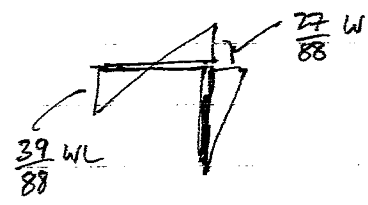
$$H = \frac{-\int \quad \times \quad dx}{\int \quad \times \quad dx} = \frac{\left(\frac{3WL}{4}\right) L \times \frac{1}{2} \times \frac{3L}{2}}{\left(\frac{3L}{2}\right) L^2 + \frac{1}{3}.L^2 \times L}$$

$$= \frac{\frac{9WL^3}{16}}{\left(\frac{3}{2}+\frac{1}{3}\right)L^3} = \frac{9WL^3}{16L^3\left(\frac{9+2}{6}\right)} = W\frac{9 \times 6^3}{16_8 \times 11} = \underline{\underline{\frac{27W}{88}}}$$

So BM at D (midspan) $= -\frac{3WL}{4} + \frac{27WL}{88} = -\frac{3WL}{4}\left(1 - \frac{9}{22}\right) = -\frac{3WL}{4}\left(\frac{22-9}{22}\right) = -\frac{3(13)}{88}WL$

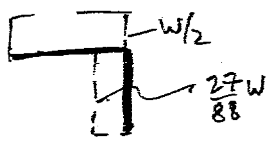
$$= -\frac{39}{88}WL \quad \text{sagging.}$$

BM at C $= \frac{27}{88}WL$




BMD.

Shear Force.



Q3 cont'd.

b) For whole web: $\sigma_y = 355$ MPa.

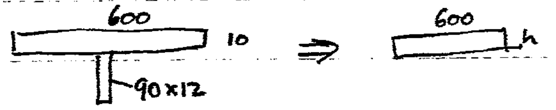
$$\lambda_{web} = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}} = \frac{900}{10} = 90 > 56 \quad \text{so need stiffeners} \quad (DS4).$$

Stiffeners: $\lambda_{stiff} = \frac{b}{t}\sqrt{1} = \frac{90}{12} = 7.5 \leq 8$ so stiffeners are compact (DS4).

Need $I_{xx}$, $Z_e$ and $A$ for cross-section.

Estimate via smeared section:

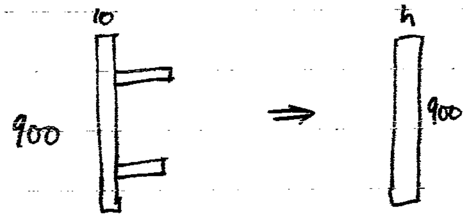
flange:




$600h = 600 \times 10 + 90 \times 12$

$\Rightarrow \underline{h = 11.8 \text{ mm}}$

web

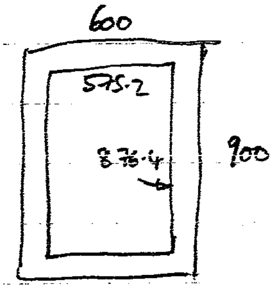



$900h = 900 \times 10 + 2(90 \times 12)$

$\Rightarrow \underline{h = 12.4 \text{ mm}}$

and $600 - 2 \times 12.4 = 575.2$ mm

$900 - 2 \times 11.8 = 876.4$ mm $\Rightarrow$




$$I_{major} = \left(\frac{bd^3}{12}\right)_{ext} - \left(\frac{bd^3}{12}\right)_{int} = \frac{1}{12}\left[600(900)^3 - 575.2(876.4)^3\right] = 4184 \times 10^6 \text{ mm}^4$$

$$A = 600(900) - (575.2)(876.4) = 35{,}895 \text{ mm}^2$$

$$Z_e = \frac{I_{major}}{y_{max}} = \frac{4184 \times 10^6 \text{ mm}^4}{450 \text{ mm}} = 9.298 \times 10^6 \text{ mm}^3$$

Stress at ~~centre~~ midspan of beam, in extreme fibre

$$\sigma_{max} = \frac{|M|}{Z} + \frac{P}{A} = \frac{39}{88}\frac{(1000 \times 10^3 \text{ N})\, 6 \times 10^3 \text{ mm}}{9.298 \times 10^6 \text{ mm}^3} + \frac{27}{88}\frac{(1000 \times 10^3)\text{ N}}{35{,}895 \text{ mm}^2}$$

$$= 286 \text{ N/mm}^2 + 8.55 \text{ N/mm}^2$$

$$= \underline{295 \text{ MPa}}$$

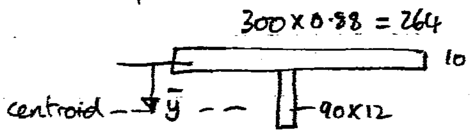
b) i) Top flange: $\lambda = \frac{300}{10}\sqrt{1} = 30 > 24$ for compression $\therefore$ $\underline{\underline{\text{not}}}$ compact

between stiffeners

Q3 cont'd.

Not compact, so use $b_e = K_c b$. DS4(a) for $\lambda = 30 \Rightarrow K_c \sim 0.88$

so, effective T-section for compression flange is




$$A_T = 264 \times 10 + 90 \times 12 = 3720 \text{ mm}^2.$$

$$A_T \bar{y} = (90 \times 12) \times (45+5) = 54000 \text{ mm}^3$$

$$\Rightarrow \bar{y} = \frac{54000}{3720} = 14.52 \text{ mm}$$

$$I_T = \frac{1}{12}(264)(10)^3 + (264)(10)(14.52)^2 + \frac{1}{12}(12)(90)^3 + 90(12)\left[50 - 14.52\right]^2$$

$$= 22{,}000 + 556{,}600 + 729{,}000 + 1{,}359{,}500$$

$$= \underline{\underline{2.667 \times 10^6 \text{ mm}^4}}$$

$\bar{\lambda}$ for DS1 $= \sqrt{\dfrac{\text{Plastic}}{\text{Elastic}}}$

$$N_{pl} = 3720 \text{ mm}^2 \times (355 \text{ N/mm}^2) = 1321 \text{ kN}$$

$$N_{elastic} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 (210 \times 10^3)(2.667 \times 10^6)}{(1000)^2} \quad \frac{\text{N/mm}^2 \text{ mm}^4}{\text{mm}^2}$$

$$= 5528 \text{ kN}.$$

$$\bar{\lambda} = \sqrt{\frac{\text{Plastic}}{\text{Elast}}} = \sqrt{\frac{1321}{5528}} = 0.49.$$

Curve: ~~Welded box → curve (b)~~. Welded I-section: (DS2)

$t_f \leq 40$ mm, z-z → Curve (c).

DS1, for $\bar{\lambda} = 0.49 \rightarrow \chi = \underline{0.85}$.

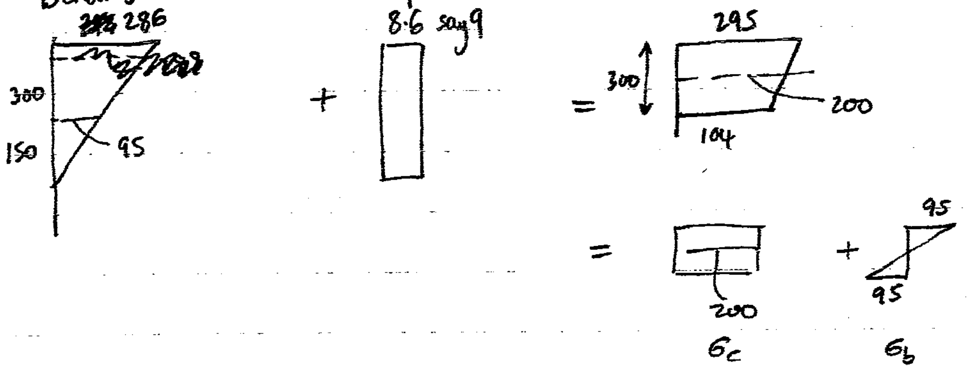
so $\sigma_{cr} = 0.85 \times (355 \text{ MPa}) = \underline{\underline{302 \text{ MPa}}}$

c.f. 295 MPa from earlier calc.

Apparently OK, (since 302 MPa > 295 MPa), but too close, given that basic section properties not reduced in previous calcs.

Q3 cont'd.).

b ii). Most heavily stressed panel is top panel.




$\lambda_{panel} = \frac{300}{10}\sqrt{1} = 30$ as before.

DS4 $\Rightarrow$ $K_c = 0.88$ , $K_b = 1.22$ , $K_\tau = 1.0$.

$\phi = \frac{1000}{300} = 3.3$, use $\phi = 3$.

Strength: $\sigma \leq \sqrt{\sigma_Y^2 - 3\tau^2}$ ?

$\tau = 0$ at centre ? (Max BM ? )

No: $\underset{\text{shearforce}}{\tau = W/2} \times \frac{1}{\text{area of webs}} = \frac{1000 \times 10^3 \text{ N}}{2(2 \times 900 \times 10) \text{ mm}^2} = 27.8$ MPa.

$295 < \sqrt{(355)^2 - 3(27.8)^2} = 351$ so OK.

Stability check: $\frac{\sigma_c}{\sigma_{cc}} + \left(\frac{\sigma_b}{\sigma_{bc}}\right)^2 + \left(\frac{\tau}{\tau_c}\right)^2 \leq 1$

$$\frac{200}{(0.88)(355)} + \left(\frac{95}{(1.22)(355)}\right)^2 + \left(\frac{27.8}{(355/\sqrt{3})}\right)^2$$

$$0.64 + 0.05 + 0.02$$

$$= 0.71 < 1 \quad \therefore \text{ OK.}$$

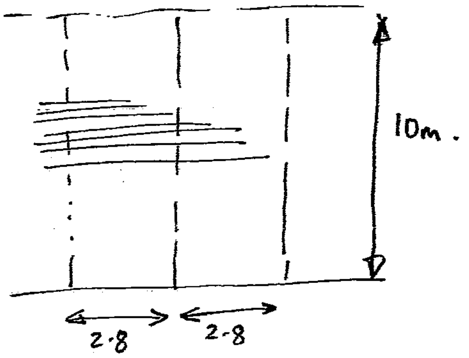
(As one would expect ... given that compression flange barely makes it.).

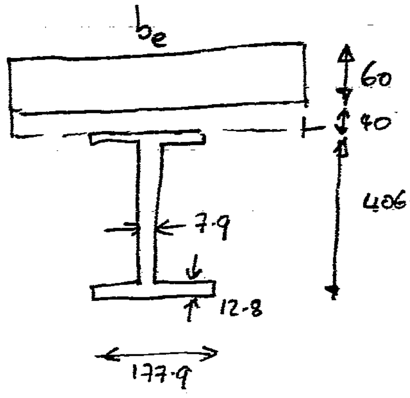
Candidates were asked to determine the adequacy of a plate girder section for a specified set of loads. The main errors were: about half of the candidates did not correctly calculate the normal stresses due to bending and axial compression, despite being given the bending moment distribution; miscalculation of $I$ for the T-section; ignoring the shear stresses in the stability calculation; misreading the appropriate knockdown curve. Not a very popular question.

4D10

Q4. a)

PLAN




SDB: $A_{steel} = 76.5\,cm^2$ ; mass/m = 60.1 kg/m $\quad I_{major} = 21600\,cm^4$.

Check for compactness: $\lambda_{flange} = \left(\frac{177.9 - 7.9}{2}\right)\frac{1}{12.8}\sqrt{\frac{275}{355}} = 5.84$ ($< 8$ ∴ OK).

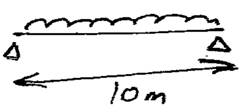
$\lambda_{web} = \left(\frac{406.4 - 2\times 12.8}{7.9}\right)\sqrt{\frac{275}{355}} = 42.4$ ($< 56$ ∴ OK).

Effective concrete: smallest of $b = 2.8m$ or $\frac{span}{4} = 2.5m$ $\Rightarrow b_e = 2.5m$. (DS6)

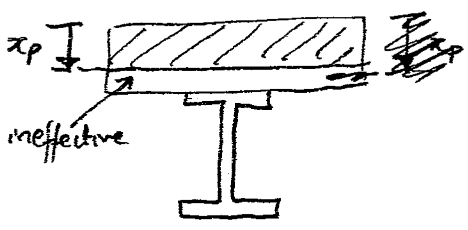
Loads:
- Slab $= 24\,kN/m^3 \times \left(0.06 + \frac{0.04}{2}\right) \times 2.8m = 5.38\,kN/m$
- UB $= 0.59\,kN/m$
- Services $3\,kN/m^2 \times 2.8m = 8.4\,kN/m$
- Imposed load $5.5\,kN/m^2 \times 2.8m = 15.4\,kN/m$.

Total permanent load $= (5.38 + 0.59 + 8.4) = 14.4\,kN/m$
$\times 1.35$ factor $\Rightarrow 19.4\,kN/m$.
imposed load $= 15.4 \times 1.5 \Rightarrow 23.1\,kN/m$

$\left.\right\}$ $42.5\,kN/m$




$$BM = \frac{wl^2}{8} = \frac{(42.5)10^2}{8} = 531\,kNm$$




Assume Neutral Axis in Concrete at depth $x_p$.

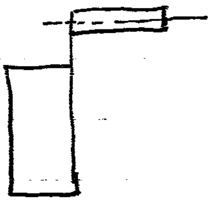
Axial Equilib: $A_s\sigma_y = 0.6 f_{cd} b_e x_p$.

$$(76.5\times10^{-4})(275\times10^6) = 0.6(30\times10^6)(2.5)x_p$$

$$\rightarrow x_p = \frac{(76.5\times10^{-4})(275)}{0.6(30)(2.5)} = 46.8\,mm$$

(within concrete ✓). ($< 60mm$)

Q4a) Cont'd.



← Take moments about here

$$M_{design} = A_s \sigma_y \left(\frac{D}{2} + h_c - \frac{x_p}{2}\right)$$

$$= (76.5\times10^{-4})(275\times10^{6})\times\left[\underbrace{\frac{0.406}{2} + 0.1 - \frac{0.0468}{2}}_{0.28}\right]$$

$$= \underline{\underline{588.2 \text{ kNm}}}$$

so $M_D > M_{\substack{max \\ applied}}$ $(588.2 > 531 \checkmark)$.

b) Studs: Shear strength $\underline{\underline{P_d = 47 \text{ kN}}}$ from DS6

Axial force in concrete $= A_s \sigma_y = (76.5\times10^{-4})(275\times10^{6}) = \underline{\underline{2104 \text{ kN}}}$

No. of studs $\geq 2\times\frac{A_s\sigma_y}{P_d} = \frac{2\times 2104}{47} = 89.5 \Rightarrow$ $\underline{\underline{\text{say } 90}}$ (even)

Spacing $= \frac{10\text{m}}{90} = 111$ mm — but this interferes with trough spacing

Put studs in pairs in the trough (but pairing reduces strength by 80%)

so need $\frac{90}{0.8} = 114$ studs, or 57 pairs, needed.

Trough spacing $= 70+10+70+10 = \underline{\underline{160\text{mm}}}$

$\Rightarrow \frac{10\text{m}}{0.16\text{m}} = 62$ troughs available

so put a pair of studs in each trough → more than enough (62 provided, 57 required).

4D10

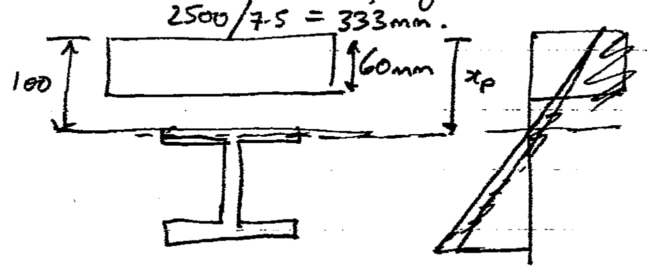
Q4 c) Short term imposed load:

$E_c = 28\,GPa$ (DS6)

Modular ratio $\frac{E_s}{E_c} = \frac{210}{28} = 7.5$.

Transform to steel section.

Assume Neutral axis in steel ~~flange~~..




$$x_p\,(333 \times 60 + 7650) = \left[333 \times 60 \times 30 + 7650 \times \left(100 + \frac{406.4}{2}\right)\right]$$

Conc steel Conc steel

$$= \frac{599{,}400 + 2{,}319{,}000}{27{,}630} = \frac{2{,}918{,}800}{27{,}630}$$

$$= \underline{105.6\text{ mm}}$$ (which is in steel flange).

$$I_{xx} = \frac{1}{12}\left[333\,(60)^3\right] + (333)(60)(105.6-30)^2 + 21600 \times 10^4 + (7650)\left(\frac{406.4}{2} - 5.6\right)^2$$

$$= 5.994 \times 10^6 + \overset{114.2}{\cancel{36.4}} \times 10^6 + 216 \times 10^6 + 298.7 \times 10^6$$

$$= 634.9 \times 10^6\ \text{mm}^4$$

$$\delta = \frac{5wL^4}{384\,E_{steel}\,I_{xx}} = \frac{5\,(15.4 \times 10^3)\,(10)^4}{384\,(210 \times 10^9)(634.9 \times 10^6)} = \underline{15\text{ mm}}$$

$$\frac{Span}{250} = \frac{10\text{m}}{\cancel{\phantom{00}}\,250} = 40\text{mm} \quad (> 15\text{mm} \quad \therefore OK).$$

A regular question answered very well by most of the candidates. The main errors were: failing to check for compactness of the supporting steel section; not understanding fully the trough spacing requirements; an incorrect choice of transformed section (by using the elastic depth from earlier rather than simply ignoring the trough region).