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1. (a) i) Integrating

$$B_2 - B_1 = \int_{T_1}^{T_2} \left(1 - \frac{T_o}{T}\right) dQ$$

From the first law of thermodynamics

$$\delta Q = \dot{m} c_p \delta T$$

Where $\dot{m}c_p$ is the heat capacity flow rate, F , hence.

$$B_2 - B_1 = \int_{T_1}^{T_2} F \left(1 - \frac{T_o}{T}\right) dT = F \left[(T_2 - T_1) - T_o \ln \left(\frac{T_2}{T_1}\right) \right]$$

The cold stream undergoes a change in availability of

$$\begin{aligned} B_{2c} - B_{1c} &= \int_{T_{1c}}^{T_{2c}} \left(1 - \frac{T_o}{T}\right) F_c dT \\ &= F_c [T - T_o \ln T]_{T_{1c}}^{T_{2c}} \\ &= F_c \left[(T_{2c} - T_{1c}) - T_o \ln \left(\frac{T_{2c}}{T_{1c}}\right) \right] \end{aligned}$$

Where F_c is the heat capacity flow rate for the cold stream. Similarly for the hot stream which is cooling down

$$B_{2h} - B_{1h} = F_h \left[(T_{2h} - T_{1h}) - T_o \ln \left(\frac{T_{2h}}{T_{1h}}\right) \right]$$

Overall, the availability balance for the heat exchanger is given by

$$0 = -B_{2c} + B_{1c} - B_{2h} + B_{1h} - T_o \Delta S_{irrev}$$

$$T_o \Delta S_{irrev} = -B_{2c} + B_{1c} - B_{2h} + B_{1h}$$

$$T_o \Delta S_{irrev} = -F_h \left[(T_{2h} - T_{1h}) - T_o \ln \left(\frac{T_{2h}}{T_{1h}}\right) \right] - F_c \left[(T_{2c} - T_{1c}) - T_o \ln \left(\frac{T_{2c}}{T_{1c}}\right) \right]$$

Comment [S1]: The inlet and outlet are labelled 1 and 2 respectively.

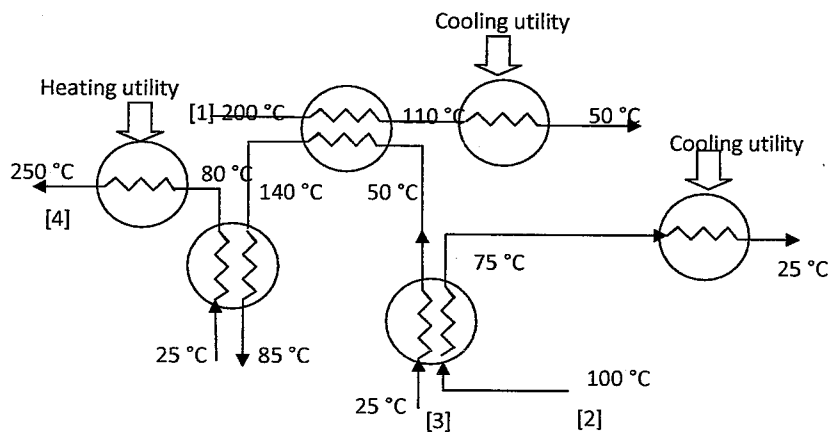
Using a first law balance over the heat exchanger

$$0 = F_h [(T_{2h} - T_{1h})] + F_c [(T_{2c} - T_{1c})]$$

$$T_o \Delta S_{irrev} = T_o F_h \ln \left(\frac{T_{2h}}{T_{1h}}\right) + T_o F_c \ln \left(\frac{T_{2c}}{T_{1c}}\right)$$

[15%]

b) A petroleum processing plant must cool its product from 200 C to 50C, but must heat up several of the raw material heating the plant. The plant is old and the heat integration, which is shown in fig. 1. is not optimal. The heat capacity flow rate for each stream is 1 kW/C



i) How much heating and cooling are required by the scheme shown in Fig. 1? Calculate the minimum amount of exergy which must be supplied by the heating utility, and the maximum which can be recovered into the cooling utility.

Heating utility required for stream [4]

$$Q_{heating} = (250 - 80) * 1 = 170 \text{ kW}$$

Cooling utility for streams 2 and 4

$$Q_{cooling} = (75 - 25) * 1 + (110 - 50) * 1$$

$$Q_{cooling} = 110 \text{ kW}$$

The availability change for a stream being heated or cooled is,

$$B_2 - B_1 = \int_{T_1}^{T_2} \left(1 - \frac{T_o}{T}\right) F dT$$

$$= -F T_o \ln\left(\frac{T_2}{T_1}\right) + F(T_2 - T_1)$$

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The change in availability of the stream is equal to the exergy change, so the exergy supply is given by

$$E_{heating} = -1 * 298.15 * \ln\left(\frac{250 + 273.15}{80 + 273.15}\right) + 1 * (250 + 273.15 - 80 - 273.15)$$

$$E_{heating} = 52.8 \text{ kW}$$

The maximum exergy recovered into the cooling utility is when the heat transfer is reversible, and is given by

$$E_{cooling} = \left[\overbrace{-1 * 298.15 * \ln\left(\frac{50 + 273.15}{110 + 273.15}\right) + 1 * (50 + 273.15 - 110 - 273.15)}^{\text{Stream 1}} \right]$$

$$+ \left[\overbrace{-1 * 298.15 * \ln\left(\frac{25 + 273.15}{75 + 273.15}\right) + 1 * (25 + 273.15 - 75 - 273.15)}^{\text{Stream 2}} \right]$$

$$= -13.0 \text{ kW}$$

The negative sign arises because the streams are cooling and losing exergy. An equivalent amount could be recovered into the cooling utility.

[15%]

iii) Assuming all the flows are reversible, and the hot and cold utilities are supplied reversibly, how much availability is lost in the current design through irreversibility?

There are two ways to approach this. The only losses due to irreversibility are in the heat exchangers due to the heat transfer over the finite temperature difference. Using the result from part (a) [and noting that the Temperature needs to be in Kelvin] these losses are:

Heat exchanger	Stream 1		Stream 2		F (kW/C)	Stream 3		F (kW/C)	Availability loss (kW)
	Tin (C)	Tout (C)	Tin (C)	Tout (C)		Tin (C)	Tout (C)		
1	25	50	100	75	1	1	3.33		
2	50	140	200	110	1	1	10.35		
3	140	85	25	80	1	1	7.88		

Therefore the total loss is 21.6 kW. An alternative approach is by an overall availability balance. The availability change for each stream is

Stream	Inlet temperature	Outlet temperature	Heat capacity flow	Heat load	Availability change
1	200	50	1	150	-36.32
2	100	25	1	75	-8.10
3	25	85	1	-60	5.33
4	25	250	1	-225	57.36
Total				-60	18.27

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The total availability change required for the heating and cooling of the streams is 18.27 kW. For these target temperatures this quantity is fixed. The overall availability balance is

$$\text{Exergy Supply} = \text{Change in availability of streams} + \text{Availability given to the cold utility} + \text{Loss}$$

i.e.

$$52.8 = 18.27 + 13.00 + \text{Loss}$$

So the loss is 21.56 kW.

[20%]

c) The heat integration is to be optimised.

i) What are the minimum amounts of heating and cooling utility required?

ii) What is minimum amount of exergy which must be supplied by the heating utility?

A grand composite curve is needed to optimise to answer these questions.

Heat
Integration
(Optimised)

Temperature Intervals (C)		Hot heat capacity flow	Cold Heat capacity flow	Total	Heat taken in	Heating utility	Cumulative	Load including heat utility
Start	End	kW/C	kW/C	kW/C	kW		kW	Kw
200	250	0	1	1	50		50	-10
100	200	1	1	0	0		50	-10
85	100	2	1	-1	-15		35	-25
50	85	2	2	0	0		35	-25
25	50	1	2	1	25		60	0

From the energy cascade, the minimum heating utility required is 60 kW. No cooling utility is required.

[15%]

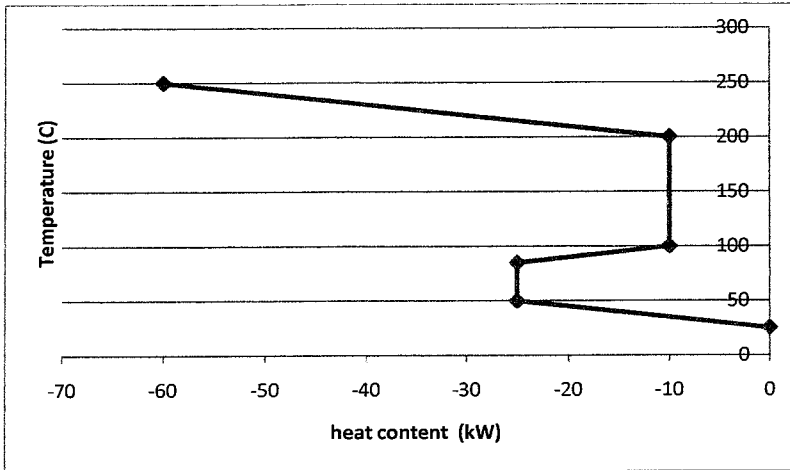


Figure 1. Grand composite curve

50 kW of heat has to be supplied between 200 and 250 C. The heat capacity flow in this interval is 1 kW/K. The exergy required to heat this composite stream is

$$\int_{T_1}^{T_2} \left(1 - \frac{T_o}{T}\right) F dT = -F T_o \ln\left(\frac{T_2}{T_1}\right) + F(T_2 - T_1)$$

$$= -1 * 298.15 * \ln\left(\frac{250 + 273.15}{200 + 273.15}\right) + 1 * (250 - 200)$$

10 kW of heating is required between 25 and 35 degrees C.

$$\int_{T_1}^{T_2} \left(1 - \frac{T_o}{T}\right) F dT = -F T_o \ln\left(\frac{T_2}{T_1}\right) + F(T_2 - T_1)$$

$$= -1 * 298.15 * \ln\left(\frac{35 + 273.15}{25 + 273.15}\right) + 1 * (35 - 25)$$

Therefore the total exergy required is 20.2 kW.

[10%]

iv) For the new heat integration scheme, what is the loss in the availability? Why would the heat integrated design result in the lowest possible availability loss?

Using the overall availability balance, and noting that the change in availability of the streams is fixed.

Exergy Supply = Change in availability of streams + Availability given to the cold utility + Loss

Loss = Exergy Supply - Availability lost to cold utility - Change in availability of stream

$$= 20.2 - 0 - 18.27 = 1.93 \text{ kW}$$

Comment [s2]: An alternative approach to this part of the question is to notice that the grand composite curve above is equivalent to a single heat exchanger exchanging heat between two streams between 100 and 35 degrees celcius. Then the irreversibility can be calculated from the formula derived in part (a)

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For these target temperatures, change in availability of the streams is fixed. The loss is then the difference in availability of the heat supplied to the hot utility and that given to the cold utility. The heat integrated design gives both the minimum amount of hot utility, and allows the lowest temperature it can be supplied at to be identified (i.e. gives the minimum possible exergy supply). Any additional hot utility will result in heat being dumped to the cold utility; however, the hot utility is always at a higher temperature than the cold utility, so you end up just dumping heat from the hot to the cold utility across a finite temperature difference, destroying availability. Therefore this design has the lowest possible availability loss.

[20%]

Most candidates were able to do part (a) , which was very straight forward. Some very good answers were given by students. In particular, some of the students spotted that in the final part of the question, the grand composite curve was equivalent to a system involving only one heat exchanger. This very insightful step led to a very elegant solution to the final part of question c.

Question 2.

a) In the context of life cycle analysis, discuss three methods of allocating burdens between co-products, and their relative advantages and disadvantages.

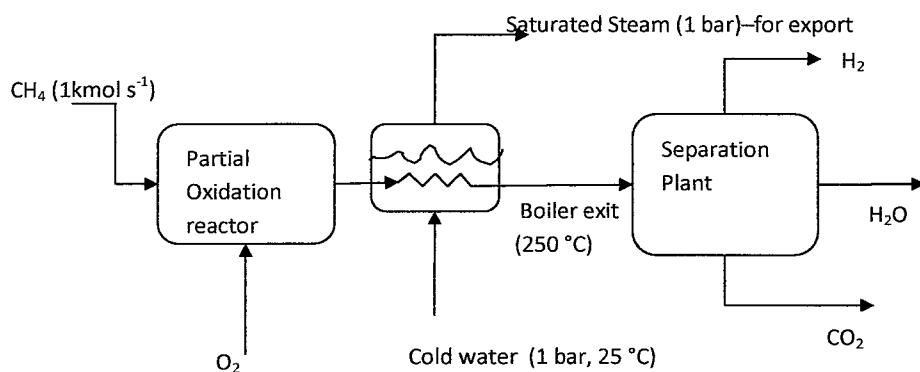
Allocation by mass or energy content simply proportions the burden according to how much of each co-product is produced. This is simple to apply, but does not reflect the value placed on one co-product relative to the other.

Allocation by price uses the market value to proportion the burden. This is also relatively simple to apply, but has the added advantage that the allocation in some way reflects the value society places on each of the co-products. For example, an almost zero value waste product would only be apportioned a very small amount of the burden, whilst the main high value product (which the process was probably built to produce) will take most of the burden.

Allocation by substitution is effectively a way of avoiding allocation. Where a co-product replaces another product which would otherwise have to be produced. The product being replaced effectively gives a measure of the environmental burden associated with the co-product. For this to be effective you need a homogeneous market, or have a LCA for the actual process which would otherwise make the co-product.

[25%]

b)



All streams at $25 \text{ }^\circ\text{C}$ and 1 bar unless otherwise stated

i) How much hydrogen is produced by the plant?

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There is no reaction on the separation plant, so the molar flows of each species leaving it are the same as those in the exit from the boiler.

$$\text{Carbon Balance: } N_{CO_2} = N_{CH_4} = 1 \text{ kmol s}^{-1}$$

Stoichiometric combustion requires 2 kmols of Oxygen per kmol of methane, so the oxygen flow is $N_{O_2} = 0.625 * 2 = 1.25 \text{ kmol s}^{-1}$

$$\text{Oxygen Balance: } 2N_{O_2} = 2N_{CO_2} + N_{H_2O}$$

$$N_{H_2O} = 2.5 - 2 * 1 = 0.5 \text{ kmol s}^{-1}$$

Hydrogen Balance:

$$4 = 2 * N_{H_2} + 2 * N_{H_2O}$$

$$N_{H_2} = 2 - .5 = 1.5 \text{ kmol s}^{-1}$$

[15%]

ii) How much steam can be exported?

Need to do an enthalpy balance over the reactor + the boiler. The heat released by the reaction of oxygen + methane, cooling to 250C is given by

$$Q = \overbrace{N_{CO_2}h_{CO_2} + N_{H_2}h_{H_2} + N_{H_2O}h_{H_2O}}^{\text{Boiler exit at 250 C}} - \overbrace{N_{CH_4}h_{CH_4} - N_{O_2}h_{O_2}}^{\text{inlet at 25 C}}$$

$$Q = 1 * -384162 + 1.5 * 6560 + 0.5 * -234083 - 1 * -74600 - 1.25 * 0$$

$$Q = -416763.5 \text{ kJ s}^{-1}$$

The negative sign implies this is a heat release (as expected)

From the steam tables, the heat absorbed by the water is

$$-Q = m(2674.9 - 104.9)$$

$$416763.5 = m(2674.9 - 104.9)$$

Giving

$$m = 162.16 \text{ kg s}^{-1}$$

[15%]

b)j) What is the global warming potential of the hydrogen AND steam produced by the plant?

Some additional data about the plant is given below.

We must take into account all the emissions to air, not just CO₂. From the data provided in table 2 and 3, the GWP associated with the supply of 1 kmol/s of natural gas is

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$$GWP = \sum_{\text{all emissions}} \text{amount} * \text{potency factor}$$

$$GWP_{CH_4} = 1.4 * 10^{-1} * 21 + 4.7 * 1 + 1.8 * 10^{-2} * 3 + 3.3 * 10^{-2} * 40 + 4.5 * 10^{-5} * 310 \\ = 9.0 \text{ kg } CO_2 \text{ equivalent per Kmol of Methane}$$

Repeating the calculation for the oxygen gives

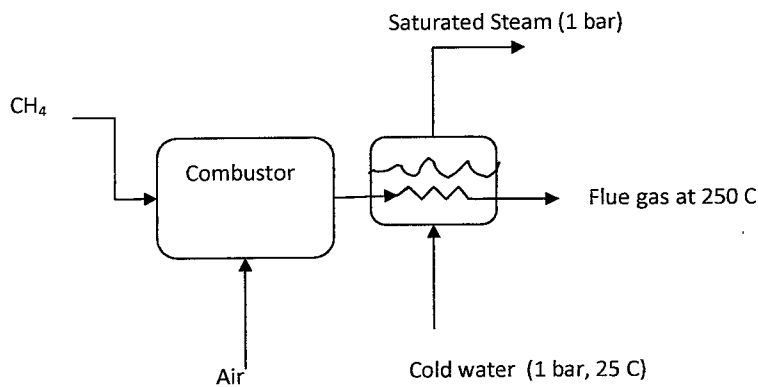
$$GWP_{O_2} = 9.12 \text{ kg of } CO_2 \text{ equivalent per Kmol of oxygen supplied}$$

The plant also produces 1 kmol s⁻¹ of CO₂, equivalent to 44 kg s⁻¹ of CO₂. Therefore the total GWP is

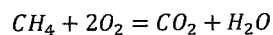
$$GWP_{total} = 9.0 + 44 + 1.25 * 9.12 = 64.4 \text{ kg}_{CO_2} \text{ s}^{-1}$$

[20%]

ii) What is the global warming potential of the hydrogen if allocation by substitution is used to account for the co-product of steam? You may assume that the steam would otherwise have to be provided by a steam boiler, which burns methane stoichiometrically in air.



We need to repeat the calculation but now with stoichiometric combustion of the methane



$$Q = \overbrace{N_{CO_2}h_{CO_2} + N_{H_2O}h_{H_2O} + N_{N_2}h_{N_2}}^{\text{Boiler exit at 250 C}} - \overbrace{N_{CH_4}h_{CH_4} + N_{O_2}h_{O_2} + N_{N_2}h_{N_2}}^{\text{inlet at 25 C}}$$

$$Q = 1 * -384162 + 2 * -234083 + 2 * \frac{0.79}{0.21} * 6597 - (-74600) - 2 * 0 - 0 * 2 * .79/.21$$

$$Q = -728093 \text{ kJ s}^{-1}$$

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So the steam flow is given by

$$-728093 = m(2674.9 - 104.9)$$

$$m = -\frac{728093}{2570}$$

$$m = 283 \text{ kg s}^{-1}$$

Therefore to supply the original 162.16 kg s^{-1} of steam, the steam generator would have to burn

$$\frac{162.2}{283} = 0.573 \text{ kmol s}^{-1} \text{ of methane}$$

The environmental burden associated with this is then

$$GWP_{steam} = .573 GWP_{CH_4} + .573 * 44 = .573 * (9.0 + 44) = 30.4 \text{ kg}_{CO_2} \text{ s}^{-1}$$

Therefore, the GWP of the hydrogen produced is

$$GWP_{H_2} = GWP_{total} - GWP_{steam} = 64.4 - 30.4 = 34.0 \text{ kg}_{CO_2} \text{ s}^{-1}$$

[25%]

Generally well done and attempted by all but 3 candidates. The descriptive part of the question was very well answered by most candidates. The heat and mass balance let some students down. Most were able to do the combined global warming potential calculation. Only the stronger students were able to answer the final part where allocation by substitution had to be applied. Most misread the question which clearly stated that the combustion was to be carried out in air, rather than pure oxygen; this was not heavily penalised.

print). The strength per unit cost, or per unit embodied energy, for wood is much lower than carbon fibre. Large blades can be manufactured from layer of wood (bamboo is excellent) which can be combined e.g. carbon fibre to make blades which can withstand the fatigue cycles, and have a very low embodied energy.

[35%]

(c) Biofuels are liquid fuels produced from energy (and sometimes food) crops. The energy contained within the crops is effectively stored solar energy, and the carbon contained in the fuel has been extracted from the air. Therefore, it could be argued that the fuel is carbon neutral. However, this does not take into account the energy needed to produce the fuel, for which a life-cycle analysis is needed. To grow the crop in the first place, there are energy inputs associated with the production of the fertilizers, farm machinery etc. In fact this can be one of the largest environmental burdens associated with the bio-fuel. The fuel must then be processed, which also requires energy. If you are making bio-ethanol, this involves a fermentation, then a very energy intensive distillation to remove the water from the ethanol. In some cases, the energy needed to process the fuel can come from the combustion of the waste products (e.g. the bagass in the case of ethanol from sugar cane). However, even this case a valid question is whether or not there was a better use of the waste, e.g. to burn in a power station and displace coal. The final lifecycle energy input for a biofuel is often very dependant on the assumptions made about how the waste and co-products are used. Therefore, a biofuel is not necessarily carbon neutral.

The other issues which arises is the quantity of fuel needed and the amount of land required to grow the crop. Any land used to grow an energy crop is land that could otherwise have been used to produce food. Most first generation biofuels use food crops, which leads to a difficult debate on food vs fuel, given that not everyone in the world has access to affordable food. The amounts of petrol and diesel used by the UK are very large; to significantly displace fossil fuels, all of the agricultural land in the UK would have to changed from food production to energy crop production. I.e. biofuels may make some contribution to the transport sector, but their contribution is likely to be small.

[30%]

Parts (a) and (c) were very well answered. Most were able to talk about sustainability in terms of natural capital external costs. Nearly all candidates scored highly when discussing biofuels in part (c). Part (b) on wind turbines was less well answered, with very few candidates talking about the design in terms of the pdf of the wind distribution, and the consequences for control of the blades and forces experienced by the structure.

3 (a)

Natural capital is the stock of goods and services provided by the environment to mankind. The price of these is often taken to be zero. An external cost, or an externality is a cost which is borne by society as a whole, rather than at the point where the activity occurs. An example of this would be SO₂ produced by power-stations, which causes acid rain. The cost of the SO₂ release was borne by countries downwind of the powerstations, whose forests and rivers were polluted by acid rain. In the US they opted for the cap and trade system where the power utilities had to pay for the privilege of releasing SO₂, thus attaching a price to the release of the pollutant, and a value to the service provided by the environment. In this case the price bore no relation to the actual damage caused, but did provide an incentive to reduce the SO₂ emissions.

In an ideal world, unsustainable behaviour (i.e. the irreversible depletion of natural capital) would have a price attached to it, which reflected the true cost of the behaviour. E.g. for a finite resource such as a metal ore, the price of the ore would go up as the ore became more scarce, the market would react to the price rise by using the metal ore efficiently or recycling or by finding an alternative. In practice the price tends to reflect the cost of extraction of the resource and not actual stock remaining, so that changes in price do not act as an accurate signal to preserve the resource. As prices rise, the commodity may perversely become more valuable, which provides an incentive to invest in technologies which more efficiently extract the resource.

For pollutants, the capacity of the environment can be depleted/overwhelmed leading to a degradation in natural capital. In this case, the environment is seen as a free good and there is no price attached to the release of the pollutant. The cost is external, and there is no incentive to avoid the pollution.

[35%]

(b) Wind turbines extract energy from the wind and directly produce energy. The wind is intermittent, so the loading on the wind turbine blades, and the power which is produced varies. The power produced is proportional to the cube of the wind speed, however, very high wind speeds have to be avoided to prevent excessively high bending moments on the root of the blade. Also, in terms of the distribution of windspeed, the wind will only be very strong for short durations during the year. Designing the blades to extract energy during the periods of very high wind speed does not make sense, since this doesn't happen very often the blades have to be massively overdesigned. The problem is made worse by the fact that the loading is not constant due to gusts of wind, so the peak loading could be higher than the average. Two ways to solve this problem are pitch control to reduce the loading on the blades during gusts, this is difficult on a second by second basis, but ok if you want to avoid high wind average wind speeds, or to design the shape of the blade so that a large proportion of it stalls when exposed to a high wind velocity. In the latter case, there is no active control system. Thus, the shape of the blade (an airfoil which twists to change the angle of attack as you move away from the root) is critical.

The gusty nature of the wind means that the design of the blades is limited by fatigue (i.e. many cycles of loading and unloading). Composite, fibrous materials are good at resisting fatigue. Hence, early designs of blades were manufactured from e.g. carbon fibre and resins. Wood is a naturally occurring fibrous material, is cheap, and has a low embodied energy (and negative carbon foot