4M16 2011 Solutions

Q1

(a) (i) The becquerel is a measure of the activity of a radionuclide and has the units of disintegrations per second.

(ii) The gray is a measure of the energy absorbed and has the units of $\mathrm{J\,kg^{-1}}$.

(iii) The sievert is a measure of the damage done to living tissue and also has units of $\mathrm{J\,kg^{-1}}$.

(iv) The sievert is obtained by multiplying the energy absorbed in Gy by two weighting factors, one to account for the type of radiation, with α being the most damaging, and the other to account for the organs receiving the radiation, the latter factor being unity for whole body radiation.

The becquerel on its own does not give an indication of potential damage, firstly, because it gives no indication of the energy of the particular radiation, and, secondly, because the extent of damage also depends on the type of radiation, hence the need for the weighting factor. [25%]

(b) There are three basic methods of protection: time, distance and shielding. Time is an obvious way of reducing exposure and time limits are normally imposed to ensure that people cannot spend too long in an area of high radiation. Distance is a valuable factor since the dose due to a point source is inversely proportional to the square of the distance from the source. In practice it is unlikely that the necessary protection can be provided by time and distance alone so shielding with radiation absorbing/attenuating materials is usually required. [15%]

(c) Assume there are no radioactive daughter products to account for.

(i) Activity after 5 years is given by: $A = A_0 \exp(-\lambda t)$

The decay constant $$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5.272} = 0.1315\ \mathrm{yr^{-1}}$$

$$A = 1\times 10^{15} \exp(-0.1315\times 5) = 5.182\times 10^{14}\ \mathrm{Bq}$$ [10%]

(ii) Assuming the springs can be treated as a point source, the unshielded flux at distance R is given by

$$\phi = \frac{2A}{4\pi R^2}$$

(note the 2 in the numerator is because there are two γ emissions per decay)

Thus, at a distance $R = 0.3$ m, i.e. at the exterior of the lead shield but accounting only, at this stage, for the effects of geometry:

$$\therefore\quad \phi = \frac{2\times 5.182\times 10^{14}}{4\pi (0.3)^2} = 9.164\times 10^{14}\ \mathrm{m^{-2}s^{-1}}$$

The shielded flux is then given by

$$\phi(t) = \phi_0 \exp(-Ut)$$

where t is the thickness of the shield and U is the exponential attenuation coefficient

$$\therefore\quad \phi = 9.164\times 10^{14} \exp(-0.046\times 300) = 9.307\times 10^{8}\ \mathrm{m^{-2}s^{-1}}$$

The average γ-ray energy $$E = \frac{1}{2}(1.17 + 1.33) = 1.25\ \mathrm{MeV}$$

So the surface dose rate

$$D = 1.6\times 10^{-13}\frac{E\Sigma\phi}{\rho} = 1.6\times 10^{-13}\times\frac{1.25\times 3\times 9.307\times 10^{8}}{10^{3}} = 5.584\times 10^{-7}\ \mathrm{Gy\,s^{-1}}$$

$$\therefore \quad D = 5.584 \times 10^{-7} \times 3600 = 2.010 \text{ mGy hr}^{-1}$$

The weighting factor for γ-rays = 1, so the surface dose rate

$$\therefore \quad D = 1 \times 2.010 = 2.010 \text{ mSv hr}^{-1}$$ [45%]

Given the legislative dose limit for workers in the UK is 20 mSv yr^{-1} this dose rate is a bit on the high side and therefore more shielding may be desirable. [5%]

A popular and generally well-done question. Common errors were: in part (b), stating that shielding should be used wherever possible (rather than wherever necessary/appropriate); in part (c), failure to state/justify assumptions (especially the neglect of radioactive daughter products); in part (c)(ii), not accounting for spatial/geometry effects when estimating the dose rate.

Q2

(a) Number of uranium atoms per unit volume

$$N_U = 0.1\frac{\rho}{A}$$

where A is the mass of a uranium atom.

$$\therefore\quad N_U = 0.1\times\frac{18900}{238\times1.661\times10^{-27}} = 4.781\times10^{27}\ \mathrm{m}^{-3}$$

$$\therefore\quad N_{\text{U-235}} = 0.0072\times4.781\times10^{27} = 3.442\times10^{25}\mathrm{m}^{-3}$$

$$\therefore\quad N_{\text{U-238}} = 0.9928\times4.781\times10^{27} = 4.747\times10^{27}\mathrm{m}^{-3}$$

$$\Sigma_a = \sum_i \Sigma_{ai} = \sum_i N_i\left(\sigma_{c_i}+\sigma_{f_i}\right)$$

$$\therefore\quad \Sigma_a = 3.442\times10^{25}\times(107+580)\times10^{-28} + 4.747\times10^{27}\times(2.75+0)\times10^{-28}+1$$

$$\therefore\quad \Sigma_a = 4.67\ \mathrm{m}^{-1}$$

Only U-235 is fissile $\quad\therefore\quad \Sigma_f = 3.442\times10^{25}\times580\times10^{-28} = 2.00\ \mathrm{m}^{-1}$

$$\eta = \frac{\nu\Sigma_f}{\Sigma_a} = \frac{2.43\times2.00}{4.67} = 1.0407 \text{ (only just greater than 1)}$$ [20%]

(b)

$$D\nabla^2\phi + (\eta-1)\Sigma_a\phi = 0$$

For rectangular parallelepiped geometry

$$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$\therefore\quad \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{(\eta-1)\Sigma_a}{D}\phi = 0$$

Let $\frac{(\eta-1)\Sigma_a}{D} = B_m^2$ the material buckling.

Assume

$$\phi(x,y,z) = X(x)Y(y)Z(z)$$

$$\therefore\quad YZ\frac{\partial^2 X}{\partial x^2}+XZ\frac{\partial^2 Y}{\partial y^2}+XY\frac{\partial^2 Z}{\partial z^2}+B_m^2XYZ = 0$$

$$\therefore\quad \frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}+B_m^2 = 0$$

This implies that

$$\frac{1}{X}\frac{d^2X}{dx^2}+\alpha^2 = 0;\quad \frac{1}{Y}\frac{d^2Y}{dy^2}+\beta^2 = 0;\quad \frac{1}{Z}\frac{d^2Z}{dz^2}+\gamma^2 = 0$$

with

$$\alpha^2+\beta^2+\gamma^2 = B_m^2$$

These are SHM equations, so their general solutions are of the form

$$X = A\cos(\alpha x) + C\sin(\alpha x) \text{ etc.}$$

If the origin of the coordinate system is at the centre of the core, then symmetry demands that only cosine terms appear, so the solution for ϕ is

$$\phi = \phi_0 \cos(\alpha x)\cos(\beta y)\cos(\gamma z)$$

where ϕ_0 is the flux at the centre of the core.

The minimum volume core of this geometry will be <u>cubic</u> of side length L, say.

If extrapolation distances can be ignored, the boundary condition will be that the flux goes to zero at the edges of the core, i.e.

$$\phi = 0 \text{ at } x = \pm\frac{L}{2};\ y = \pm\frac{L}{2};\ z = \pm\frac{L}{2}$$

$$\therefore \quad \cos\left(\frac{\alpha L}{2}\right) = 0 \quad \Rightarrow \quad \frac{\alpha L}{2} = \frac{\pi}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{L}$$

and similarly $$\beta = \frac{\pi}{L} = \gamma$$

so $$\phi = \phi_0 \cos\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi y}{L}\right)\cos\left(\frac{\pi z}{L}\right)$$

The criticality condition is $$\alpha^2 + \beta^2 + \gamma^2 = B_m^2$$

$$\therefore \quad 3\left(\frac{\pi}{L}\right)^2 = B_m^2$$ [50%]

(c) $$3\left(\frac{\pi}{L}\right)^2 = B_m^2 \quad \Rightarrow \quad L = \frac{\sqrt{3}\pi}{B_m}$$

$$B_m^2 = \frac{(\eta - 1)\Sigma_a}{D} = \frac{(1.0407 - 1)\times 4.67}{0.05} = 3.801\ \text{m}^{-2}$$

$$\therefore \quad L = \frac{\sqrt{3}\pi}{\sqrt{3.801}} = 2.79\ \text{m}$$ [10%]

(d) The capture cross-section of H_2O is <u>much</u> larger than that of D_2O (the hydrogen nuclei in H_2O can readily accept another neutron). Thus, the change of coolant/moderator will lead to an increase in the macroscopic absorption cross-section of the non-fuel core.

The scattering cross-section of H_2O is also larger than that of D_2O. Thus, the change of moderator increases Σ_s and therefore reduces D, which is inversely proportional to Σ_s.

With the change of moderator, Σ_a increases to $5.67\ \text{m}^{-1}$.

$$\therefore \quad \eta = \frac{\nu\Sigma_f}{\Sigma_a} = \frac{2.43 \times 2.00}{5.67} = 0.857$$

As η is less than unity it will not be possible to establish criticality in a reactor with this core composition however large the reactor. For a light water cooled and moderated reactor to be feasible η must be increased by enriching the fuel to increase the proportion that is fissile. [20%]

Another popular and generally well-done question. Part (a) was done surprisingly badly – perhaps candidates were caught by surprise as this particular topic has not been examined for a few years. A surprisingly large number of candidates did not recognize (by inspection) that the minimum volume core would be cubic. A number of candidates incorrectly thought that the neglect of extrapolation distances implied there would be no neutron leakage. Many candidates overlooked the impact on the average number of neutrons released per neutron absorbed of the proposed change to light water in part (d).

Q3

(a) The most important factors governing the choice of coolant are:

- Neutron capture cross-section
- Moderating characteristics
- Boiling point (if a liquid)
- Heat capacity
- Corrosive properties

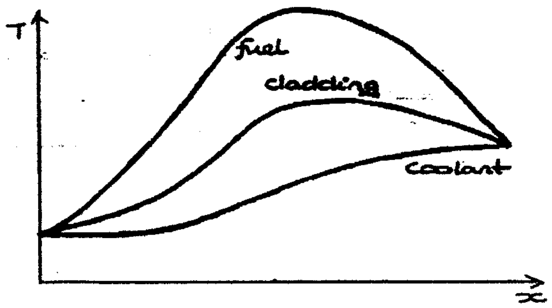
Other possible factors: cost, availability, toxicity, density, thermal expansivity, how easily it leaks

Current practice:

Magnox reactors, AGRs:	CO_2
PWRs and BWRs:	H_2O
CANDU reactors:	D_2O
Fast breeder reactors:	liquid Na

[30%]

(b) (i)



Not to scale

[10%]

(ii) The maximum coolant temperature will occur at the exit to the channel.

From the SFEE $\dot{m} c_p \left[T_{c_{out}} - T_{c_{in}}\right] = P$

$$\therefore \quad T_{c_{out}} = T_{c_{in}} + \frac{P}{\dot{m} c_p} = 250 + \frac{6\times10^6}{150\times10^3} = 290\ ^\circ\text{C}$$

[10%]

The maximum temperature in a fuel pin will occur on the centre line.

As $L = L'$ (power is zero at the ends of the fuel channel), Ginn's equation becomes

$$\theta = \frac{T - T_{c_{1/2}}}{T_{c_{out}} - T_{c_{1/2}}} = \sin\left(\frac{\pi x}{2L}\right) + Q\cos\left(\frac{\pi x}{2L}\right) \quad (1)$$

with $$Q = \frac{\pi \dot{m} c_p}{UA} = \frac{\dot{m} c_p}{4 r_o L U}$$

Here $$r_o = \frac{12.0}{2} + 0.4 = 6.4 \text{ mm and } L = 3\text{ m}$$

For the fuel centre line $$\frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f}$$

$$\therefore \quad \frac{1}{U} = \frac{1}{35\times10^3} + \frac{0.4\times10^{-3}}{13} + \frac{6.4\times10^{-3}}{25\times10^3\times6\times10^{-3}} + \frac{6.4\times10^{-3}}{2\times3}$$

$$\therefore \quad \frac{1}{U} = 1.169\times10^{-3}\ \text{m}^2\text{KW}^{-1}$$

<u>Per pin</u> $$\dot{m}c_p = \frac{150\times10^3}{37} = 4054\ \mathrm{W K^{-1}}$$

$$Q = \frac{\dot{m}c_p}{4r_o LU} = \frac{4054}{4\times6.4\times10^{-3}\times3}\times1.169\times10^{-3} = 61.69$$

From (1) $$T = T_{c_{1/2}} + \left(T_{c_{out}} - T_{c_{1/2}}\right)\left[\sin\left(\frac{\pi x}{2L}\right) + Q\cos\left(\frac{\pi x}{2L}\right)\right]$$

$$T_{c_{1/2}} = \tfrac{1}{2}\left(T_{c_{in}} + T_{c_{out}}\right) = \frac{250+290}{2} = 270\ ^\circ\mathrm{C}$$

$$T = 270 + 20\left[\sin\left(\frac{\pi x}{2L}\right) + Q\cos\left(\frac{\pi x}{2L}\right)\right]$$

The maximum temperature occurs where

$$\frac{dT}{dx} = 0 = 20\times\frac{\pi}{2L}\left[\cos\left(\frac{\pi x}{2L}\right) - Q\sin\left(\frac{\pi x}{2L}\right)\right]$$

$$\therefore\quad \tan\left(\frac{\pi x}{2L}\right) = \frac{1}{Q} \quad\Rightarrow\quad x = \frac{2L}{\pi}\tan^{-1}\frac{1}{Q}$$

$$\therefore\quad x = \frac{2\times3}{\pi}\tan^{-1}\frac{1}{61.69} = 0.031\ \mathrm{m}\ \text{(i.e. “just” past half way)}$$

Substituting for x

$$T = 270 + 20\left[\sin(0.01621) + 61.69\cos(0.01621)\right] = 1504\ ^\circ\mathrm{C}$$ [50%]

A slightly less popular question and overall the least well done. Many candidates wasted time by deriving general results for the location and value of the maximum temperature that were not actually required to answer part (b)(ii). There were also some curious interpretations of the information provided about the assembly geometry.

Q4

(a) Many fission products are unstable. Some decay by neutron emission. Unlike the neutrons emitted promptly in fission, these neutrons are emitted some time after the fission reaction that produced the relevant fission product (at a time dependent on the decay constant of the fission product in question). These neutrons are in consequence known as *delayed neutrons*.

Delayed neutrons have a very significant, beneficial effect on reactor dynamics. They increase the average neutron lifetime and hence lengthen the dominant time constant governing the dynamic behaviour of the neutron population. [15%]

(b) (i) The reactor is being operated in steady state, hence

$$\frac{dn}{dt}=\frac{\rho-\beta}{\Lambda}n_0+\lambda c=0 \quad (1)$$

and

$$\frac{dc}{dt}=\frac{\beta}{\Lambda}n_0-\lambda c=0 \quad (2)$$

Rearranging (2) $\therefore \; c=\frac{\beta}{\lambda\Lambda}n_0$

Adding (1) and (2) $\frac{\rho}{\Lambda}n_0=0 \;\Rightarrow\; \rho=0$

[It is also acceptable to state the latter result without proof.] [15%]

(ii)

$$\frac{dn}{dt}=\frac{\rho-\beta}{\Lambda}n+\lambda c \quad (3)$$

$$\frac{dc}{dt}=\frac{\beta}{\Lambda}n-\lambda c \quad (4)$$

Taking Laplace transforms (with p as the transform variable)

$$p\bar{n}-n_0=\frac{\rho-\beta}{\Lambda}\bar{n}+\lambda\bar{c} \quad (5)$$

$$p\bar{c}-c_0=\frac{\beta}{\Lambda}\bar{n}-\lambda\bar{c} \quad (6)$$

From (b) $c_0=\frac{\beta}{\lambda\Lambda}n_0$, so substituting in (6)

$$p\bar{c}-\frac{\beta}{\lambda\Lambda}n_0=\frac{\beta}{\Lambda}\bar{n}-\lambda\bar{c} \;\Rightarrow\; \bar{c}(p+\lambda)=\frac{\beta}{\lambda\Lambda}(\lambda\bar{n}+n_0)$$

Then substituting for $\bar{c}$ in (5)

$$p\bar{n}-n_0=\frac{\rho-\beta}{\Lambda}\bar{n}+\frac{\beta}{\Lambda}\frac{(\lambda\bar{n}+n_0)}{(p+\lambda)}$$

$$\therefore\; \bar{n}\left[p-\frac{\rho-\beta}{\Lambda}\right]=n_0\left[1+\frac{\beta}{\Lambda(p+\lambda)}\right]+\frac{\beta}{\Lambda}\frac{\lambda\bar{n}}{(p+\lambda)}$$

$$\therefore\; \bar{n}\left[p+\frac{\beta-\rho}{\Lambda}\right](p+\lambda)=n_0\left[p+\lambda+\frac{\beta}{\Lambda}\right]+\frac{\beta\lambda}{\Lambda}\bar{n}$$

$$\therefore\; \bar{n}\left[p^2+p\left(\lambda+\frac{\beta-\rho}{\Lambda}\right)+\lambda\left(\frac{\beta-\rho}{\Lambda}\right)-\frac{\beta\lambda}{\Lambda}\right]=n_0\left[p+\lambda+\frac{\beta}{\Lambda}\right]$$

$$\therefore\; \bar{n}\left[p^2+p\left(\lambda+\frac{\beta-\rho}{\Lambda}\right)-\frac{\lambda\rho}{\Lambda}\right]=n_0\left[p+\lambda+\frac{\beta}{\Lambda}\right]$$

To find the system time constants, solve

$$p^2 + p\left(\lambda + \frac{\beta - \rho}{\Lambda}\right) - \frac{\lambda\rho}{\Lambda} = 0$$

Substituting the values given

$$\therefore \quad p^2 + p\left(0.1 + \frac{0.007 - 0.003}{10^{-4}}\right) - \frac{0.1 \times 0.003}{10^{-4}} = 0$$

$$\therefore \quad p^2 + 40.1p - 3 = 0 \quad \Rightarrow \quad p = 0.07467 \text{ or } -40.175 \text{ s}^{-1}$$

Therefore the dominant (positive) time constant

$$T_+ = \frac{1}{p_+} = \frac{1}{0.07467} = 13.39 \text{ s}$$ [45%]

(iii) In the *prompt jump approximation* the neutron population is assumed to stay in equilibrium with the precursor population. So if $\frac{dn}{dt} = 0$ (3) gives

$$\frac{\rho - \beta}{\Lambda} n + \lambda c = 0 \quad \Rightarrow \quad n = \frac{\lambda\Lambda}{\beta - \rho} c$$

Substituting for n in (4)

$$\frac{dc}{dt} = \frac{\beta\lambda}{\beta - \rho} c - \lambda c = \frac{\rho\lambda}{\beta - \rho} c$$

By inspection

$$p = \frac{\rho\lambda}{\beta - \rho} = \frac{0.003 \times 0.1}{0.007 - 0.003} = 0.075 \text{ s}^{-1}$$

$$T_+ = \frac{1}{p_+} = \frac{1}{0.075} = 13.33 \text{ s}$$

Thus, the prompt jump approximation gives a good estimate of the exact time constant. From a safety point of view, the prompt jump approximation underestimates T_+ – the approximation is *conservative*: the real system will respond more slowly than predicted. [25%]

The least popular question but done well by many. The only common failing was the inability of candidates to realize that in part (b)(ii) they could just solve the quadratic equation for the inverse time constants and instead attempting to find an approximate solution, with varying degrees of success.