

ENGINEERING TRIPOS PART IIB

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Friday 29 April 2011 9 to 10.30

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Module 4A11

TURBOMACHINERY II

*Answer not more than two questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

1 (a) Write down the Meridional Streamline Curvature Equation, explain the physical meanings of each term. Describe and comment on the assumptions used in deriving this equation. Show that the Meridional Streamline Curvature Equation can be reduced to the Simple Radial Equilibrium Equation (SREE), stating any further assumptions required:

$$\frac{\partial h_o}{\partial r} - T \frac{\partial s}{\partial r} = V_m \frac{\partial V_m}{\partial r} + \frac{V_\theta}{r} \frac{\partial(rV_\theta)}{\partial r} \quad [20\%]$$

(b) A low speed, axial flow, single stage air turbine test rig has cylindrical casing and hub lines with hub-to-tip ratio of 0.6 and a rotor tip diameter of 1.0 m. The flow is axial and uniform at the stage inlet and exit. The inlet velocity is  $50.0 \text{ m s}^{-1}$  and the flow leaving the stator is at  $60^\circ$  from the axial and is uniform along the blade height. The design stage reaction at mid-span is 0.5. The aerodynamic loss along the span can be assumed to be uniform and the density of the air is  $1.2 \text{ kg m}^{-3}$ .

(i) Using the SREE, estimate the pressure and the axial velocity differences between the casing and hub in the axial gap between the stator and the rotor. [40%]

(ii) Using the information obtained in (i) above, or otherwise, estimate the stage reaction at the hub and at the casing. [20%]

(iii) Based on the above results, comment on the validity of the use of the SREE. Also, comment on whether the SREE over- or under-estimates the spanwise flow variations in the axial gap between the stator and the rotor. [10%]

(c) Two different blade designs are used for the above turbine test rig to investigate the effects of aspect ratio on blade performance. The high aspect ratio design has a narrow blade chord with an aspect ratio of 3 for both the rotor and the stator blade rows. The low aspect ratio design has blade rows with an aspect ratio of 1. The non-dimensional inter-blade row gap is kept the same as 20% of axial chord for both stages. All blades are radially stacked. Explain for which design you would expect the simple radial equilibrium equation to give better approximations for the spanwise flow distributions at the inter-blade row gap? [10%]

2 (a) The specific entropy production through a weak shockwave may be approximated as  $\Delta s \approx R \frac{\gamma+1}{12\gamma^2} \left( \frac{\Delta p}{p_1} \right)^3$ , where  $R$  is gas constant,  $\gamma$  the ratio of specific heat capacities,  $\Delta p$  the pressure rise across the shockwave and  $p_1$  the static pressure upstream of the shock. Show that the loss of isentropic efficiency due to the

shockwave compression process can be estimated as  $\Delta \eta_{shock} \approx \frac{\gamma+1}{12\gamma^2} \left( \frac{\Delta p}{p_1} \right)^2$ . [30%]

(b) (i) The tip section of a transonic air compressor rotor with upstream relative Mach number 1.3 has been designed to produce a static pressure ratio  $p_2 / p_1 = 2.125$ , of which 85% is estimated to be due to shockwave compression ( $(p_2 / p_1)_{shock} = 1.806$ ) and the remaining 15% due to diffusion downstream of the shock. It is also estimated that the entropy increase due to the downstream diffusion is about 50% of that due to the shockwave. Using the result obtained in (a), estimate the isentropic efficiency of the rotor section, assuming the loss of efficiency is entirely due to the shockwave and the diffusion. Comment on the result you obtain. Take the ratio of specific heat capacities  $\gamma = 1.4$  for air. [30%]

(ii) The rotor is tested and it is found that, for the tip section, both the measured static pressure rise and the isentropic efficiency are significantly lower than the design values as in (b) (i). The rotor tip is unshrouded. Discuss possible sources for the discrepancies and suggest corresponding remedies that might increase the overall performance. [20%]

(c) Discuss the loss mechanisms for shrouded and unshrouded rotor tips. Comment on the suitability of using a shrouded tip for a transonic compressor rotor for which the tip clearance can be effectively controlled. [20%]

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3 A designer is asked to produce a 50% reaction, repeating stage, axial flow turbine design with a flow coefficient  $\phi = 0.5$  and a stage loading coefficient  $\psi = 2.0$ .

(a) Stating any assumptions, calculate the mean-line flow angles at inlet and exit of the stator and rotor in both the absolute and relative frames of reference. Hence sketch the velocity triangles. [25%]

(b) Sketch a surface pressure distribution for the mean-radius section of the stator that represents a good blade design. State which aspects of the pressure distribution are beneficial. [15%]

(c) At the endwalls, the inlet flow to the stator deviates from the angle calculated in part (a) causing the stator to be at either positive or negative incidence.

(i) Sketch likely pressure distributions for the stator, showing the effect of positive and negative incidence. [10%]

(ii) The endwall boundary layer from the rotor upstream of the stator causes the flow on the stator inlet endwall to be “skewed”. Explain what is meant by boundary layer skew and why it occurs. For a point in the boundary layer with an axial velocity equal to 50% of the mean-line value, calculate the stator inlet flow angle. [10%]

(iii) The rotor upstream of the stator is shrouded and the resultant shroud-leakage jet also creates a local change of flow angle at the stator inlet. Stating any assumptions made, estimate this flow angle. [10%]

(iv) Explain how sweep can be used to reduce the sensitivity of the stator blade to incidence. [10%]

(d) Apart from computational speed, state two additional advantages of the throughflow method, as compared to a three-dimensional finite volume Navier-Stokes solver, for turbomachinery design. Explain how a throughflow method might be extended to include the effects of skew and shroud leakage discussed in part (c). [20%]

**END OF PAPER**