ENGINEERING TRIPOS PART IIB

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Friday 6 May 2011 2.30 to 4

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Module 4C7

# RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*Candidates may bring their notebooks to the examination.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

aas04

1 (a) The autocorrelation function $R_{xx}(\tau)$ of a stationary random process $x(t)$ is given by

$$R_{xx}(\tau) = \left(\frac{\pi S_0}{\gamma}\right) e^{-\gamma|\tau|}$$

where $S_0$ and $\gamma$ are constants.

(i) Find the mean squared value of $x(t)$, and derive an expression for the double-sided spectral density $S_{xx}(\omega)$. [20%]

(ii) The random process $x(t)$ is the output of a linear system which is excited by a white noise force $F(t)$ which has double sided spectral density $S_0$. Deduce that the governing differential equation must be

$$A\dot{x} + Bx = F(t)$$

and find the constants $A$ and $B$ in terms of $\gamma$. [20%]

(iii) Calculate the mean squared velocity $\mathrm{E}[\dot{x}^2]$ and comment on the physical relevance of your result. [20%]

(b) A linear system has the equation of motion

$$a_N \frac{\mathrm{d}^N x}{\mathrm{d}t^N} + a_{N-1} \frac{\mathrm{d}^{N-1} x}{\mathrm{d}t^{N-1}} + a_{N-2} \frac{\mathrm{d}^{N-2} x}{\mathrm{d}t^{N-2}} + \ldots + a_0 x = F(t)$$

where $F(t)$ is white noise excitation. Consider the mean squared value of the $m$th derivative of $x(t)$ :

$$\mathrm{E}\left[\left(\frac{\mathrm{d}^m x}{\mathrm{d}t^m}\right)^2\right]$$

For what range of values of $m$ does this quantity have a finite value? Comment on the practical implications of this result. [40%]

2 An item of equipment of mass $M$ is mounted inside a spacecraft launch vehicle via a spring-damper system of stiffness $K$ and damper rate $C$. The equation of motion of the system can be written as

$$M\ddot{x} + C\dot{x} + Kx = F(t)$$

where $x(t)$ is the displacement of the equipment and the force $F(t)$ arises from acoustic excitation of the vehicle and can be approximated as white noise with double-sided spectral density $S_0$.

(a) The equipment will suffer impact damage if the response $x(t)$ exceeds the clearance $b$ between the equipment and the spacecraft sidewall. Derive an expression for the probability that impact damage will occur for a launch of duration $T$ seconds. [35%]

(b) For a given design, the impact probability is found to be too high. The design can be improved by increasing the clearance $b$, or by modifying the spring-damper system. The effective "cost" of the design is found to be proportional to the product $b^3CK$. Explain how the design should be modified to reduce the impact probability in the most cost efficient way. [30%]

(c) The stress in the equipment mount is given by $S = Kx$, and the S-N law governing fatigue failure of the mount has the form

$$N = \alpha S^{-\beta}$$

where $\alpha$ and $\beta$ are constants with $\beta > 1$. Derive an expression for the fatigue damage accumulated by the equipment mount for a launch of duration $T$. If the predicted fatigue damage it too great by a factor of 2, explain how you would modify the design of the mount to produce an acceptable result. [35%]

3 The free vibration behaviour of a particle is described by the following non-linear equation

$$\ddot{x} + k_0 x + k_1 x^2 - k_2 x^3 = 0$$

where $k_0$, $k_1$ and $k_2$ are real, positive constants.

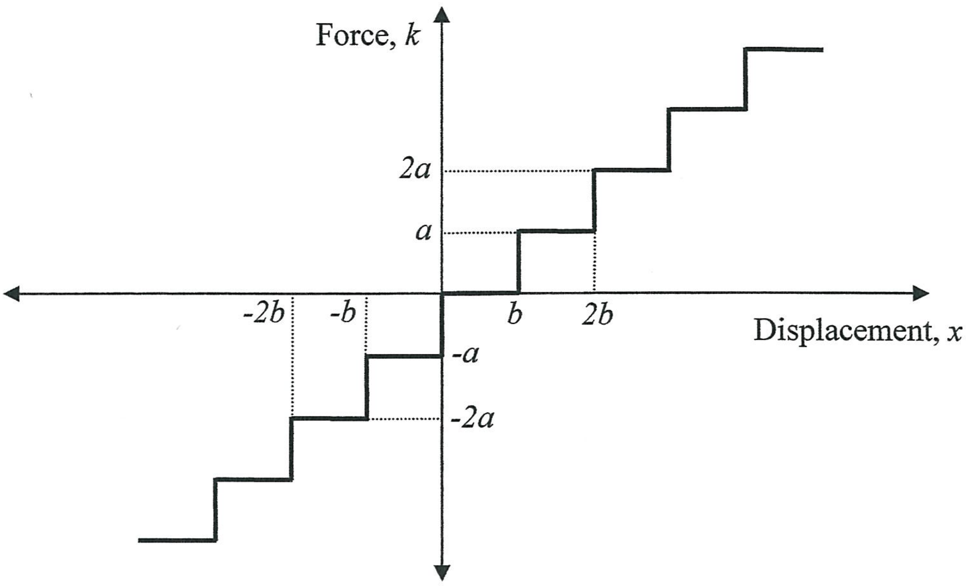
(a) Identify the equilibrium or singular points of the system. [20%]

(b) Derive the type and stability of each singular point. [40%]

(c) Sketch the behaviour of the system in the phase plane. [30%]

(d) Sketch the restoring force as a function of vibration amplitude and show that this sketch agrees with the behaviour described in the phase plane. [10%]

4 An undamped non-linear system of unit mass has a force-displacement characteristic given by a staircase function as shown below




(a) Sketch the input and output waveforms when this system is sinsusoidally driven with amplitude $\alpha$, where $b < \alpha < 2b$. [20%]

(b) Derive the Describing Function for this system for input amplitude $\alpha < 2b$. [50%]

(c) If the system is driven by a force $A\cos\omega t$, determine an approximate relation between response amplitude $\alpha$ and $\omega$ for $\alpha < 2b$. [30%]

**END OF PAPER**