ENGINEERING TRIPOS PART IIB

Friday 29 April 2011 2.30 to 4

Module 4C8

APPLICATIONS OF DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

DC/03

1 A 'bicycle' model of a vehicle, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The vehicle moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients C at both the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure.

(a) Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + 2C\frac{v}{u} + (a-b)C\frac{\Omega}{u} = C\delta$$

$$I\dot{\Omega} + (a-b)C\frac{v}{u} + \left(a^2 + b^2\right)C\frac{\Omega}{u} = aC\delta$$

State your assumptions. [30%]

(b) An autonomous vehicle is found to have a critical speed above which its motion becomes unstable. It is proposed to eliminate this problem by steering the front wheels in response to yaw rate, so that $\delta = -K\Omega$, where K is a positive constant.

(ii) Derive a characteristic equation that can be used to analyze the stability of the vehicle. [30%]

(ii) Determine the conditions for which forward motion of the vehicle is stable. Sketch a graph of the variation of the critical speed with K, showing salient values. Compare the critical speed of the controlled vehicle to that of the uncontrolled vehicle. [40%]

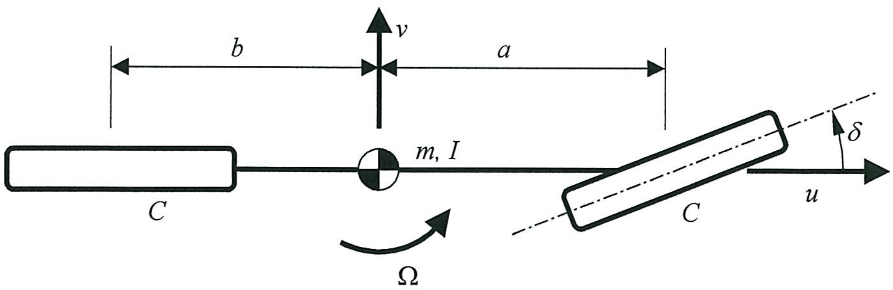

Fig. 1

2. The linear creep equations governing wheel forces in rolling contact are given by:

$$X = -C_{11}\xi$$
$$Y = -C_{22}\alpha - C_{23}\psi$$
$$N = C_{32}\alpha - C_{33}\psi$$

(All terms are defined on the data sheet).

(a) Use the 'brush' model with longitudinal and lateral bristle stiffnesses K_x, and K_y, to derive values for C_{22} and C_{32} for a rectangular contact area having dimensions $2l \times 2h$, a normal load Z and uniform contact pressure. State your assumptions. [40%]

(b) Explain the source of the C_{23} and C_{33} coefficients in the linear creep equations. Give examples of where they arise in vehicle dynamics. [30%]

(c) Explain which of the linear creep coefficients are needed to perform simple analyses of the stability of:

(i) cars
(ii) railway bogies. [30%]

3. (a) Describe, with the aid of sketches, the five parameters used to specify the path of a satellite orbiting around the earth. Your explanation should include definitions of the terms: *First Point of Aries*, and *Line of Nodes*. What additional parameter is needed to compute the position of a satellite, assuming that the orbit is elliptical? [35%]

(b) GPS satellites orbit the earth in a near-circular orbit with a period of exactly 12 sidereal hours, at an inclination of 55° to the equatorial plane. For a receiver sited at the North Pole, calculate:

(i) the maximum elevation that each satellite will reach;

(ii) the fraction of its orbit for which each GPS satellite will be visible to the receiver;

(iii) the average number of satellites that would be at an elevation of more than 30°, for the receiver. [55%]

(c) A project has been proposed to monitor changes in the thickness of the polar ice cap by means of GPS receivers placed on its surface. Comment briefly on the accuracy which might be expected from these measurements, in the light of the information calculated in part (b), above. [10%]

4. (a) A satellite orbits the Earth in a near-circular polar orbit (i.e. the Earth's rotational axis lies in the plane of the orbit). Using terms up to and including the J_2 term for the Earth's gravitational potential from the data sheet, calculate the radial and tangential forces on the satellite as a function of its position, and hence show that the equations which govern the satellite's motion are:

$$\ddot{r} - r\dot{\theta}^2 = -(\mu/r^2) + (3\lambda/4r^4)(1 + 3\cos 2\theta)$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = (3\lambda/2r^4)\sin 2\theta$$

where $\lambda = \mu J_2 R^2$, r and θ are the polar co-ordinates of the satellite, and the other symbols have their usual meanings. What do these equations tell us about the conservation (or otherwise) of the satellite's moment of momentum about the centre of the Earth? [65%]

(b) The NOAA-19 weather satellite (launched in 2009) has a mass of 1400 kg, and orbits the earth in a near-circular polar orbit at a mean altitude of 870 km. What is the magnitude and frequency of the fluctuation in radial force that it experiences during its orbit? At what point on its orbit is this force at a maximum? [35%]

END OF PAPER