ENGINEERING TRIPOS PART IIB

Monday 2 May 2011 9 to 10.30

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A voltage controlled parallel-plate actuator is comprised of a movable electrode separated by an air-gap of magnitude g from a fixed electrode as shown in Fig. 1. The movable electrode is supported by a spring of stiffness k and a voltage V is applied between the electrodes. The overlap area between the plates is A and inertia of the moving electrode can be neglected.

(a) Write down the force equilibrium equations for the moving electrode. [10%]

(b) Write down the equations for the onset of pull-in instability and derive an expression for the pull-in voltage. [30%]

(c) The voltage V is now connected across a circuit comprising the electro-static actuator and a fixed capacitor C_S connected in series. Derive the condition for actuator stability and show that there exist values of C_S for which the actuator is stable for travel over the entire gap. [60%]

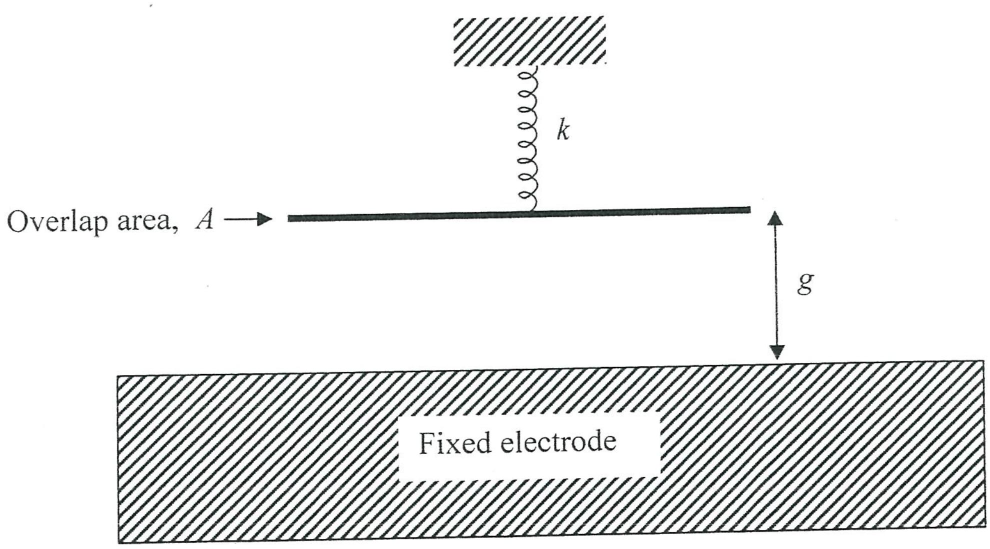

Fig. 1

2 A vibratory micromechanical gyroscope uses a 5 μg polysilicon proof mass supported off the substrate by crab-leg flexures. The thickness of the structural polysilicon layer is 2 μm. The resonant frequencies for motion along the drive and sense axes are 10 kHz and 11 kHz respectively.

(a) The proof mass is actuated along the drive direction by an electrostatic comb drive. Derive an expression for the electrostatic force generated by a comb drive actuator as a function of voltage V applied between the electrode and the proof mass, the number of electrode gaps N and other geometrical parameters defining the system. [20%]

(b) What is the minimum magnitude of the AC voltage that is to be applied across the electrodes in the comb drive actuator to generate resonant motion of amplitude equal to 5 μm along the drive direction assuming a Quality Factor of 100 for the driven mode. You should assume a minimum gap between electrodes of 1 μm, a maximum DC voltage of 10 V and the number of electrode gaps $N = 10000$. [20%]

(c) The proof mass is driven to a sinusoidal displacement of amplitude 1 μm at a frequency of 1000 Hz along the drive axis. Estimate the deflection of the proof mass along the sense axis in response to an externally applied rotation rate of 100 deg s^{-1} about the sensitive axis of rotation assuming negligible damping. [20%]

(d) A differential parallel-plate capacitor structure is used to pick off the motion of the proof mass along the sense axis. The nominal sense capacitance is 100 fF and the nominal gap spacing between each pair of fixed and movable electrodes comprising the differential parallel-plate sense capacitor is 1 μm. Estimate the change in capacitance for the conditions outlined in (c) above. [20%]

(e) Describe the concept of mode-matching in a vibratory micromechanical gyroscope and the consequent trade-off between bandwidth and sensitivity. Describe a solution to overcome the bandwidth-sensitivity trade-off for this device. [20%]

3 A molecular separation device is shown in top view in Fig. 2. The device consists of a separation column where differentially charged species separate via electrophoresis. The separation column is a microfluidic channel etched into the glass substrate with fluidic ports and electrodes suitably patterned. The channels are filled with an electrolyte buffer solution and continuous flow can be assumed. The width of the separation column is 100 μm and the depth of the channel is 100 μm. An electrical double layer is formed at the channel surface with an associated Debye length L_D of 1 nm. The viscosity η of the solution is 1.5×10^{-3} kg m^{-1} s^{-1}. The 1-D Navier Stokes equation for fluid velocity $U(z)$ is given by

$$\frac{d^2 U}{d z^2} = \frac{\sigma_w E_x}{\eta L_D} \exp\left(\frac{-z}{L_D}\right)$$

Here σ_w is the channel surface charge density, E_x is the applied electric field along the length of the channel and z is the distance away from one surface of the channel.

(a) Show that the flow velocity U can be written in the form

$$U = U_0 \left(1 - \exp\left(\frac{-z}{L_D}\right)\right)$$

Note that $U(z)$ satisfies the no-slip boundary condition and write down an expression for U_0 as a function of σ_w, η, L_D and E_x. Comment on the sign of U_0. [30%]

(b) The channel surface charge density is -0.1 C m^{-2} and a potential of 100 V is applied initially between ports 1 and 2 while ports 3 and 4 are held at ground. The distance between ports 1 and 2 is 10 mm. Estimate the volumetric flow rate and the time taken for a plug of solution to travel from port 1 to port 2. [20%]

(c) Two differentially charged molecules are separated by electrophoresis in the separation column. During the separation process, a potential of 100 V is applied between ports 3 and 4 while ports 1 and 2 are held at ground. Ports 3 and 4 are spaced by a distance of 50 mm and the electrophoretic mobilities of the molecules in solution differ by 10^{-9} m^2 V^{-1} s^{-1}. Calculate the relative separation distance between the molecules when the bulk solution has traveled a distance of 20 mm down the separation column. [20%]

(cont.

(d) Estimate the width of the separation band for the case in (c) if the diffusion constant of the molecular species is 50 $\mu m^2\ s^{-1}$. Comment on this result and describe the device parameters that should be optimised to obtain a good separation. [30%]

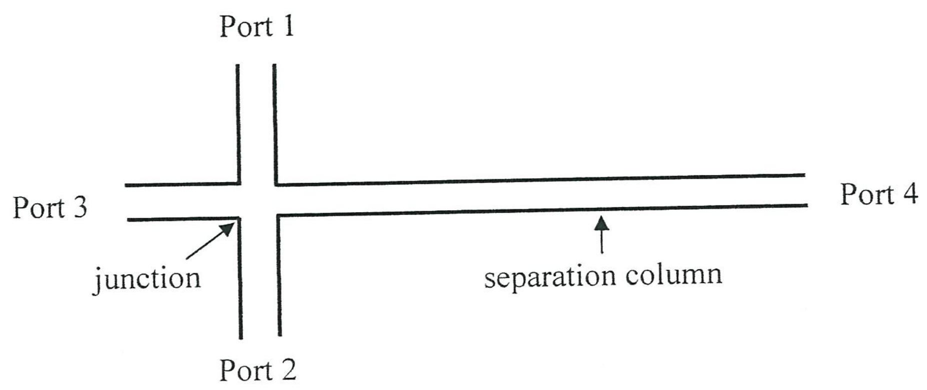

Fig. 2

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4 Two micromachined beams of masses m_1 and m_2 with associated spring constants k_1 and k_2 and damping constants b_1 and b_2 respectively are separated by an air-gap. A voltage V is applied across the beams to actuate them by closing the air-gap. The air-gap is equal to magnitude g when the applied voltage $V = 0$ V and the overlap area A of the beams remains constant.

(a) Write down the force equilibrium equations for the motion of the beams in response to the applied voltage. Linearise these equations for small displacements relative to the gap. [20%]

(b) Draw a lumped electrical circuit model for the linearised system. [20%]

(c) Qualitatively describe the pull-in behaviour observed for large applied voltages for the cases where:

(i) $k_1 >> k_2$

(ii) $k_1 \approx k_2$ [20%]

(d) Write down an equation expressing the pull-off condition as a function of the applied voltage V assuming that adhesive forces act during contact. The work of adhesion between the surfaces is equal to w and the effective contact radius is equal to R. [20%]

(e) Describe a device application to which the above modeling applies. [20%]

END OF PAPER