ENGINEERING TRIPOS PART IIB

Thursday 5 May 2011 9 to 10.30

Module 4C16

ADVANCED MACHINE DESIGN

*Answer **all** questions.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Module 4C16 data sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

djc02

1 Figure 1 shows schematically the cam mechanism used to lift the inlet valve in a car internal combustion engine. The cam has base circle radius 20 mm and tip circle radius 5 mm, with the tip circle centre offset from the base circle centre by 30 mm. The face width of the cam is 10 mm. Cam and follower are both made of steel. The flat faced follower is held down by a spring which exerts a force of 50 N when contact is on the base circle and 600 N when at the point of highest lift. The effective mass of the follower and valve train is 0.1 kg.

(a) Show that the contact force per length of contact between cam and follower is 5 kN/m for contact on the base circle and 46.8 kN/m for contact at the point of maximum lift, at a cam rotational speed of 2000 rpm. [30%]

(b) Assess the tribological conditions for contact on the base circle and for contact at the point of maximum lift, making use of the design data given in Table 1 and the lubrication map given in Fig. 2. How will the lubrication conditions at these points in the cycle be affected by the cam speed and the spring force? [70%]

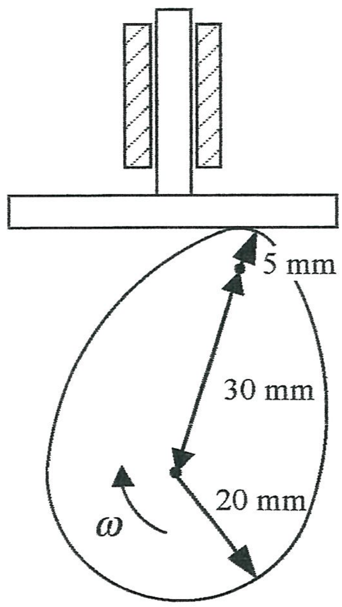

Fig. 1

Cam speed	2000 rpm
Cam and follower combined R_q roughness	1.0 μm
Viscosity η_0 of mineral oil lubricant	0.1 Pa s
Pressure viscosity coefficient α	2×10^{-8} Pa^{-1}
Effective Young's Modulus E^*	115 GPa

Table 1

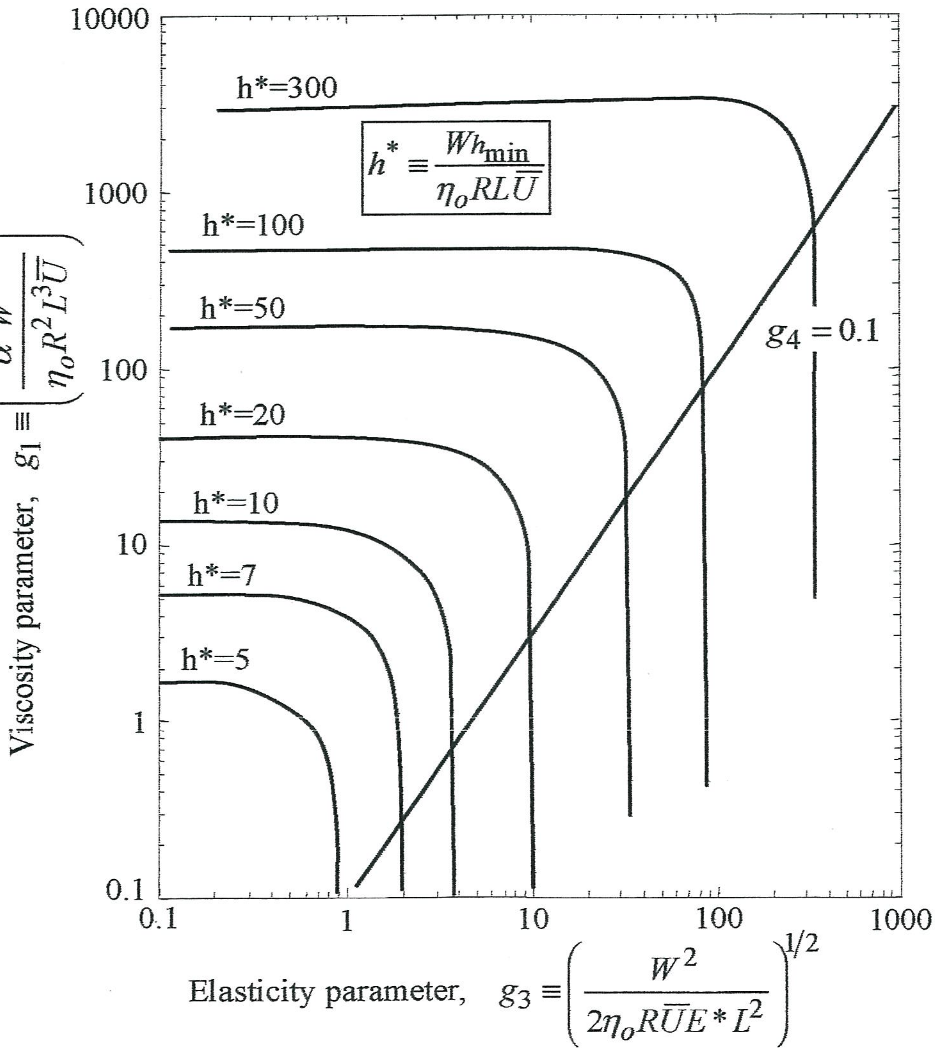

Fig. 2

2 Table 2 presents theoretical predictions for the dimensionless load carrying capacity W^* of a long journal bearing, where W^* is defined by

$$W^* = \frac{W/L}{\eta D\omega}\left(\frac{c}{R}\right)^2$$

with W the load, L the length of the bearing, η the oil viscosity, R and D the bearing radius and diameter and c the clearance.

(a) Outline how such predictions for W^* are obtained, explaining the concepts of the attitude angle ψ and eccentricity ratio ε and detailing appropriate fluid exit boundary conditions which can be used. [25%]

(b) Use the data of Table 2 to find the minimum film thickness in the bearing for the following conditions: $W = 500$ kN, $L = 0.4$ m, $R = 0.05$ m, $c = 50$ μm, $\eta = 0.05$ Pa s and $\omega = 200$ rad s^{-1}. [15%]

(c) What value of c should be chosen to maximise the minimum film thickness, with other conditions as specified in part (b)? [40%]

(d) Use the Petrov approximation to estimate the power dissipated in the bearing. How could this estimate be used to assess the influence of thermal effects on the minimum film thickness? [20%]

ε	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
W^*	0.458	0.882	1.304	1.767	2.318	3.048	4.165	6.284	12.47

Table 2

3 A simple parallel hybrid drive for a road vehicle consists of a *constant power* internal combustion engine driving the wheels through a continuously variable transmission (CVT). Energy is stored in a flywheel that is connected to the wheels through another CVT. The total torque driving the wheels is the sum of the torques from the CVTs of the engine and the flywheel. There are no friction brakes; deceleration is achieved by recovering energy into the flywheel.

(a) The driving force f required to move the vehicle is given by $f = Ma + C$ where M is the mass of the vehicle, a is the acceleration and C is a constant force. The vehicle undergoes a drive cycle consisting of constant acceleration from zero speed to speed V in T seconds, followed by constant deceleration back to zero speed in another T seconds.

(i) For one drive cycle (duration $2T$) sketch as a function of time the energy needed to provide the driving force f. Hence show that, providing $MV > CT$, the maximum energy occurs at time T and is equal to $(MV + CT)\,V/2$. [40%]

(ii) If the speed of the flywheel is the same at the beginning and the end of the drive cycle, and if the minimum flywheel speed during the cycle is equal to half the beginning speed, sketch the energy stored in the flywheel as a function of time. Annotate your sketch with expressions for the beginning energy and the minimum energy. [30%]

(b) The flywheel consists of a solid disc spinning about a central axis perpendicular to the plane of the disc. Derive an expression for the specific energy (energy per unit mass) of the flywheel in terms of the density ρ and allowable stress σ_a of the material. You may assume that the maximum stress σ in a solid disc of radius R rotating at angular speed ω is given by $\sigma = 0.5\rho R^2\omega^2$. [30%]

END OF PAPER

ENGINEERING TRIPOS Part IIB

Modules 4C16 Data Sheet

HYDRODYNAMIC LUBRICATION

Viscosity: temperature and pressure effects

$\eta = \eta_0$ at $p = 0$ and $T = T_0$

Vogel formula

$$\eta = \eta_0 \exp\left\{\frac{b}{T + T_c}\right\}$$

Barus equation

$$\eta = \eta_0 \exp\{\alpha p\}$$

Roelands equation

$$\eta = \eta_0 \exp\left\{\ln\left(\frac{\eta_0}{\eta_r}\right)\left[\left(1 + \frac{p}{p_r}\right)^{\beta}\left(\frac{T_0 + T_r}{T + T_r}\right) - 1\right]\right\}$$

p_r, T_r and η_r are reference values

Viscous pressure flow

Rate of flow q_x per unit width of fluid of viscosity η down a channel of height h due to pressure gradient $\frac{dp}{dx}$

$$q_x = -\frac{h^3}{12\eta}\frac{dp}{dx}$$

Reynolds' Equation for a steady configuration

1–D flow:

$$\frac{dp}{dx} = 12\eta\overline{U}\left\{\frac{h - h^*}{h^3}\right\}$$

$\overline{U}$ is the entraining velocity so that $|\overline{U}h^*|$ is flow per unit width through the contact.

2–D flow:

$$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right\} = 12\overline{U}\frac{\partial h}{\partial x}$$

ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where R_1, R_2 are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where E_1, E_2 and ν_1, ν_2 are Young's moduli and Poisson's ratios.

	Line contact (width $2b$; load W' per unit length)	Circular contact (diameter $2a$; load W)
Semi contact width or contact radius	$b = 2\left\{\frac{W'R}{\pi E^*}\right\}^{1/2}$	$a = \left\{\frac{3WR}{4E^*}\right\}^{1/3}$
Maximum contact pressure ("Hertz stress")	$p_0 = \left\{\frac{W'E^*}{\pi R}\right\}^{1/2}$	$p_0 = \frac{1}{\pi}\left\{\frac{6WE^{*2}}{R^2}\right\}^{1/3}$
Approach of centres	$\delta = \frac{2W'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right)-\frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right)-\frac{1}{2}\right\}\right]$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{W^2}{E^{*2}R}\right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{W'}{2b} = \frac{\pi}{4}p_0$	$\bar{p} = \frac{W}{\pi a^2} = \frac{2}{3}p_0$
Mean shear stress	$\tau_{max} = 0.30p_0$ at $x = 0$, $z = 0.79b$	$\tau_{max} = 0.31p_0$ at $r = 0$, $z = 0.48a$ for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0$

Mildly elliptical contacts

If the gap at zero load is $h = \frac{1}{2}Ax^2 + \frac{1}{2}By^2$, and $0.2 < A/B < 5$

Ratio of semi-axes $b/a \cong (A/B)^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (AB)^{-1/2}$ or better $R_e = [AB(A+B)/2]^{-1/3}$.

For **approach** use circular contact equation with $R = (AB)^{-1/2}$ (**not** R_e)

ELASTOHYDRODYNAMIC LUBRICATION

Formulae for line contact film thickness

$\bar{U}$ is the entraining velocity, R is the effective radius of curvature and E^* is the contact modulus (see elastic contact stress formulae).

Rigid isoviscous (Kapitza)

$$h_c = 4.9\frac{\bar{U}\eta_0 RL}{W}$$

Ertel-Grubin

$$\frac{\bar{h}}{R} = 1.37\left(\frac{\eta_0\alpha(2\bar{U})}{R}\right)^{3/4}\left(\frac{E^* RL}{W}\right)^{1/8}$$

Dowson and Higginson

$$\frac{\bar{h}}{R} = 1.6(2\alpha E^*)^{0.54}\left(\frac{\bar{U}\eta_0}{2E^* R}\right)^{0.7}\left(\frac{W/L}{2E^* R}\right)^{-0.13}$$

EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where } R = \frac{A}{S}$$

MPFS, DJC, JAW
January 2009

4C16 2011 Answers

1 (a) -

(b)

	base circle	tip circle
p_0	95.7 MPa	586 MPa
g_1	0.77	66.9
g_3	0.16	2.28
h^*	3	30
h_{min}	2.5 μm	1.18 μm

2 (a) -

(b) 35.6 μm

(c) 55.0 μm

(d) 12.6 kW

3 (a) (i) -

(ii) energy at start $2MV^2/3$

minimum energy $MV^2/6$

(b) $\sigma_a/2\rho$