ENGINEERING TRIPOS PART IIB

Monday 2 May 2011 9 to 10.30

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SKH02

1 Consider a reinforced concrete chimney of uniform cross-section. Assume that this chimney can be idealized as a single degree of freedom structure with a height of 100 m, a bending stiffness EI of 8×10^{13} Nm^2, and a mass M of 4×10^6 kg, as shown in Fig. 1(a). Assume that the chimney is clamped at the base.

(a) Estimate the natural frequency of the system and determine the maximum shear in the structure due to the impulse of horizontal ground acceleration shown in Fig. 1(b). [15%]

(b) You are asked to design a base isolation system for the chimney (without damping), as shown in Fig. 1(c). The mass of the base is 2/3 of that of the superstructure ($M_b = 2M/3$). Determine the spring stiffness, k_b, required to reduce the natural frequency of the structure to 20% of that found in part (a). Estimate the maximum shear in the superstructure due to the impulse shown in Fig. 1(b). (Hint: Assume that the superstructure remains rigid, $EI = \infty$, for this initial design). [20%]

(c) Determine the actual natural frequencies and mode shapes of the isolated structure. [30%]

(d) Determine the actual maximum shear in the superstructure of the isolated structure. Comment on the relative contributions of each mode to the response, and on the effectiveness of the base isolation. [35%]

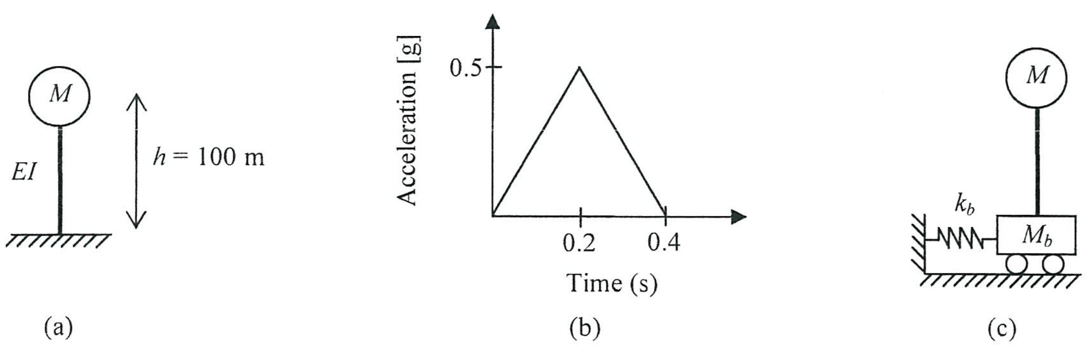


Fig. 1

2 (a) Explain how you would determine the natural frequency of a distributed mass system. [20%]

(b) A 15 m high building could be considered to act as a cantilever beam with flexural stiffness *EI* of 6×10^7 Nm^2 and self-weight *m* of 50,000 kgm^{-1}, as shown in Fig. 2. This building is expected to undergo flexural vibrations with a mode shape as follows:

$$\bar{u}_n(x) = 1 - \cos\frac{n\pi x}{30}$$

where *n* indicates the mode of vibration. Determine the first and second mode natural frequencies for this beam. [40%]

(c) Comment on the suitability of the mode shape function above in representing the deformation of the building. [10%]

(d) Explain the three methods by which the responses in each of the building's vibration modes might be superposed and comment on the advantages and disadvantages of each method. [30%]

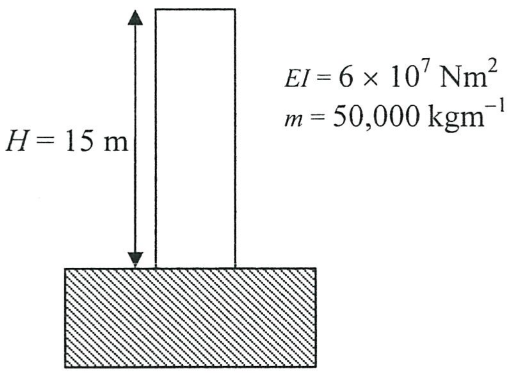


Fig. 2

3 (a) Explain briefly what soil characteristics are necessary for liquefaction to be a risk and what measures can be taken to prevent it from occurring. [20%]

(b) A machine tool is mounted on a concrete pad foundation that has plan dimensions of 2 m × 2 m and is 0.5 m thick. The concrete has a unit weight of 24 kNm^{-3}. The base of the foundation pad is to be embedded to a depth of 0.5 m below the ground surface. The site consists of a loose, sandy layer with a saturated unit weight of 19 kNm^{-3} and a void ratio of 0.8. The Poisson's ratio of sand may be taken as 0.3. The water table at this site is at the ground surface. By considering a reference plane 1 m below the underside of the pad foundation, calculate the horizontal and rotational stiffness offered by the foundation. [30%]

(c) The machine tool mounted on the foundation has a mass of 20 tonnes at an average height of 1 m above the ground surface. Estimate the natural frequency of the machine-foundation system for both horizontal and rocking vibrations. [25%]

(d) If the machine tool runs at a speed of 750 rpm, comment on the suitability of this foundation system for this machine tool. [10%]

(e) What measures could be taken to improve the performance of the foundation system in minimising unwanted vibration of the machine tool? [15%]

4 Explain briefly:

(a) the resolution of D'Alembert's paradox for the drag force on a body in a fluid. [10%]

(b) why bubbles rising in beer do not accelerate at around 1000g. [10%]

(c) flutter. [30%]

(d) how rain can cause cables to undergo aerodynamic instabilities in the wind. [10%]

(e) the different characteristics of the pressure profiles generated by high explosives and by gas blasts. [20%]

(f) aerodynamic admittance. [20%]

END OF PAPER

--

Engineering Tripos Part IIB/EIST Part II FOURTH YEAR

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$ $\quad \omega = 2\pi f$

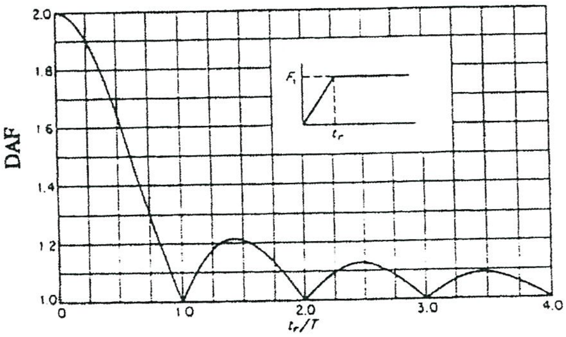
Modal analysis of simply-supported uniform beams

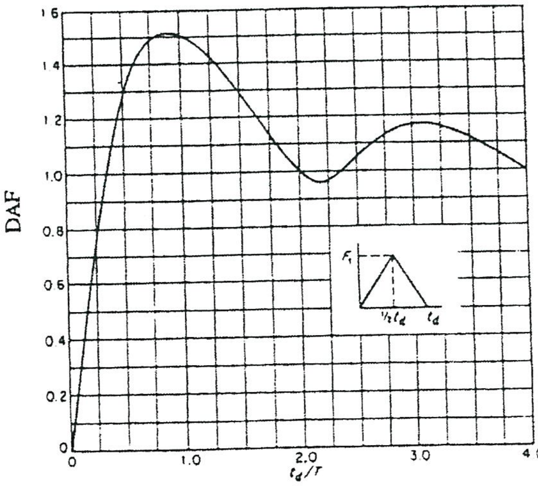
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

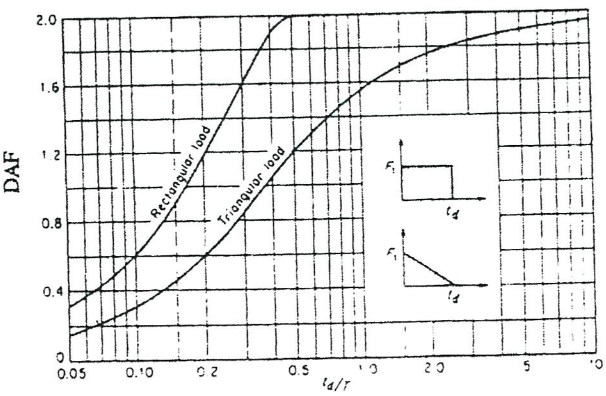
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

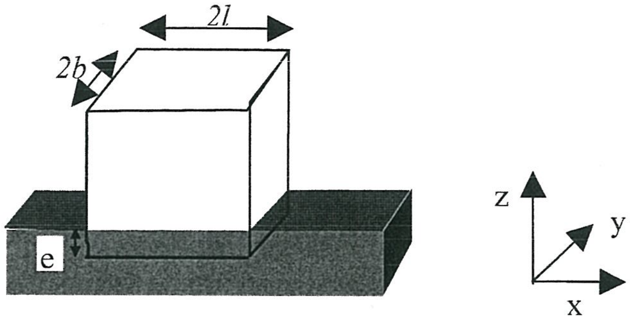
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^{3}}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^{3}}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^{3}\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma_v' \frac{(1+2K_o)}{3}$$

where σ_v' is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3-e)^2}{(1+e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \, . \, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

where

$$\tau_{max} = \left[\left(\frac{1 + K_o}{2} \sigma'_v \sin \phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma'_v \right)^2 \right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006

IIB 2011 4D6 ANSWERS

Q1:	a)	f=1.23 Hz Vmax=25.5 MN	
	b)	Kb=16MN/m Vmax=5.9 MN	
	c)	f1=0.25 Hz f2=1.97 Hz	
	d)	V1=6.1 MN V2=0.45 MN	
Q2	b)	M1=170,070 kg K1=54 kN/m f1=0.09 Hz	M2=1,125,000 kg K2=866 kN/m f2=0.14 Hz
Q3	c)	frock=7.4 Hz,	fh=15.5 Hz