ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2011 2.30 to 4

Module 4D7

CONCRETE AND MASONRY STRUCTURES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet (4 pages).
(ii) The Cumulative Normal Distribution Function (1 page).

STATIONERY REQUIREMENTS
Single-sided script paper
Graph Paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

JML/CJB04

1 (a) Describe briefly at least three failures of concrete structures in the past fifty years, with differing primary cause(s) of failure. In each instance outline the reason(s) for the failure and discuss the implications for and effects on Codes of Practice, and whether the failure could have been avoided if the designer had used larger factors of safety on the main loadings. [45%]

(b) The probability density function for the maximum value of a certain loading on a certain structure during its design life is made up of straight lines as shown in Fig. 1.

(i) Show that the maximum probability density ϕ_m is $1/(4\sigma)$ and that the characteristic value of this loading is $L_m + 2.106\sigma$. [10%]

(ii) The strength of such structures against this loading has a probability density function of the same form as in Fig. 1, with the same magnitude σ but mean value S_m. Determine the probability of failure under this loading for a structure designed to have characteristic strength equal to the characteristic value of this loading. How could an engineer ensure that the probability of failure is zero? [30%]

(iii) Comment briefly on how these results differ from those obtained when both probability density functions are the normal distribution with standard deviation σ. [15%]

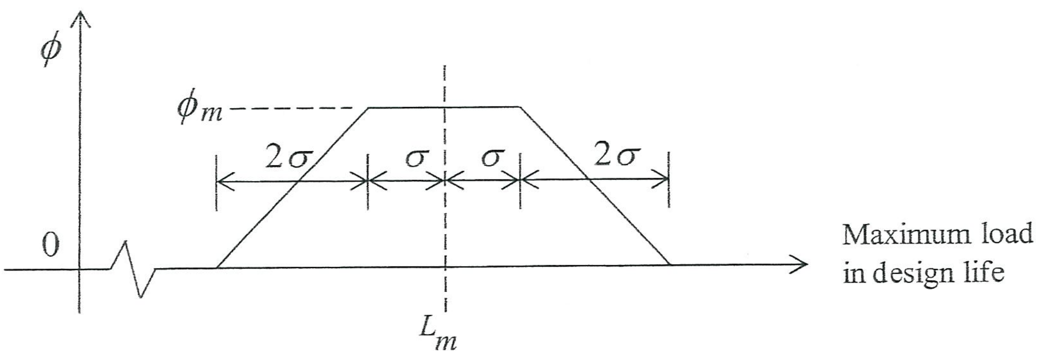


Fig. 1

2 (a) Write brief notes on the effects that the amount, form and behaviour of any water present have on the strength, behaviour and performance of typical reinforced concrete structures, during the casting, curing, hardening and subsequent life of the structure. [50%]

(b) Seven years after the construction of a concrete structure, with specified cover 30 mm to the steel reinforcement, a phenolphthalein indicator test on a core taken from an exposed surface gave a deep pink colour beyond 10 mm from the surface. Concrete dust samples were taken from the same region of the structure at a depth of 12 mm four and eight years after construction, and showed chloride concentration per unit weight of cement 0.2% and 0.6% respectively.

(i) Without considering detailed solutions of the diffusion equation, obtain three simple bounds on the age at which corrosion of the reinforcement will be initiated. [15%]

(ii) Refine your estimate of the time to corrosion initiation by considering a solution to the diffusion equation. [25%]

(iii) What factors will affect the rate of corrosion after initiation? [10%]

3 An experiment is conducted where a simply-supported reinforced concrete beam is subjected to two vertical point loads at locations one third distance from each end of the beam. The point loads are of equal magnitude. The concrete cross-section is rectangular with a width of 200 mm and an overall height of 330 mm. The concrete stiffness is 20 GPa and the concrete tensile strength is 4 MPa. All steel reinforcement has a Young's modulus of elasticity of 200 GPa and a design yield stress of 475 MPa. In the constant moment region, the longitudinal reinforcement is located in a single layer where the centroid is at a distance of 30 mm from the bottom surface of the beam. In the shear spans, vertical steel stirrups with a diameter of 8 mm are located at 100 mm spacings. The applied loads are increased gradually and, when the moment in the constant moment region reaches 17.4 kNm, first cracking is observed.

(a) At an applied moment of 48.6 kNm, the neutral axis depth at a cracked cross-section in the constant moment region is measured to be 102.2 mm. At this stage, the concrete and the steel can be assumed to behave elastically and the concrete can be assumed to carry no tension.

(i) By considering a cracked section, calculate the percentage of internal longitudinal steel reinforcement in the constant moment region. [15%]

(ii) If an initial approximation is made whereby, in calculating curvature, the contribution of the steel to the *uncracked* second moment of area is assumed to be negligible, predict the average curvature through the constant moment region. [25%]

(iii) If the contribution of the steel to the uncracked second moment of area is taken into account then find the corresponding uncracked second moment of area. Comment on the implications in connection with the approximation made in part (ii). [25%]

(b) A similar beam is being designed to sustain a factored design shear load where each of the point loads has a magnitude of 150 kN. Use the variable-angle truss analogy to calculate the range of strut angles that would result in a safe design. The factored design compressive stress in a concrete strut can be taken as 8 MPa. [35%]

4 (a) An axial load, N, of 600 kN is applied to the centre of a 325 mm × 325 mm square reinforced concrete column. The internal longitudinal reinforcement consists of 4 No. 32 mm diameter bars as shown in Fig. 2. The cover distance to the centroid of the reinforcement steel is 40 mm. The factored design yield stress of the longitudinal steel is 410 MPa. The concrete has a factored design compressive cube strength of f_{cd} = 30 MPa. The concrete is assumed to fail in compression at a uniform stress of 0.6 f_{cd}. The column is stocky and all the steel can be assumed to yield ($\varepsilon_y/\varepsilon_{cm}$ is small).

(i) If the column is subjected to uni-axial bending about the x-x axis, find the maximum bending moment that the section can withstand. [15%]

(ii) In addition to bending about the x-x axis, the column is also subjected to bending about the y-y axis. When the bending moment about the y-y axis is fixed at 12 kNm, find the location of the neutral axis at failure and the corresponding value of M_{xx}. Discuss how approximations of bi-axial failure surfaces can be used in design. [55%]

(b) Compare and contrast a load-bearing masonry building versus a reinforced concrete frame structure with infill masonry walls in terms of constructability, structural, fire, aesthetic, acoustic and thermal performance. [30%]

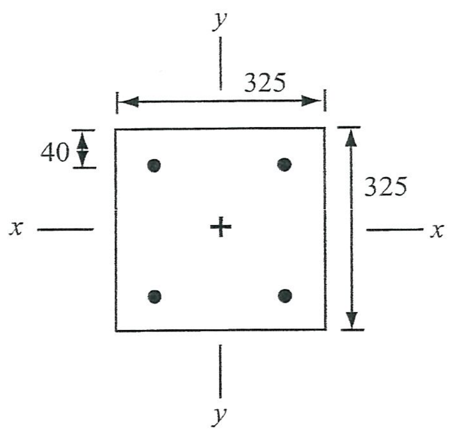


Fig. 2

END OF PAPER

Engineering Tripos Part IIB FOURTH YEAR

Module 4D7 : Concrete and masonry structures

Formula and Data Sheet

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

Material variability, partial safety factors and probability of failure

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually γ_m is 1.15 for steel, 1.5 for concrete; and γ_f is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Convolution integral $$P_f = \int_{-\infty}^{+\infty} f_S(x)\, F_R(x)\, dx$$

Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$$\left[3.15\frac{W}{C} - 1.14h\right] \Big/ \left[1 + 3.15\frac{W}{C}\right]$$

for hydration degree h :

and gel/space ratio (gel volume / gel + capillaries) $2.14h \big/ \left[h + 3.15\, {}^{W}\!/_{C} + a\right]$.

Mechanical properties of concrete

Cracking strain typically 150×10^{-6}, strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

Durability considerations

Present value of some future good: $S_i/(1 + r)^i$ for stepped,

or $S_i/\exp(r_c t_i)$ for continuous discounting

where $(1 + r) = \exp(r_c)$

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\{2k(H+h_c)/\Delta n\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

solution $c = c_o(1-\mathrm{erf}(z)),\ z = x/2\sqrt{Dt}$

Table of erf (z) :

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
erf (z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	
z	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Passivation for pH > 12 and Cl^- < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 μA/cm^2, resistivity > 100 k Ω cm, half-cell potential > –200 mV (but probable for < –350 mV).

SLS : cracking
Steel minimum area $A_s f_y \geq f_t A_{ct}$
for pure tension, to produce multiple cracks.

Then, limitation of crack width to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$
Where crack spacing $s_{r,\max} = 3.4c + 0.425 k_1 k_2 \phi / \rho_{p,eff}$

with k_1 0.8 for high bond, 1.6 for plain bars;
k_2 1.0 for tension, 0.5 for bending.

SLS : deflection
Interpolated curvature

$$\alpha = \zeta\alpha_{II} + (1-\zeta)\alpha_I$$

where $\zeta = 1 - \beta(\sigma_{sr}/\sigma_s)^2$

β is 1.0 for short-term, 0.5 for sustained load,
σ_{sr} is steel stress, for cracked section, but using loads which first cause cracking at the section considered.
σ_s is current steel stress, calculated for cracked section.

ULS : moment and axial force

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel ε_y of course depends on strength, as roughly f_y/E. Initial calculations often use uniform stress of $0.6 f_{cd}$ on the compression zone at failure. With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d(1 - 0.5\, x/d)/\gamma_s;$$

where
$$x/d = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$$b_w + l_o/5 \le b_{actual},$$

where l_o is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$$e_2 = \frac{l_o^2}{\pi^2}\kappa_m$$

where κ_m is curvature at mid-height at failure and l_o is effective length.

Eurocode multiplies by further factors K_r and K_φ,

where $$K_r = \left(\frac{n_u - n}{n_u - n_{bal}}\right) \le 1$$

Shear in reinforced concrete

For ***unreinforced*** webs at ULS, shear strength in EC2 is

$$V_{Rd,c} = [\frac{0.18}{\gamma_c} k(100\rho_1 f_{ck})^{1/3} + 0.15\sigma_{cp}]b_w d$$
$$\ge (v_{\min} + 0.15\sigma_{cp})b_w d$$

where: $k = 1 + \sqrt{200/d} \le 2.0$ a factor that varies with effective depth, d (with d in mm),
ρ_l is the reinforcement ratio of anchored steel= $A_s/b_w d$ but $\rho_l \le 0.02$.
$v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$

For ***reinforced*** webs at ULS, shear strength in EC2 is

- Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where:
$$f_{c,\max} = 0.6(1 - f_{ck}/250) f_{cd}$$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d)(\cot\theta)/(s\gamma_s)$$

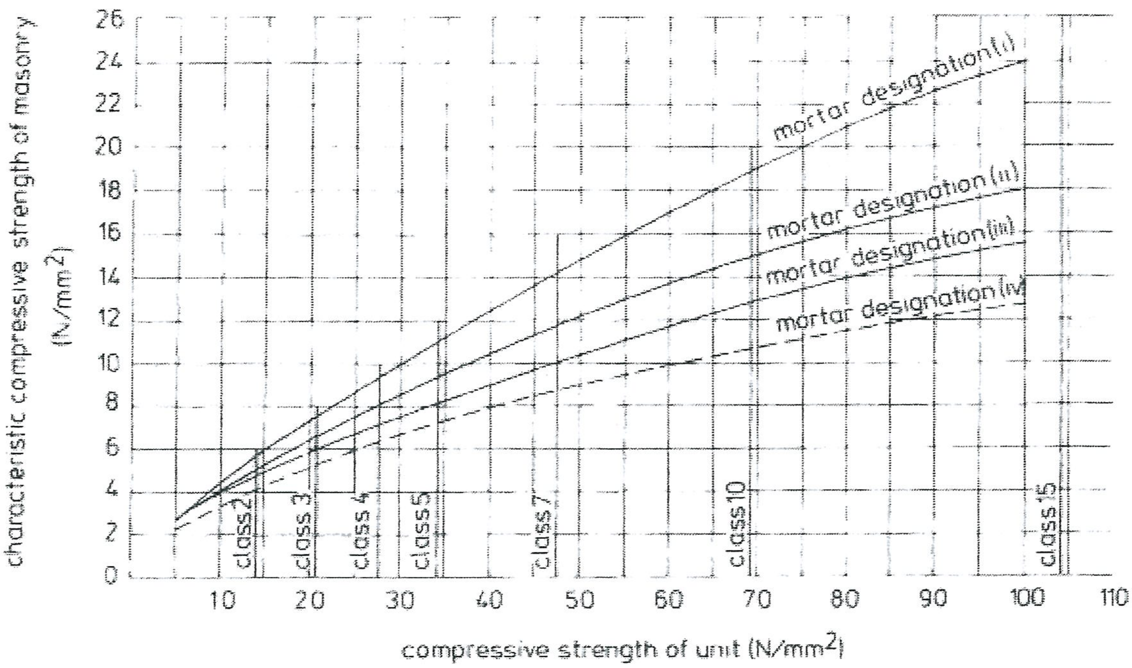
Torsion at ULS

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd}\sqrt{\frac{A_w \cdot \sum A_l}{s \cdot u}}$$

where $\sigma_c < \nu \cdot f_{cd}$

Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

Figure 5.6(a) Characteristic compressive strength, f_k, of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

CRM/JML Nov 2009

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}dx \quad \text{FOR } 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^{2}0097$	$\cdot 9^{2}0358$	$\cdot 9^{2}0613$	$\cdot 9^{2}0863$	$\cdot 9^{2}1106$	$\cdot 9^{2}1344$	$\cdot 9^{2}1576$
2·4	$\cdot 9^{2}1802$	$\cdot 9^{2}2024$	$\cdot 9^{2}2240$	$\cdot 9^{2}2451$	$\cdot 9^{2}2656$	$\cdot 9^{2}2857$	$\cdot 9^{2}3053$	$\cdot 9^{2}3244$	$\cdot 9^{2}3431$	$\cdot 9^{2}3613$
2·5	$\cdot 9^{2}3790$	$\cdot 9^{2}3963$	$\cdot 9^{2}4132$	$\cdot 9^{2}4297$	$\cdot 9^{2}4457$	$\cdot 9^{2}4614$	$\cdot 9^{2}4766$	$\cdot 9^{2}4915$	$\cdot 9^{2}5060$	$\cdot 9^{2}5201$
2·6	$\cdot 9^{2}5339$	$\cdot 9^{2}5473$	$\cdot 9^{2}5604$	$\cdot 9^{2}5731$	$\cdot 9^{2}5855$	$\cdot 9^{2}5975$	$\cdot 9^{2}6093$	$\cdot 9^{2}6207$	$\cdot 9^{2}6319$	$\cdot 9^{2}6427$
2·7	$\cdot 9^{2}6533$	$\cdot 9^{2}6636$	$\cdot 9^{2}6736$	$\cdot 9^{2}6833$	$\cdot 9^{2}6928$	$\cdot 9^{2}7020$	$\cdot 9^{2}7110$	$\cdot 9^{2}7197$	$\cdot 9^{2}7282$	$\cdot 9^{2}7365$
2·8	$\cdot 9^{2}7445$	$\cdot 9^{2}7523$	$\cdot 9^{2}7599$	$\cdot 9^{2}7673$	$\cdot 9^{2}7744$	$\cdot 9^{2}7814$	$\cdot 9^{2}7882$	$\cdot 9^{2}7948$	$\cdot 9^{2}8012$	$\cdot 9^{2}8074$
2·9	$\cdot 9^{2}8134$	$\cdot 9^{2}8193$	$\cdot 9^{2}8250$	$\cdot 9^{2}8305$	$\cdot 9^{2}8359$	$\cdot 9^{2}8411$	$\cdot 9^{2}8462$	$\cdot 9^{2}8511$	$\cdot 9^{2}8559$	$\cdot 9^{2}8605$
3·0	$\cdot 9^{2}8650$	$\cdot 9^{2}8694$	$\cdot 9^{2}8736$	$\cdot 9^{2}8777$	$\cdot 9^{2}8817$	$\cdot 9^{2}8856$	$\cdot 9^{2}8893$	$\cdot 9^{2}8930$	$\cdot 9^{2}8965$	$\cdot 9^{2}8999$
3·1	$\cdot 9^{3}0324$	$\cdot 9^{3}0646$	$\cdot 9^{3}0957$	$\cdot 9^{3}1260$	$\cdot 9^{3}1553$	$\cdot 9^{3}1836$	$\cdot 9^{3}2112$	$\cdot 9^{3}2378$	$\cdot 9^{3}2636$	$\cdot 9^{3}2886$
3·2	$\cdot 9^{3}3129$	$\cdot 9^{3}3363$	$\cdot 9^{3}3590$	$\cdot 9^{3}3810$	$\cdot 9^{3}4024$	$\cdot 9^{3}4230$	$\cdot 9^{3}4429$	$\cdot 9^{3}4623$	$\cdot 9^{3}4810$	$\cdot 9^{3}4991$
3·3	$\cdot 9^{3}5166$	$\cdot 9^{3}5335$	$\cdot 9^{3}5499$	$\cdot 9^{3}5658$	$\cdot 9^{3}5811$	$\cdot 9^{3}5959$	$\cdot 9^{3}6103$	$\cdot 9^{3}6242$	$\cdot 9^{3}6376$	$\cdot 9^{3}6505$
3·4	$\cdot 9^{3}6631$	$\cdot 9^{3}6752$	$\cdot 9^{3}6869$	$\cdot 9^{3}6982$	$\cdot 9^{3}7091$	$\cdot 9^{3}7197$	$\cdot 9^{3}7299$	$\cdot 9^{3}7398$	$\cdot 9^{3}7493$	$\cdot 9^{3}7585$
3·5	$\cdot 9^{3}7674$	$\cdot 9^{3}7759$	$\cdot 9^{3}7842$	$\cdot 9^{3}7922$	$\cdot 9^{3}7999$	$\cdot 9^{3}8074$	$\cdot 9^{3}8146$	$\cdot 9^{3}8215$	$\cdot 9^{3}8282$	$\cdot 9^{3}8347$
3·6	$\cdot 9^{3}8409$	$\cdot 9^{3}8469$	$\cdot 9^{3}8527$	$\cdot 9^{3}8583$	$\cdot 9^{3}8637$	$\cdot 9^{3}8689$	$\cdot 9^{3}8739$	$\cdot 9^{3}8787$	$\cdot 9^{3}8834$	$\cdot 9^{3}8879$
3·7	$\cdot 9^{3}8922$	$\cdot 9^{3}8964$	$\cdot 9^{4}0039$	$\cdot 9^{4}0426$	$\cdot 9^{4}0799$	$\cdot 9^{4}1158$	$\cdot 9^{4}1504$	$\cdot 9^{4}1838$	$\cdot 9^{4}2159$	$\cdot 9^{4}2468$
3·8	$\cdot 9^{4}2765$	$\cdot 9^{4}3052$	$\cdot 9^{4}3327$	$\cdot 9^{4}3593$	$\cdot 9^{4}3848$	$\cdot 9^{4}4094$	$\cdot 9^{4}4331$	$\cdot 9^{4}4558$	$\cdot 9^{4}4777$	$\cdot 9^{4}4988$
3·9	$\cdot 9^{4}5190$	$\cdot 9^{4}5385$	$\cdot 9^{4}5573$	$\cdot 9^{4}5753$	$\cdot 9^{4}5926$	$\cdot 9^{4}6092$	$\cdot 9^{4}6253$	$\cdot 9^{4}6406$	$\cdot 9^{4}6554$	$\cdot 9^{4}6696$
4·0	$\cdot 9^{4}6833$	$\cdot 9^{4}6964$	$\cdot 9^{4}7090$	$\cdot 9^{4}7211$	$\cdot 9^{4}7327$	$\cdot 9^{4}7439$	$\cdot 9^{4}7546$	$\cdot 9^{4}7649$	$\cdot 9^{4}7748$	$\cdot 9^{4}7843$
4·1	$\cdot 9^{4}7934$	$\cdot 9^{4}8022$	$\cdot 9^{4}8106$	$\cdot 9^{4}8186$	$\cdot 9^{4}8263$	$\cdot 9^{4}8338$	$\cdot 9^{4}8409$	$\cdot 9^{4}8477$	$\cdot 9^{4}8542$	$\cdot 9^{4}8605$
4·2	$\cdot 9^{4}8665$	$\cdot 9^{4}8723$	$\cdot 9^{4}8778$	$\cdot 9^{4}8832$	$\cdot 9^{4}8882$	$\cdot 9^{4}8931$	$\cdot 9^{4}8978$	$\cdot 9^{5}0226$	$\cdot 9^{5}0655$	$\cdot 9^{5}1066$
4·3	$\cdot 9^{5}1460$	$\cdot 9^{5}1837$	$\cdot 9^{5}2199$	$\cdot 9^{5}2545$	$\cdot 9^{5}2876$	$\cdot 9^{5}3193$	$\cdot 9^{5}3497$	$\cdot 9^{5}3788$	$\cdot 9^{5}4066$	$\cdot 9^{5}4332$
4·4	$\cdot 9^{5}4587$	$\cdot 9^{5}4831$	$\cdot 9^{5}5065$	$\cdot 9^{5}5288$	$\cdot 9^{5}5502$	$\cdot 9^{5}5706$	$\cdot 9^{5}5902$	$\cdot 9^{5}6089$	$\cdot 9^{5}6268$	$\cdot 9^{5}6439$
4·5	$\cdot 9^{5}6602$	$\cdot 9^{5}6759$	$\cdot 9^{5}6908$	$\cdot 9^{5}7051$	$\cdot 9^{5}7187$	$\cdot 9^{5}7318$	$\cdot 9^{5}7442$	$\cdot 9^{5}7561$	$\cdot 9^{5}7675$	$\cdot 9^{5}7784$
4·6	$\cdot 9^{5}7888$	$\cdot 9^{5}7987$	$\cdot 9^{5}8081$	$\cdot 9^{5}8172$	$\cdot 9^{5}8258$	$\cdot 9^{5}8340$	$\cdot 9^{5}8419$	$\cdot 9^{5}8494$	$\cdot 9^{5}8566$	$\cdot 9^{5}8634$
4·7	$\cdot 9^{5}8699$	$\cdot 9^{5}8761$	$\cdot 9^{5}8821$	$\cdot 9^{5}8877$	$\cdot 9^{5}8931$	$\cdot 9^{5}8983$	$\cdot 9^{6}0320$	$\cdot 9^{6}0789$	$\cdot 9^{6}1235$	$\cdot 9^{6}1661$
4·8	$\cdot 9^{6}2067$	$\cdot 9^{6}2453$	$\cdot 9^{6}2822$	$\cdot 9^{6}3173$	$\cdot 9^{6}3508$	$\cdot 9^{6}3827$	$\cdot 9^{6}4131$	$\cdot 9^{6}4420$	$\cdot 9^{6}4696$	$\cdot 9^{6}4958$
4·9	$\cdot 9^{6}5208$	$\cdot 9^{6}5446$	$\cdot 9^{6}5673$	$\cdot 9^{6}5889$	$\cdot 9^{6}6094$	$\cdot 9^{6}6289$	$\cdot 9^{6}6475$	$\cdot 9^{6}6652$	$\cdot 9^{6}6821$	$\cdot 9^{6}6981$

Example: $\Phi(3{\cdot}57) = \cdot 9^{3}8215 = 0{\cdot}9998215$.