ENGINEERING TRIPOS PART IIB

Wednesday 4 May 2011 2.30 to 4.00

Module 4F2

ROBUST AND NONLINEAR SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

gv02

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1 (a) For a dynamical system $\dot{x} = f(x)$ explain what is meant by

(i) An *equilibrium point*, [10%]

(ii) An *invariant set*. [10%]

(b) State *LaSalle's Theorem* and explain why it is sometimes needed in addition to Lyapunov's Theorems. [30%]

(c) The motion of a mass suspended on a nonlinear spring and with nonlinear damping may be modelled by the equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -f(x_2) - g(x_1) \end{aligned}$$

where x_1 is the position of the mass, and the functions f and g satisfy the conditions

$$\sigma f(\sigma) \geq 0 \quad \text{and} \quad \sigma g(\sigma) \geq 0 \quad \text{for} \quad |\sigma| \leq \sigma_0,$$

$f(0) = 0$, $g(0) = 0$, and $f(\sigma) \neq 0$, $g(\sigma) \neq 0$ if $\sigma \neq 0$.

(i) Find the equilibrium point of this system. [10%]

(ii) By considering the function

$$V(x_1, x_2) = \frac{x_2^2}{2} + \int_0^{x_1} g(\sigma) d\sigma$$

show that this equilibrium point is asymptotically stable. [40%]

2 (a) Find the describing function of the nonlinearity $f_1(e) = e^3$.
(You may use the fact that $4\sin^3\theta = 3\sin\theta - \sin(3\theta)$.) [10%]

(b) Consider a linear system with transfer function

$$G(s) = \frac{k}{(s-1)(s+5)^2}$$

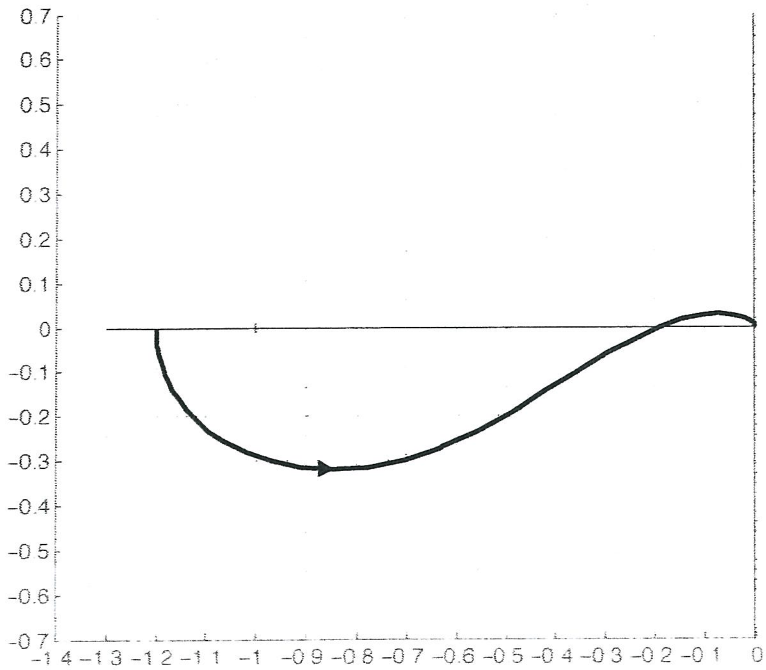
with $k > 0$. Figure 1 shows the Nyquist plot of this system when $k = 30$. Use the describing function method to investigate the existence and characteristics of any limit cycles when a system with this transfer function is connected to the nonlinearity $f_1(e) = e^3$ in a negative feedback loop, as shown in Fig. 2. [30%]

(c) Sketch the input-output characteristic of the nonlinearity

$$f_2(e) = \left(\frac{1}{1+|e|} + 1\right) e$$

Without further calculation, explain how the results of part (b) would change if the nonlinearity $f_1(e)$ were replaced by $f_2(e)$. [30%]

(d) State the *Circle Criterion* for nonlinear feedback systems, and comment on its applicability to the feedback loop shown in Fig. 2 when the nonlinear element is either $f_1(e)$ or $f_2(e)$. [30%]


0.7
0.6
0.5
0.4
0.3
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-0.1
-0.2
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-0.4
-0.5
-0.6
-0.7
-1.4 -1.3 -1.2 -1.1 -1 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0

Fig. 1

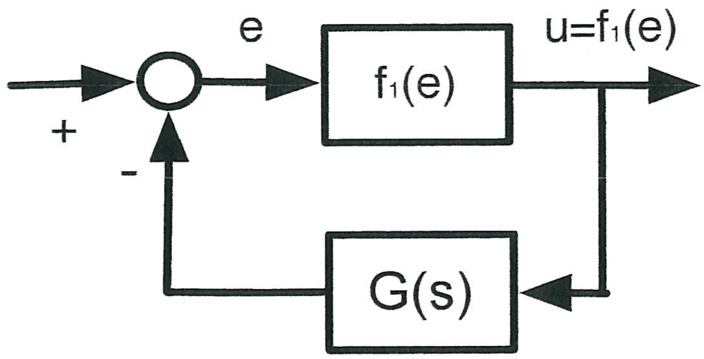
e
u=f1(e)
f1(e)
+
-
G(s)

Fig. 2

3 (a) Define the H_∞ norm of a transfer function matrix and discuss its interpretation in terms of the relationship between the input and output of a linear system. [30%]

(b) Calculate the H_∞ norm of any of the following transfer functions which are in H_∞:

(i) $G_1(s) = \dfrac{s-1}{2s+3}$

(ii) $G_2(s) = \dfrac{1}{s^2+0.1s+1}$

(iii) $G_3(s) = \dfrac{s}{s-1}$

(iv) $G_4(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+1} \\ \frac{2}{s+1} & \frac{1}{s+1} \end{bmatrix}$

[30%]

(c) If $P(s) = \sum_k \lambda_k P_k(s)$ for $k = 1,2,\ldots,n$ where the P_k are stable transfer functions and the λ_k are positive constants satisfying $\sum_k \lambda_k \leq 1$, show that $\|P\|_\infty \leq \max_k \|P_k\|_\infty$. State a sufficient condition under which a feedback controller C stabilizes any P within this set. Is this condition necessary (give an example)? [40%]

4 (a) Find the transfer function from $\begin{bmatrix} w \\ z \end{bmatrix}$ to $\begin{bmatrix} y \\ u \end{bmatrix}$ in Fig. 3, where P and C are transfer function matrices of compatible dimensions. [30%]

(b) Given P and C, as in Fig. 3, state the necessary and sufficient condition under which $\|(I - CP_\delta)^{-1}\|_\infty < \gamma$ for all P_δ of the form $P_\delta = (I + \Delta)P$, $\Delta \in H_\infty$, $\|\Delta\|_\infty < \varepsilon$. [40%]

(c) Assuming the condition derived in (b) is satisfied, characterize in terms of P all those systems $\hat{P}$ for which $\|\hat{P}(I - C\hat{P})^{-1}C\|_\infty < 1/\varepsilon$. [30%]

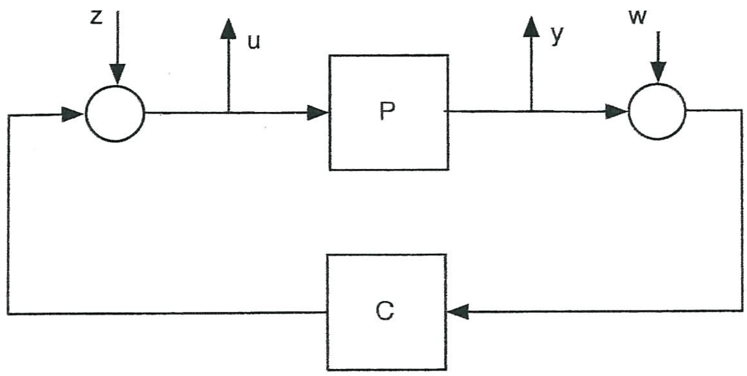

Fig. 3

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