ENGINEERING TRIPOS PART IIB

Monday 9 May 2011 9 to 10.30

Module 4F5

ADVANCED WIRELESS COMMUNICATIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

agif03

1 (a) Consider the channel shown in Fig. 1.

(i) Find the channel capacity of the channel. [15%]

(ii) How does the channel capacity depend on p? [10%]

(iii) What is the capacity-achieving input distribution? [10%]

(b) Show that

$$\Pr\{i(\bar{\mathbf{X}},\mathbf{Y}) > \log_2 \beta\} \leq \frac{1}{\beta}$$

where

$$i(\mathbf{x},\mathbf{y}) = \log_2 \frac{P_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x})}{P_{\mathbf{Y}}(\mathbf{y})}$$

and where $\bar{\mathbf{X}}$ and $\mathbf{Y}$ are independent. [25%]

(c) Hence show that the average random coding error probability of a code with M codewords can be upper-bounded as

$$\bar{P}_e \leq \Pr\{i(\mathbf{X},\mathbf{Y}) \leq \log_2 \beta\} + \frac{M-1}{\beta}.$$

[25%]

(d) Using this bound, show the achievability part of Shannon's capacity theorem. [15%]

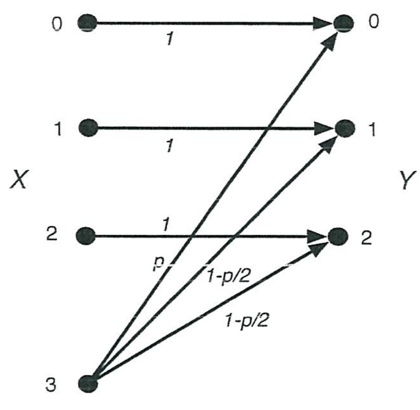

Fig. 1

2 Pulse-position modulation (PPM) is a modulation scheme that maps the information bits onto the position of a pulse within the signalling period T_s. In particular, M-PPM uses the following signals for transmission

$$x_m(t) = \begin{cases} 1 & (m-1)\frac{T_s}{M} \leq t \leq m\frac{T_s}{M} \\ 0 & \text{otherwise.} \end{cases}$$

for $m = 1, \ldots, M$.

(a) What is the dimension of the signal space? [10%]

(b) Find a suitable orthonormal basis for the signal space. [20%]

(c) Write down the vector representation as a function of the basis. [20%]

(d) Find the energy of each signal vector and the corresponding average energy. [10%]

(e) Draw the block diagram of the optimal receiver. [15%]

(f) Write down the equations of the optimal detector and use the union bound to find an upper bound to the probability of error. [15%]

(g) Explain why the union bound is not a probability, and explain why it is still a useful bound that characterises the error probability. [10%]

3 Consider the code whose generator matrix is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Write down the corresponding parity-check matrix in systematic form and the parity-check equations. What is the code rate? [20%]

(b) Write down the variable and check-node degree distribution polynomials (node perspective) of the code, interpreted as a low-density parity-check code, i.e., $\Lambda(x)$ and $P(x)$. [10%]

(c) Find the minimum distance of the code. [20%]

(d) The code is used to transmit information over a binary symmetric channel (BSC) with crossover probability $\varepsilon < 0.5$.

(i) Show that finding the maximum likelihood (ML) codeword is equivalent to finding the codeword at minimum Hamming distance from the received word $\mathbf{y}$. [25%]

(ii) The code is used to transmit 9 independent bits of information. Find the ML estimate of the information bit stream if the received word is

$$\mathbf{y} = [1\,0\,1\,1\,1\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,1\,1\,1\,0\,1\,0].$$

[25%]

4 (a) Consider transmission over a fading channel with the multipath intensity profile given in Fig. 2 and Doppler spectrum following the Jakes model given by

$$S_H(\xi) = \begin{cases} \frac{1}{\pi f_m \sqrt{1-(\xi/f_m)^2}} & |\xi| \leq f_m \\ 0 & |\xi| > f_m \end{cases}$$

where f_m is the maximum Doppler frequency.

(i) What is the coherence bandwidth of the channel? [10%]

(ii) What is the coherence time of the channel if the carrier frequency is $f_c = 10$ MHz and the mobile user is moving at a velocity $v = 10$ km h^{-1}? [15%]

(iii) Will the channel introduce frequency or time selectivity if codewords of duration $T_x = 30$ ms using signals of bandwidth $B_x = 1.5$ MHz are employed for transmission. [15%]

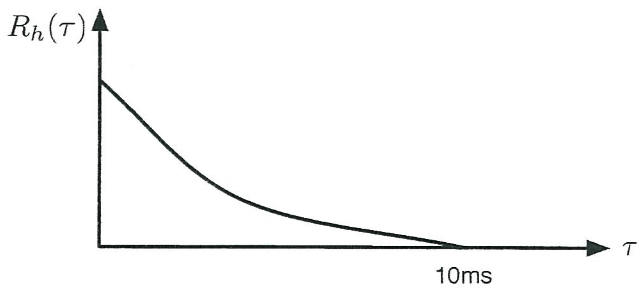

Fig. 2

(b) Consider a multiple-input single-output (MISO) channel described by

$$y = \mathbf{h}^T \mathbf{x} + z$$

where $\mathbf{h} = (h_1, \ldots, h_{n_t}) \in \mathbb{C}^{n_t}$ is the vector of fading coefficients, with n_t being the number of transmit antennas. Assume that the entries of $\mathbf{h}$ are zero mean, unit variance complex Gaussian random variables.

(i) Suppose that at every time step one symbol is transmitted, each time from a different antenna. What is the diversity order of this scheme? What is the rate? [20%]

(ii) Suppose that a repetition code is employed, and that at every time step one symbol is transmitted, each time from a different antenna. What is the diversity order of this scheme? What is the rate? [20%]

(iii) Suppose now that $n_t = 2$ and that Alamouti's scheme is employed, i.e.,

$$[y_1 \; y_2] = [h_1 \; h_2] \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + [n_1 \; n_2].$$

What is the diversity order achieved by this scheme? What is the rate? [20%]

END OF PAPER