ENGINEERING TRIPOS PART IIB

Wednesday 11 May 2011 9 to 10.30

Module 4F6

SIGNAL DETECTION AND ESTIMATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

wjf02

1 (a) Describe, in detail, how *Maximum Entropy* methods may be used to assign probability distributions. [30%]

(b) Given the first moment of a distribution (from experimental measurements, for example) show, using Lagrange multipliers, that the distribution having *Maximum Entropy* is an exponential distribution. [40%]

(c) Show that the entropy of this distribution is:

$$H = \ln \sigma + 1$$

where σ is the standard deviation of the distribution. [30%]

2 (a) Define *Fisher Information* and show that for an unbiased estimator, $\hat{\theta}(x)$, of a parameter θ, the variance associated with $\hat{\theta}(x)$ satisfies

$$var(\hat{\theta}(x)) \geq I_\theta^{-1}$$

where I_θ is the *Fisher Information* for the scalar parameter θ. [40%]

(b) Derive the following condition for an efficient unbiased estimator:

$$\frac{\partial \ln p(x|\theta)}{\partial \theta} = I_\theta(\hat{\theta}(x) - \theta)$$

where $p(x|\theta)$ is the likelihood function and $\hat{\theta}(x)$ is an estimator for θ. [30%]

(c) Describe how this equation leads to the *Neyman-Fisher* factorization theorem. [30%]

3 (a) Describe the role of numerical Bayesian methods in statistical inference and give examples for the cases of parameter estimation and model selection. [20%]

(b) Outline how one would obtain expressions for the marginal probability density functions for the intercept, c, and slope, m, of a straight line in the presence of white Gaussian noise, n_i, if the line can be modelled as

$$d_i = mx_i + c + n_i \; .$$

[40%]

(c) Show how the Gibbs sampler is applied to this problem assuming the noise variance is known and demonstrate how, in this case, the problem reduces to that of being able to sample from simple distributions. Derive expressions for these distributions. [40%]

4 (a) Describe how the *Maximum a posteriori* (MAP) and Bayes criteria can be applied to detection theory. Discuss the advantages and disadvantages of the Neyman-Pearson decision rule over the above two criteria. [25%]

(b) Describe how the threshold of the Neyman-Pearson decision rule may be obtained from the *Receiver Operator Characteristic* (ROC) curve. [15%]

(c) It is required to detect a line given by

$$s(n) = A + B\, n$$

where $n = 0, 1, 2, ..., N-1$, and is corrupted by additive white Gaussian noise of variance σ^2 where A and B are known.

(i) Show that the data may be written in the form of a *general linear model.* [30%]

(ii) Determine the Neyman-Pearson detector for this problem. [30%]

END OF PAPER