ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Monday 2 May 2011 2.30 to 4

Module 4M16

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4M16 data sheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

gtp03

1 (a) Define the following terms/units used to measure radioactivity and its effects on living tissue, giving units for items (i) to (iii):

(i) becquerel;

(ii) gray;

(iii) sievert;

(iv) weighting factors.

Why does the becquerel on its own not give an indication of the damage caused to living tissue? [25%]

(b) What basic principles are used to minimise exposure when working with ionising radiation? [15%]

(c) Magnox fuel elements have a nimonic spring at the top of each element to ensure proper location in the fuel channels. The springs are removed from the elements before transport to Sellafield for reprocessing and stored on site in shielded vaults. Nimonic contains about 20% cobalt, which during irradiation in the reactor picks up neutrons to form Co-60. In order to remove the active material from site on decommissioning, it will be necessary to transport the springs to an interim storage in a lead-shielded cylindrical container. The transport will not take place until five years after the shutdown of the reactor.

Using the information below and stating any assumptions made, estimate:

(i) the activity of the Co-60 just before transport; and [10%]

(ii) the surface dose on the container in $\mathrm{Sv\,hr^{-1}}$ and comment on the latter result. [50%]

Data

Activity of the springs at reactor shutdown: 1×10^{15} Bq

Thickness of lead shielding: 300 mm

Co-60 has a half-life of 5.272 yr and each decay releases two γ-rays of energies 1.17 MeV and 1.33 MeV respectively.

The macroscopic absorption cross-section for γ-rays in human tissue is $3\ \mathrm{m^{-1}}$.

The density of human tissue can be taken as $1000\ \mathrm{kg\,m^{-3}}$.

The exponential attenuation coefficient for γ-rays in lead is $0.046\ \mathrm{mm^{-1}}$.

$$D = 1.6\times10^{-13}\,\frac{E\Sigma\phi}{\rho}$$

where D is the dose rate in $\mathrm{Gy\,s^{-1}}$, E is the average γ-ray energy in MeV, Σ is the macroscopic absorption cross-section in $\mathrm{m^{-1}}$, ϕ is the γ-ray flux in $\mathrm{m^{-2}s^{-1}}$, and ρ is the density of human tissue in $\mathrm{kg\,m^{-3}}$.

2 You have been asked to use one-group diffusion theory to investigate the feasibility of a natural uranium fuelled, heavy water cooled and moderated reactor. The isotopic abundance of U-235 in natural uranium is 0.0072. A colleague has estimated that the uranium will occupy 10% of the volume of the core of such a reactor. The density of uranium is $18{,}900\ \mathrm{kg m^{-3}}$.

(a) Assuming that the combined macroscopic absorption cross-section of everything within the core other than the uranium is $1.0\ \mathrm{m}^{-1}$ and using the following data, show that the macroscopic absorption and fission cross-sections of the contents of the core are $4.67\ \mathrm{m}^{-1}$ and $2.00\ \mathrm{m}^{-1}$ respectively.

Data: U-235: $\sigma_c = 107$ barns, $\sigma_f = 580$ barns; U-238: $\sigma_c = 2.75$ barns, $\sigma_f = 0$ barns

Hence, given that the average number of neutrons released in a U-235 fission ν is 2.43, calculate η, where η is the average number of neutrons released per neutron absorbed in the core. [20%]

(b) The one-group neutron diffusion equation for a steady-state, source-free, homogeneous system can be written as

$$D\nabla^2\phi + (\eta - 1)\Sigma_a\phi = 0$$

in which ϕ is the neutron flux, Σ_a is the macroscopic absorption cross-section and D is the diffusion coefficient.

Stating any assumptions made, find the solution of this equation for a minimum volume system of rectangular parallelepiped geometry and the corresponding criticality condition. [50%]

(c) What will the dimensions of this system be if it is constructed with the core composition specified in (a)? Take the corresponding value of D to be 0.05 m. [10%]

(d) If the heavy water moderator and coolant is replaced by light water, the combined macroscopic absorption cross-section of everything within the core other than the uranium increases to $2.0\ \mathrm{m}^{-1}$ while the diffusion coefficient D reduces to 0.02 m.

Explain these changes qualitatively and discuss their implications for the feasibility of a natural uranium fuelled, light water moderated and cooled reactor. [20%]

3 (a) What factors determine the choice of coolants for nuclear reactors? List the major considerations and indicate current practice for major reactor types. [30%]

(b) A typical CANDU reactor fuel channel is rated at 6.0 MW(th). The channel contains 12 assemblies arranged end-to-end. Each assembly contains 37 fuel pins. A fuel pin consists of a solid UO_2 pellet of 12.0 mm diameter with a zircaloy cladding 0.4 mm thick. The axial power distribution is cosinusoidal over the 6.0 m active length, falling to zero at the ends. The coolant enters the channel at 250 °C.

(i) Without performing any calculations, sketch the form of the variation along the channel of the coolant temperature, the cladding surface temperature and the fuel pin temperature. [10%]

(ii) Find the locations and values of the maximum temperature in the coolant and in the fuel pins. [60%]

Data:	Channel coolant capacity	$\dot{m}c_p = 150\ \mathrm{kW\,K^{-1}}$
	Heat transfer coefficient to coolant	$h = 35\ \mathrm{kW m^{-2} K^{-1}}$
	Thermal conductivity of cladding	$\lambda_c = 13\ \mathrm{W m^{-1} K^{-1}}$
	Bond heat transfer coefficient, cladding to fuel	$h_b = 25\ \mathrm{kW m^{-2} K^{-1}}$
	Thermal conductivity of fuel	$\lambda_f = 3\ \mathrm{W m^{-1} K^{-1}}$

4 (a) Explain what is meant by the term *delayed neutrons*. Why are delayed neutrons so important to nuclear reactor dynamics? [15%]

(b) In a 'lumped' model of the kinetic behaviour of a source-free reactor operating at lower power, the equations for the neutron population $n(t)$ and the precursor population $c(t)$ can be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where all symbols have their usual meanings.

(i) A reactor has been operating at a neutron flux level n_0 for a very long time. What are the corresponding values of ρ and c in terms of n_0, β, λ and Λ? [15%]

(ii) The reactor is now subject to a step increase in reactivity of $\Delta\rho = 0.003$. Find the dominant time constant for the resulting excursion predicted by this model for the case where $\beta = 0.007$, $\lambda = 0.1\,\mathrm{s}^{-1}$ and $\Lambda = 10^{-4}$ s. [45%]

(iii) Compare the result from (ii) with that given using the *prompt jump approximation*. What are the safety implications of using the estimate given by the latter? [25%]

END OF PAPER

Data Sheet S/15

Revised January 2010

MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	299.792458×10^{6} $\mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi\times10^{-7}$ $\mathrm{H\,m^{-1}}$
Planck constant	h	6.626176×10^{-32} $\mathrm{J\,s}$
Boltzmann constant	k	1.380662×10^{-23} $\mathrm{J\,K^{-1}}$
Elementary charge	e	1.6021892×10^{-19} C

Definitions

Unified atomic mass constant	u	1.6605655×10^{-27} kg (931.5016 MeV)
Electron volt	eV	1.6021892×10^{-19} J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	10^{-28} $\mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

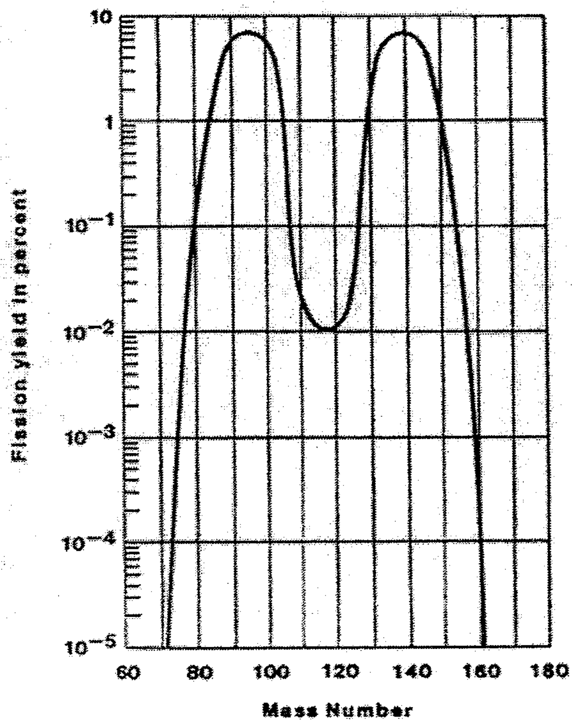
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow[33\text{ s}]{}$ Rb-90 $\xrightarrow[2.7\text{ min}]{}$ Sr-90 $\xrightarrow[28\text{ yr}]{}$ Y-90 $\xrightarrow[64\text{ hr}]{}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow[23\text{ min}]{}$ Te-131 $\xrightarrow[24\text{ min}]{}$ I-131 $\xrightarrow[8\text{ days}]{}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow[6.7\text{ hr}]{}$ Xe-135 $\xrightarrow[9.2\text{ hr}]{}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn.

Pm-149 $\xrightarrow[54\text{ hr}]{}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn.

Se-87 $\xrightarrow[17\text{ s}]{}$ Br-87 $\xrightarrow[55\text{ s}]{}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow[78\text{ min}]{}$ Rb-87}

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where $n =$ neutron concentration

$c =$ precursor concentration

$\beta =$ delayed neutron precursor fraction $= \sum \beta_i$

$\lambda =$ average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1 J_0(r)+A_2 Y_0(r)$$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2=D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\ \text{for small } x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity