ENGINEERING TRIPOS PART IIB 2012
4A3 TURBOMCHINERY 1

Dr N R Atkins

1) a) Standard proof: $\psi = 2(1 - \Lambda - \phi \tan\alpha_1)$ GIVE it!

b) i) $\Lambda = 0.5$, $\alpha_1 = 0$

$$\psi = 2(1-0.5) = 1$$

$$\Delta h_0 \text{ stage} = \psi U_m^2 = 1 \times 180^2 = 32.4 \text{ kJ kg}^{-1}$$

so Number of stages required $\frac{32.4}{350} = 10.8$ so 11 stages

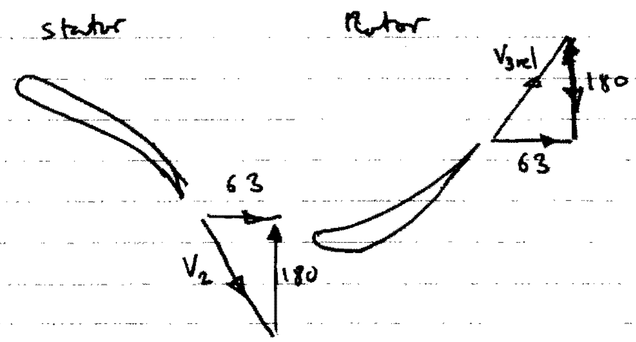
ii) $\Lambda = 0.3$, $\alpha_1 = -22°$, $\phi = 0.3$

$$\psi = 2(1 - 0.3 - 0.3\tan -22) = 1.64$$

$$\Delta h_0 \text{ stage} = \psi U_m^2 = 1.64 \times 180^2 = 53.2 \text{ kJ kg}^{-1}$$

so Number of stages required $\frac{53.2}{350} = 6.6$ so 7 stages

c)



50% reaction design

Flow coefficient $\phi = \frac{V_x}{U_m}$ so $V_x = 180 \times 0.35 = 63 \text{ ms}^{-1}$

So $V_2 = V_{3rel} = \sqrt{\{63^2 + 180^2\}} = 190.7 \text{ ms}^{-1}$

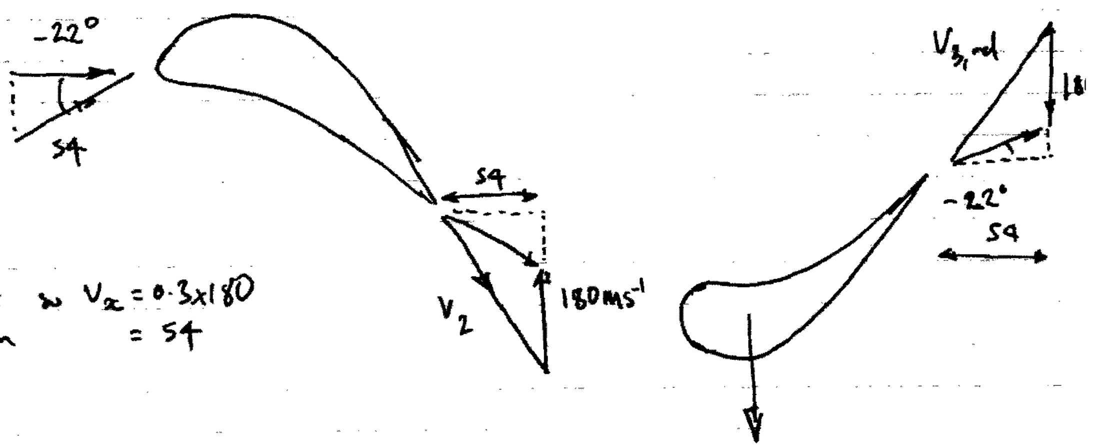
so $\eta_{tt} = 1 - 0.05 \left(2 \times 190.7^2\right) / 1 \times 180^2 = 88.8\%$

$$\eta_{ts} = \eta_{tt} - \text{exit K.E.}$$

$$= 88.8 - \frac{\frac{1}{2}\,63^2}{180^2}$$

$$= 82.65\ \%$$

<u>Low reaction design</u>



$\phi = \frac{V_x}{U_m}$ so $V_x = 0.3 \times 180$
$= 54$

$$\Delta h_0 = U \Delta V_\theta \quad \text{so} \quad \Delta V_\theta = 1.64 \times 180 = 295.2\ \text{ms}^{-1}$$

$$V_{\theta_1} = V_x \tan\alpha_1 = -21.82\ \text{ms}^{-1}$$

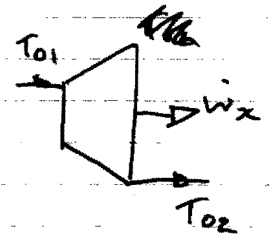
$$V_{\theta_2} = \Delta V_\theta + V_{\theta_1} = 273.4\ \text{ms}^{-1} \quad V_2 = 278.7\ \text{ms}^{-1}$$

$$V_{3,rel} = \sqrt{\{54^2 + (180 + 21.82)^2\}} = 208.9\ \text{ms}^{-1}$$

$$\eta_{tt} = 1 - 0.05\frac{(278.7^2 + 208.9^2)}{1.64 \times 180^2} = 88.6\ \%$$

Stage exit K.E. $= \frac{1}{2}(54^2 + 21.82^2)$

$$\eta_{ts} = \eta_{tt} - \frac{\frac{1}{2}(54^2 + 21.82^2)}{1.64 \times 180^2} = 85.4\ \%.$$

d) 

whole stage.

$$T_{02} = T_{01} - \frac{\Delta h_0}{c_p}$$

$$= (540 + 273.15) - \frac{350}{1.8}$$

$$= 618.7\,K$$

$$\frac{P_{03}}{P_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}\frac{1}{\eta_p}}$$

$\eta_p = 88.8\,\%$ $\qquad \frac{\gamma}{\gamma-1}\frac{1}{\eta_p} = 4.861$

$$\frac{P_{02}}{P_{01}} = 0.2634$$

$$\frac{T_{02is}}{T_{01}} = (0.2634)^{\frac{\gamma-1}{\gamma}} = 0.735$$

$$T_{02is} = 612.5\,K$$

$$\eta_{is} = \frac{T_{01} - T_{02}}{T_{01} - T_{02is}} = 90.2\,\%$$

Examiner's note:

A very popular and straightforward question, well-answered by most candidates. Almost all candidates were able to complete the proof and calculate the number of stages. Candidates began to struggle a little more with the velocity diagrams. When calculating the stage efficiencies, the most common mistake was to use the overall work output and an Euler work equation approach which often led to errors. The candidates who spotted that they had already calculated the stage-loading coefficient s for each design in the previous part of the question found it much easier.

2)

$M_1 = 0.68 \quad \alpha_1 = 42° \quad \alpha_2 = 18° \quad P_{01} = 1\times10^5, \; T_{01} = 300K.$

$$\frac{P_1}{P_{01}} = \left[1 + \frac{\gamma-1}{2} M_1^2\right]^{-\frac{\gamma}{\gamma-1}} = 0.7338$$

$$\frac{P_{01} - P_{02}}{P_{01} - P_1} = 0.035 \qquad \frac{P_{02}}{P_{01}} = -0.035\left[1 - \frac{P_1}{P_{01}}\right] + 1 = 0.9907$$

$\gamma = 1.4, \; c_p = 1005$

$$\frac{\dot{m}}{A_1}\frac{\sqrt{c_p T_{01}}}{P_{01}} = 1.1544 \Rightarrow \frac{\dot{m}}{A_1} = \frac{1.1549 \times 1\times10^5}{\sqrt{1005 \times 300}} = 210.24 \text{ kg s}^{-1}\text{m}^{-2}$$

$$A_1 = A_{x_1} \cos 42 \rightarrow \frac{\dot{m}}{A_{x_1}} = \cancel{}210.2 \times \cos 42 = 156.24 \text{ kg s}^{-1}\text{m}^{-2}$$

Continuity:

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 P_{02}} = \frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 P_{01}} \cdot \frac{P_{01}}{P_{02}} \cdot \frac{A_1}{A_2}\sqrt{\frac{T_{02}}{T_{01}}}$$

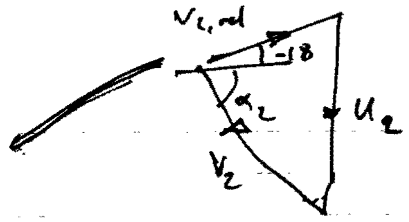
$$\text{so } \sqrt{\frac{T_{02}}{T_{01}}} = 1, \quad \frac{A_1}{A_2} = \frac{A_{x_1} \cos 42}{A_{x_1} \times \cos 18} = 0.7814$$

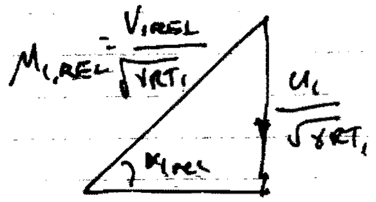
$$\text{so } \frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 P_{02}} = 0.8937 \rightarrow M_2 = 0.453$$

$$\frac{P_2}{P_1} = \frac{P_2}{P_{02}} \cdot \frac{P_{02}}{P_{01}} \times \frac{P_{01}}{P_1} \qquad \frac{P_2}{P_{02}} = 0.867$$

$$\text{so } \frac{P_2}{P_1} = 1.1705 \;//$$

c)





$$\frac{V_1}{\sqrt{\gamma R T_1}} = M_1 = M_{1,rel} \cos \alpha_{1,rel}$$

$$V_1 = M_1 \sqrt{\gamma R T_1} \qquad \gamma R = (\gamma - 1) C_p \qquad C_p T_1 = C_p T_{01} - \tfrac{1}{2} V_1^2$$

$$V_1^2 = M_{1rel}^2 \cos^2 42 \left((\gamma-1) C_p T_{01} - \frac{(\gamma-1)}{2} V_1^2 \right)$$

$$V_1 = 171.2 \text{ ms}^{-1}$$

$$U_1 = V_1 \tan 42 = 154.13 \text{ ms}^{-1}$$

$$V_{1rel} = 230.3 \text{ ms}^{-1}$$

SFEE $\quad T_{01,rel} = T_{01} \cancel{- \frac{U_1 V_{\theta 1}}{C_p}} + \frac{1}{2} \frac{U_1^2}{C_p} = 300 + \frac{154.13}{2 \times 1005} = \underline{\underline{312 K}}$

d) In relative frame $\quad \frac{p_2}{p_1} = 1.1705 \quad M_{2,rel} = 0.453$

Constant radius. $\quad T_{02,rel} = T_{01,rel} = 312.0 \qquad T_2 = \frac{T_{02,rel}}{1 + \frac{\gamma-1}{2} M_{2rel}^2} = 305.7$

$$V_{2,rel} = \sqrt{\left\{ 2 \times 1005 \left(T_{02,rel} - T_2 \right) \right\}} = 112.5 \text{ ms}^{-1}$$

$$\alpha_2 = \tan^{-1} \left[\frac{V_{2,rel} \sin \alpha_{2,rel} + U}{V_{2,rel} \cos \alpha_{2,rel}} \right] = \underline{\underline{48^\circ}}$$

d) $V_3 = V_1 = 171.2\,m/s \quad \alpha_1 = 0$

$$V_2 = \frac{V_{2rel}\cos\alpha_{2rel}}{\cos\alpha_2} = 144.0\,ms^{-1}$$

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} \qquad \Delta h_{rotor} = \frac{1}{2}\left(230.3^2 - 112.5^2\right)$$

$$= \cancel{6.1\times10^{6}}\; 2.02\times10^4$$

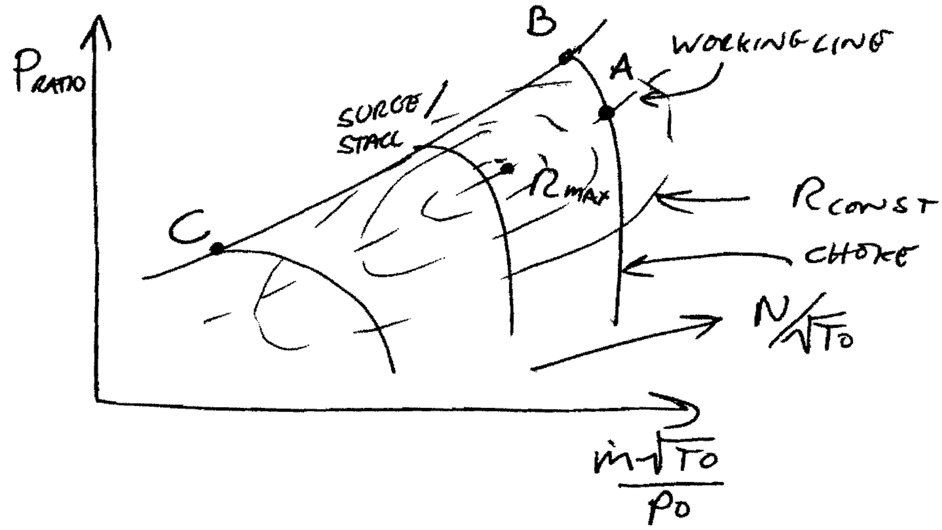
$$\Delta h_{stage} = U_2 V_{\theta 2}$$
$$= 154.1 \times 144 \sin 48^\circ$$
$$= 1.366\times10^4$$

$$\Lambda = 0.68$$

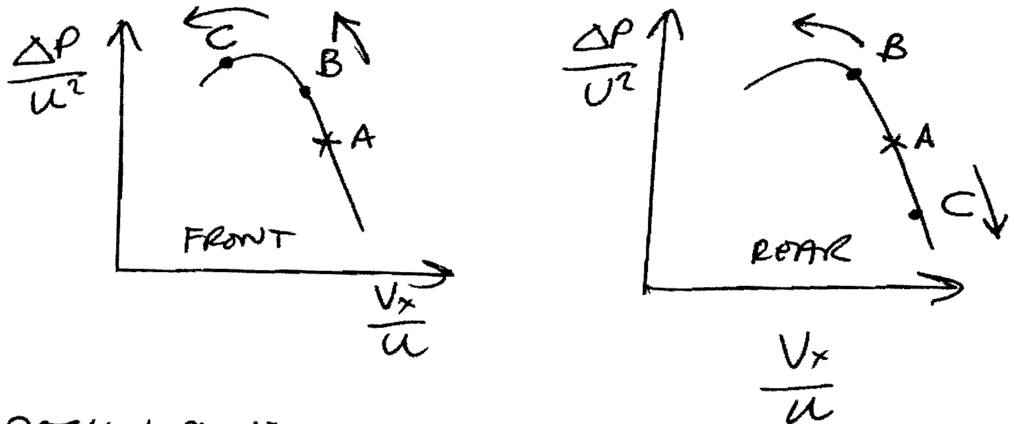
Examiner's note:

Another very popular question. Almost all candidates tackled the first part very well, calculating the correct exit Mach number from the tables in the Databook. However many were not able to manipulate the pressure ratios. Part c) was well answered. For this section the answers were given and the candidates had to show their methodology. However there were almost no successful attempts to the next part. In this case they had to find the absolute exit flow angle. This time the answer was not given - but the calculation itself was no complicated.

3 a



b



A – DESIGN POINT

B – HIGH SPEED CROSSING STALL LINE / SURGE LINE
INCREASED ΔP IN FRONT STAGES REDUCES V_x INTO NEXT STAGE SINCE ρ ~~IS~~ IN CREASED RELATIVE TO DESIGN, ΔP ~~OF~~ NEXT STAGE RISES MORE.
HENCE REAR STAGES $\frac{V_x}{U}$ MUCH LESS AND STALL

C – LOWER SPEED CROSSING STALL LINE / SURGE LINE
LOWER U GIVES SMALLER RISE IN ρ THAN AT DESIGN $\therefore$ 2ND STAGE LESS HEAVILY LOADED THAN FIRST. REAR STAGES CHOKE WHILE FRONT STALL.

(3) (c) THROTTLE IS CHOKED AS PRATIO > CHOKING VALU

$$\therefore \frac{\dot{m}\sqrt{C_p T_{03}}}{A P_{03}} = \text{CONST}$$

AT PRATIO = 12 $\dot{m}$ = 80 kg s^{-1}

FOR $\dot{m}$ = 50 kg/s

$$\frac{\dot{m}_{(50)}\sqrt{\cancel{C_p} T_{03(50)}}}{\cancel{A} P_{03(50)}} = \frac{\dot{m}_{(80)}\sqrt{\cancel{C_p} T_{03(80)}}}{\cancel{A} P_{03(80)}}$$

$$\frac{P_{03(50)}}{P_{03(80)}} = \frac{50}{80}\sqrt{\frac{T_{03(50)}}{T_{03(80)}}} \quad (2)$$

$$\frac{T_{03(50)}}{T_{03(80)}} = \frac{P_{03(50)}}{P_{03(80)}}^{\frac{\gamma-1}{R\gamma}} = \left[\frac{P_{03(50)}}{P_{03(80)}}\right]^{\frac{0.4}{1.4\times0.88}} \quad (1)$$

SUB (1) INTO (2)

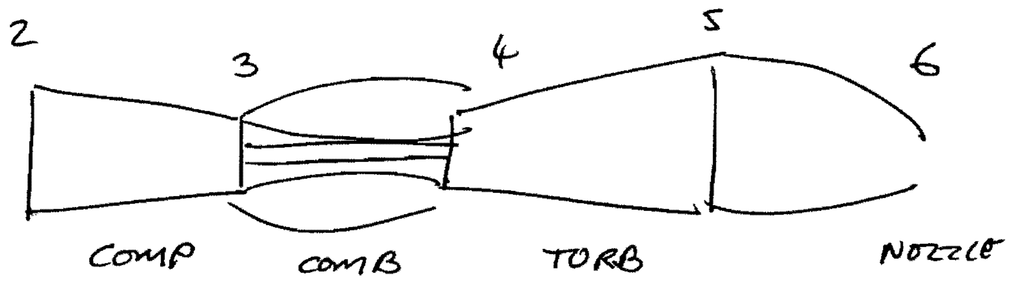
$$\frac{P_{03(50)}}{P_{03(80)}} = \frac{50}{80}\left[\frac{P_{03(50)}}{P_{03(80)}}\right]^{0.3247\times0.5}$$

$$\left[\frac{P_{03(50)}}{P_{03(80)}}\right]^{[1-0.3247\times0.5]} = \frac{50}{80} \qquad \frac{P_{03(50)}}{P_{03(80)}} = 0.570$$

$$\therefore P_{03(50)} = 0.5706\times12 = \underline{\underline{6.847}}$$

PRESSURE RATIO OF COMPRESSOR AT $\dot{m}$=50 IS $\underline{\underline{6.8472}}$

3 d



$$\left(\frac{\dot m\sqrt{C_p T_o}}{A P_o}\right)_4 = \left(\frac{\dot m\sqrt{C_p T_o}}{A\ P_o}\right)_6 = \text{CONST}$$

NB ASSUME NOZZLE OF TURBINE PLUS EXIT NOZZLE CHOKED

NB ASSUME NO PRESSURE LOSS OR HEAT TRANSFER IN EXIT NOZZLE [VALID

$$\frac{T_{o4}}{T_{o5}} = \left(\frac{P_{o4}}{P_{o5}}\right)^{\eta\frac{\gamma-1}{\gamma}}$$

NB ASSUME polytropic EFFICIENCY TURBINE FIXED [VALID AT HIGHER SPEEDS]

$$\Rightarrow \quad \frac{T_{o4}}{T_{o5}} = \text{CONST} \qquad \frac{P_{o4}}{P_{o5}} = \text{CONST}$$

$$T_{o4} - T_{o5} = T_{o4}\left(1 - \frac{T_{o5}}{T_{o4}}\right) = \text{CONST} \times T_{o4}$$

$$C_{pt}\left(T_{o4} - T_{o5}\right) = C_p\left(T_{o3} - T_{o2}\right)$$

NB ASSUME C_p, C_{pt} CONSTANT THROUGH OPERATION

$$\therefore \quad T_{o3} - T_{o2} = \text{CONST} \times T_{o4}$$

$$\left(\frac{\dot m\sqrt{C_p T_o}}{A\ P_o}\right)_4 = \text{CONST}$$

NB ASSUME $\frac{P_{o4}}{P_{o3}}$ = CON

$$\therefore \quad \frac{\dot m\sqrt{C_p\left(T_{o3} - T_{o2}\right)}}{P_{o3}} = \text{CONST}$$