1) (a) (i)

The flows created when the pressure force is balanced by Coriolis force, are called Geostrophic flows. These flows are governed by

$$\frac{1}{\rho} \nabla p = -2 \underline{\Omega} \times \underline{u}$$

where p includes the contributions from hydrostatic pressure and the centrifugal force.

The boundary layer between a geostrophic flow and a solid boundary with no-slip condition is called Ekman layer. The flow within this layer is known as Ekman layer flow and involves a balance among viscous, pressure and Coriolis forces. The governing equation is written as

$$\nu \nabla^2 \underline{u} - \frac{1}{\rho} \nabla p = 2 \underline{\Omega} \times \underline{u}$$

(ii) Density driven flows:

The density difference in the atmosphere can create flows due to buoyancy effects and these flows are known as density driven flows.

Examples are plumes, gravity currents, large scale motion in the atmosphere due to thermal or density stratification, avalanches.

(b) From the data card:

$$\frac{Dq^2}{Dt} = -\overline{u_i u_k}\frac{\partial U_i}{\partial x_k} - \nu\,\overline{\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\frac{\partial u_i}{\partial x_k}}$$
$$+ \frac{\overline{f_i u_i}}{\rho} + \text{transport terms.}$$

Sources:
(1) Wind shear, (2) Buoyancy effects

Sinks:
(1) Viscous dissipation (2) Buoyancy effects.

Buoyancy effects can act as a sink or a source depending on the thermal or density stratification.

Sunny day ⇒ thermal effects, $\frac{\overline{f_i u_i}}{\rho} = \frac{g}{T}\overline{\theta u_i}$
$= \frac{g}{T} K_\theta \left(\partial T/\partial z\right)$,
(from the data card)

Some wind ⇒ wind shear effects;

$$-\overline{u_i u_k}\left(\frac{\partial U_i}{\partial x_k}\right) = K_m\left(\frac{\partial U}{\partial z}\right)^2$$ from the data card.

Compare these two effects

$$\frac{\text{Thermal}}{\text{Mechanical}} = \frac{g}{T}\frac{K_\theta\,(\partial T/\partial z)}{K_m\,(\partial U/\partial z)^2}$$

This ratio is known as Richardson number.

$$\Rightarrow \quad R_i = \frac{g}{T} \frac{K_\theta}{K_M} \frac{(\partial T/\partial z)}{(\partial U/\partial z)^2}$$

if $R_i = 0$; No Thermal Stratification – Neutrally Stable.

$\left(\frac{\partial T}{\partial z}\right) < 0 \Rightarrow$ Unstable Stratification $\Rightarrow R_i < 0$ Unstable

$\left(\frac{\partial T}{\partial z}\right) > 0 \Rightarrow$ Stable Stratification $\Rightarrow R_i > 0$ Stable.

(c) (i) <u>Synoptic Scale</u> (of the order of 10^3 km)

(1) Thermal effects
ΔT between pole & Equator
ΔT between Land & Ocean.

(2) Rotation of earth.

(ii) <u>Meso Scale</u> (of the order of $10 - 10^2$ km)

(1) Land and Sea breeze
(2) Topography
(3) Heat island – cities.

(iii) <u>Micro-Scale</u> (of the order of 1 km)

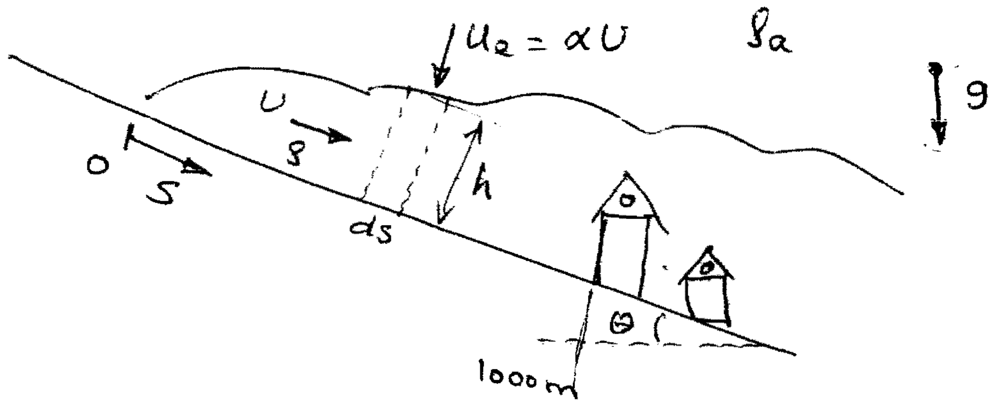
(1) wind shear, (2) building wakes.

(3) local thermal effects.

Examiner's note:

The students showed a good understanding of geostrophic and Ekman layer flows, various sources and sinks for turbulence in the atmospheric flows. They were also able to deduce the stability conditions based on Richardson number for atmospheric flows.

(2)



$\rho_a = 1.2$ kg/m^3 ; $\rho = 1.8$ kg/m^3

$\alpha = 0.05$

(a) Volume flux Conservation:

$$d(Uh) = ds\, u_e \Rightarrow \boxed{\frac{d}{ds}(Uh) = \alpha U} \quad — ①$$

mass Conservation:

$$d(\rho U h) = \rho_a u_e\, ds \Rightarrow \boxed{\frac{d}{ds}(\rho U h) = \alpha \rho_a U} \quad — ②$$

momentum Conservation:

$$d(\rho U h\, U) = ds\, h\, (\rho - \rho_a) g \sin\theta$$

$$\Rightarrow \boxed{\frac{d}{ds}(\rho U^2 h) = (\rho - \rho_a) g h \sin\theta} \quad — ③$$

using ① & ②

$$\frac{d}{ds}(\rho U h) = \rho_a \frac{d}{ds}(Uh) \Rightarrow \boxed{\frac{d}{ds}\left[(\rho - \rho_a) U h\right] = 0} \quad ④$$

for Self-Similar Solution.

$$h = A s^a \qquad U = B s^b \qquad (\rho - \rho_a) = C s^c$$

From ① $a = 1$.

From 3 $\frac{d}{ds}\left[AB^2 \, s^{a+2b}\right] = Agc \, s^{a+c}$

$\Rightarrow a+2b-1 = a+c \Rightarrow \boxed{c = 2b-1}$

From 4 $\frac{d}{ds}\left[ACB \, s^{a+b+c}\right] = 0$

$\Rightarrow a+b+c = 0 \Rightarrow \cancel{a}+b+2b-\cancel{a} = 0$
$\Rightarrow \boxed{b=0}$

$\therefore c = -1$

Now $h = As; \quad U = B; \quad (\rho-\rho_a) = c\, s^{-1}$

using 1 again, one finds $A = \alpha$.

$\therefore \boxed{h = \alpha s, \quad U = B}$

(b) The village is at 1000 m from the Source.

$\Rightarrow h = 0.05 \times 1000 = 50 \text{ m}$

(c) U = Const. at $s = 10\text{m}$; $h = 1 \text{ m}$
$\rho = 1.8 \text{ kg/m}^3$
$U = 10 \text{ m/s}$

The mass flow rate of the gas at this location

$= \rho U A = 1.8 \times 10 \times (1 \times 1) = 18 \text{ kg/s}$ Per unit wi

total Volume flux at $s = 10\text{m}$

$\dot{V}_{10} = UA = 10 \times 1 \times 1 = 10 \text{ m}^3/\text{s}$ Per unit widt

$= \dot{V}_{10,a} + \dot{V}_{10,g}$

at this location there is no air in the layer

Since $\rho = 1.8\ kg/m^3$

$$\Rightarrow \boxed{\dot{V}_{10} = \dot{V}_{10,g}}$$

Total volume flux at 1000 m

$$\dot{V}_{1000} = U \times A = 10 \times 50 \times 1 = 500\ m^3/s \text{ per unit width}$$

$$= \dot{V}_{1000,a} + \dot{V}_{1000,g}$$

$$= \dot{V}_{1000,a} + \dot{V}_{10,g}$$ (Since volume flux of the gas is to be conserved

$$\Rightarrow \dot{V}_{1000} - \dot{V}_{10} = \dot{V}_{1000,a}$$

∴ The rate of entrained air volume is

$$\dot{V}_{1000,a} = (500 - 10) = 490\ kg/s \text{ per unit width.}$$

∴ mass of air entrained $= 1.2 \times 490 = 588\ kg/s$ per unit width

mass fraction of the gas is

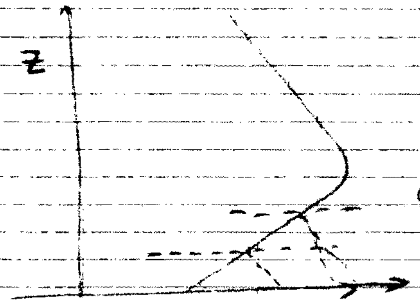
$$Y_g = \frac{\dot{m}_g}{(\dot{m}_g + \dot{m}_a)} = \frac{18}{18 + 588} = 0.0297.$$

$$\boxed{Y_g = 0.0297}$$

Examiner's note: The students were able to setup the required governing equations and deduce similarity solutions. Many students could not recognize that the volumetric rate of the entrained flow between two stations was given by the difference in the volumetric flow rates and also the chemical gas flow rate did not change with distance.

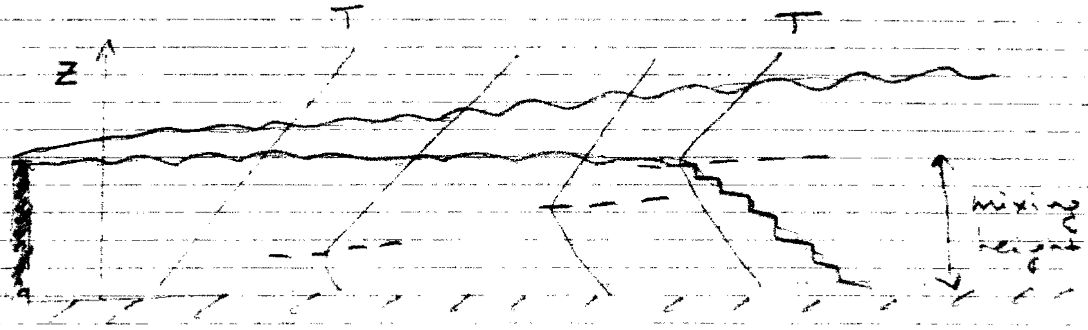
Q3

(a)



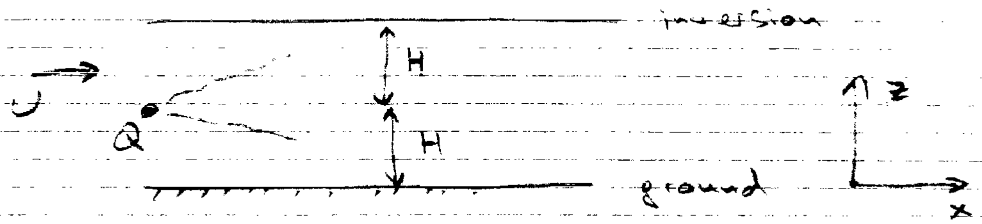
At night, radiative cooling of the surface implies colder air close to the ground than above it, resulting in a stable stratification.

At dawn, the surface begins to warm up, giving a small neutral layer up to a height called "mixing height". In this region, turbulence is not damped and therefore pollutants may be well-mixed.



Fumigation: Plume emitted at night may travel a long distance undiluted if in a stable atmosphere. At early dawn, as mixing height increases, sudden turbulence mixes the plume and mixes it so that large concentrations of pollutant may reach the surface. This is called "fumigation" and is the cause for some major accidents in history.

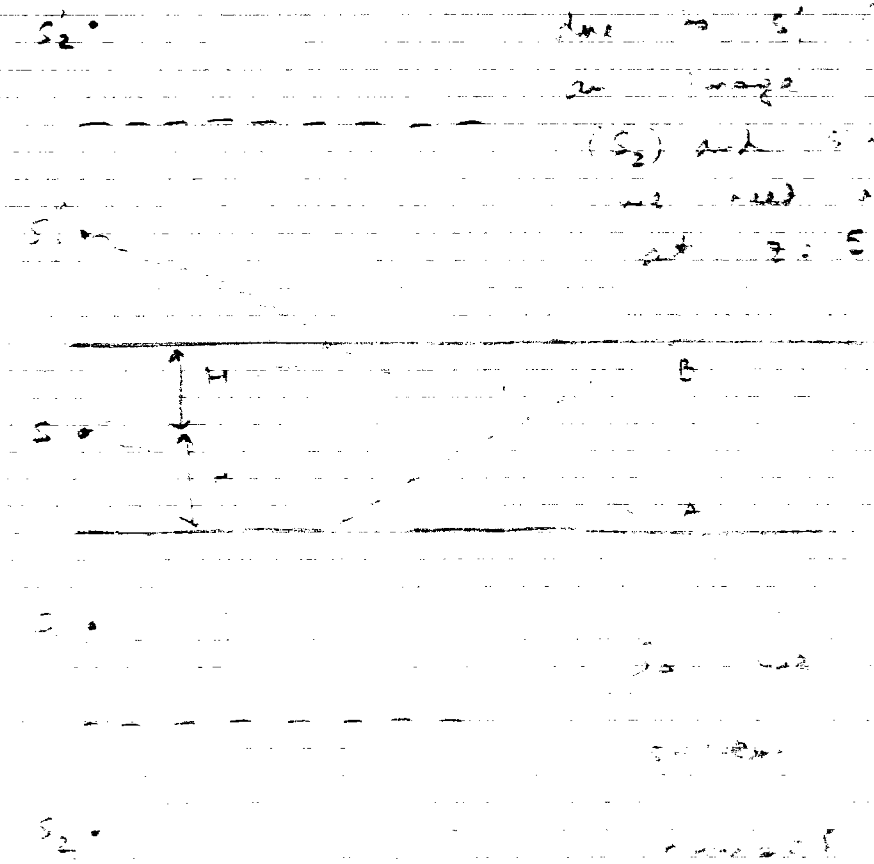
(b)



In the absence of ground and inversion,

$$\Phi(x,z) = \frac{Q}{U}\,\frac{1}{\sqrt{2\pi}\,\sigma}\,\exp\left(-\frac{(z-H)^2}{2\sigma^2}\right)$$

To take care of the ground, we need an image source S_1 at $z = -H$. But, similarly, to take care of the inversion, we need an image source at $z = 3H$ (S_1'). But to enforce $\frac{\partial \Phi}{\partial z} = 0$ at the ground due to S_1' (point A), we need an image source at $z = -3H$ (S_2) and similarly for point B, we need an image source at $z = 5H$ (S_2').



[illegible]

$z_1 = -H$ and $z_1' = 3H$

$z_2 = -3H$ and $z_2' = 5H$

$z_3 = -5H$ and $z_3' = 7H$

$\vdots \qquad \vdots$

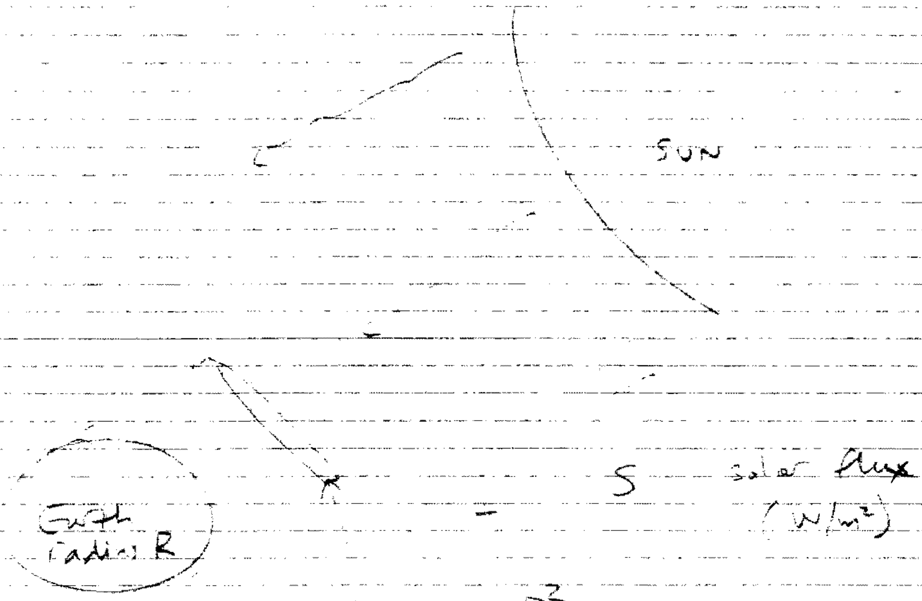
So that the pollutant is given by

$$\Phi(x, z) = \frac{Q}{U} \frac{1}{\sqrt{2\pi}\,\sigma} \left[\exp\left(-\frac{(z-H)^2}{2\sigma^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma^2}\right) + \exp\left(-\frac{(z+3H)^2}{2\sigma^2}\right) + \cdots + \exp\left(-\frac{(z-3H)^2}{2\sigma^2}\right) + \exp\left(-\frac{(z-5H)^2}{2\sigma^2}\right) + \cdots \right]$$

At close distances from the source, a few terms suffice, but at long distances many terms are needed.

Examiner's note: Few students clearly demonstrated the need for infinite number of image sources. Most candidates could describe the fumigation quite well.

Q4 (a)



Earth intercepts $\pi R^2 S$

Reflects $\alpha \pi R^2 S$ (α = albedo), i.e. receives $(1-\alpha)\pi R^2 S$

Emits $4\pi R^2 \varepsilon \sigma T_s^4$ σ Stefan-Boltzmann constant

ε emissivity

At equilibrium

$(1-\alpha) S \pi R^2 = 4\pi R^2 \varepsilon \sigma T_s^4$

$\Rightarrow T_s = \left(\frac{(1-\alpha) S}{4 \varepsilon \sigma}\right)^{1/4}$

S [illegible]

α, ε [illegible]

[illegible]

[illegible]

[illegible]

$\Rightarrow$ [illegible] T_s

A more complicated model is needed to take fully into account CO_2 and other greenhouse gases, but, approximately, presence of green-house gases affects α & ε.

(b) "Smog" is a term to indicate photochemical pollution. The primary pollutants NO & VOC's produce NO_2 and O_3, so above polluted cities we have mainly NO, NO_2, O_3 and VOC.

The reaction $NO + O_3 \xrightarrow{K_I} NO_2 + O_2$ produces NO_2.

The reactions $NO_2 \xrightarrow[h\nu]{K_{II}} O + NO$

$$O + O_2 \xrightarrow{K_{III}} O_3$$

produce O_3. The first one is active under the presence of sunlight. The overall effect is to produce high enough O_3 and NO_2 that a pollution episode may occur. The photostationary state is an approximation if we assume $\frac{d[O]}{dt} = \frac{d[O_3]}{dt} = 0$

then $\frac{d[O_3]}{dt} = -K_I[NO][O_3] + K_{II}[NO_2] = 0$ (with $\frac{d[O]}{dt} = 0$)

$$\Rightarrow [O_3] = \frac{K_{II}[NO_2]}{K_I[NO]}$$

[illegible] reaction [illegible]

The presence of VOC also participates in the cycle through $NO + VOC \rightarrow NO_2 + \ldots$

hence effectively increasing NO_2.

Smog may also contain particulate matter that cause problems in their own right, but do not directly participate in photochemistry.

Examiner's note:

The qualitative description of pollution was given very well, but few students could describe the photo-stationary state deeply.