ENGINEERING TRIPOS PART IIB 2012
4A11 TURBOMACHINERY II **Dr L Xu**

Engineering Tripos Part IIB 4A11 Cribs (2012)

1. a) MSCL: $V_m \sin\phi \dfrac{\partial V_m}{\partial m} + \dfrac{V_m^2}{R_m}\cos\phi - \dfrac{V_\theta^2}{r} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \dfrac{F_r}{\rho}$

For cylindrical flow paths, small ϕ and parallel streamlines, $\sin\phi \approx 0$, $R_m \to \infty$ and $V_m \to V_x$, the first two terms on the LHS can be neglected. Outside of the blade row (or in cases of radially stacked straight blades within the blade row) $F_r = 0$, we have $\dfrac{V_\theta^2}{r} = \dfrac{1}{\rho}\dfrac{dp}{dr}$.

Use "Tds" equation $dh - Tds = \dfrac{1}{\rho}dp$, and $h_o = h + \dfrac{1}{2}V^2 = h + \dfrac{1}{2}(V_x^2 + V_\theta^2)$ ($V_r \approx 0$ as assumed). $\dfrac{dh}{dr} = \dfrac{dh_o}{dr} - V_x\dfrac{dV_x}{dr} - V_\theta\dfrac{dV_\theta}{dr}$, $\Rightarrow$ $\dfrac{V_\theta^2}{r} = \dfrac{dh}{dr} - T\dfrac{ds}{dr} = \dfrac{dh_o}{dr} - V_x\dfrac{dV_x}{dr} - V_\theta\dfrac{dV_\theta}{dr}$

Assume uniform inlet flow and uniform work and loss along span, $\dfrac{dh_o}{dr} = 0$; and $\dfrac{ds}{dr} = 0$

$V_x\dfrac{dV_x}{dr} + V_\theta\dfrac{dV_\theta}{dr} + \dfrac{V_\theta^2}{r} = 0 \quad \Rightarrow V_x\dfrac{dV_x}{dr} + \dfrac{V_\theta}{r}\dfrac{d(rV_\theta)}{dr} = 0$ as required. Additional assumptions used to derive MSCL are inviscid steady asxi-symmetric flows. [20%]

b) $D_{hub} = 0.6\cdot 1.0 = 0.6m$, $r_{hub} = 0.3m$. Free vortex design $\Rightarrow \dfrac{dV_x}{dr} = 0$; uniform V_x at stage inlet, and at stator exit, $V_{x,2} = V_{x,1} = 50.0m/s$, free vortex design $rV_\theta = const.$

at hub, $rV_\theta = 75.0\cdot 0.3 = 22.5m/s$; at tip, $V_\theta = {rV_\theta}/{r_{tip}} = 22.5/0.5 = 45.0m/s$;

at midspan, $V_\theta = 22.5/0.4 = 56.3m/s$

i) assume kinetic energy loss across the stator is small and uniform along the blade height. Since h_o does not change across the stator (for adiabatic flows) and the spanwise V_x distribution is uniform due to free vortex design $\Rightarrow$streamlines parallel $\Rightarrow$SRE could be used. $V_{x,tip} - V_{x,hub} = 0$; as it does not vary;

$$p_{tip} - p_{hub} = \int_{hub}^{tip} dp = \rho\int_{hub}^{tip}\frac{V_\theta^2}{r}dr = \rho\int_{hub}^{tip}\frac{(rV_\theta)^2}{r^3}dr = 1.2\cdot(22.5)^2\int_{0.3}^{0.5}\frac{dr}{r^3}$$

$$= 1.2\cdot(22.5)^2\cdot 0.5\left(\frac{1}{0.09} - \frac{1}{0.25}\right) = 1.2\cdot(22.5)^2\cdot 0.5\cdot(11.4 - 4.0) = 2157pa$$

$\therefore\ p_{tip} - p_{hub} = 2157pa$ [40%]

ii) $V_1^2 = 2500(m/s)^2$; $V_{2,hub}^2 = 50^2 + 75.0^2 = 8125(m/s)^2$;
$V_{2,tip}^2 = 50^2 + 45.0^2 = 4525(m/s)^2$; $V_{2,mid}^2 = 50^2 + 56.3^2 = 5670(m/s)^2$;

$$\Delta h_{stage} = \Delta h_{stage,mid} = 0.5 \cdot (V_{2,mid}^2 - V_1^2) \cdot 2 = 3170 (m/s)^2;$$

$$\Lambda_{tip} = \frac{\Delta h_{tip,rotor}}{\Delta h_{stage}} = 1 - \frac{0.5(4525 - 2500)}{3710} = 0.68;$$

$$\Lambda_{hub} = \frac{\Delta h_{hub,rotor}}{\Delta h_{stage}} = 1 - \frac{0.5(8125 - 2500)}{3710} = 0.11$$

[20%]

c) For free vortex design V_θ increases rapidly towards the hub, reducing the local static pressure at the hub significantly, resulting in low hub reaction. This leads to high rotor inlet flow angle and increased rotor blade turning and suction surface diffusion in the hub region, with increased both profile and secondary losses and possible hub separation. The stator hub loss will also increase due to both high level of suction surface free stream velocity (profile loss) and high cross passage pressure difference (secondary loss). With the stator blade lean (with pressure surface facing down towards the hub), the blade force provides a radial component of the force to radial equilibrium and relieves the pressure gradient so the local pressure at the hub is increased. With improved hub reaction, the flow diffusion in the rotor hub can be reduced. [20%]

Examiner's note:

This question was on streamline curvature, calculation of spanwise distributions of the flow properties based on radial equilibrium and the stage reaction and its impact on the stage performance. Every candidate attempted this question and it was well answered. The streamline curvature and the radial equilibrium are well understood, overall the candidates showed better understanding of the underlying principles as well as the algebra needed for solving the numerical part of the problem.

2. a). The loading coefficient of turbomachinery is defined as $\frac{\Delta h_o}{U^2}$, this means with higher blade speed , the same non-dimensional loading can deliver more work, so that the power density through a machine is increased. For the same work output, high U means lower loading, which could lead to a design with higher efficiency and wider operating range. However for a given turbomachinery blade section, unnecessary accelerations on any part of its surface means extra diffusion the blade has to cope with for the flow to return to the desired exit condition, which in many cases means extra loss generated, with potential catastrophic flow separation, thus should be avoided. [20%]

b). Aassume for the two blades, the surface length are similar and proportional to the axial length, *i.e.*, $ds \propto dx$. The lost work of the blade per unit span in unit time is:

$T\Delta S = \sum C_s \rho C_d \int_0^1 V_\delta^3 d(x/C_x)$ for incompressible flows for air.

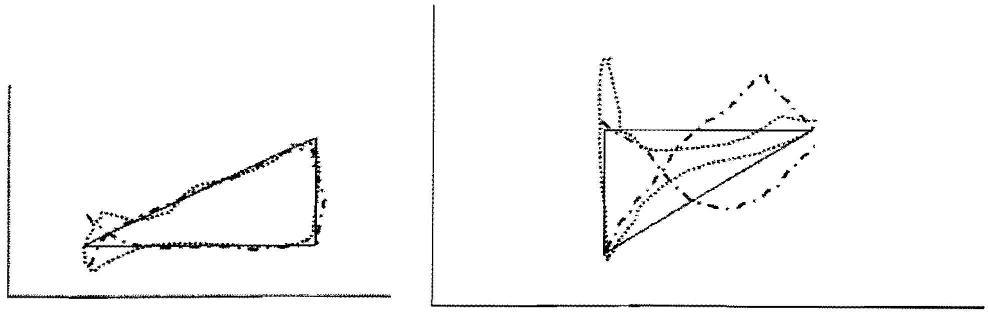
($V_2 = V_1 / \cos\beta_2 = 30/0.333 = 90.0 m/s$, so can assume incompressible). The ratio of the lost work:

$$R_{F/R} = \frac{\int V_2^3 dx + \int V^3 dx}{\int V^3 dx + \int V_1^3 dx} \; ; \; \int_0^1 V^3 dx = \int_0^1 (V_1 + \Delta V x)^3 dx = \frac{1}{4}\frac{(V_2^4 - V_1^4)}{(V_2 - V_1)} \; ;$$

$$R_{F/R} = \frac{4V_2^3 + (V_2^4 - V_1^4)/(V_2 - V_1)}{4V_1^3 + (V_2^4 - V_1^4)/(V_2 - V_1)} = \frac{4\cdot 90^3 + 90^4 - 30^4}{4\cdot 30^3 + 90^4 - 30^4} = \frac{399.6}{118.8} = 3.36 .$$

The actual ratio is expected to be slightly smaller than this value due to the loss on the suction surface of the rear-loaded blade would be slightly higher than that on the pressures surface of the front-loaded blade, due to different surface lengths. [50%]

c). The front-loaded blade will have worse incidence tolerance compared to the rear-loaded blade, as indicated in the sketches below. The dashed lines in the sketches indicate velocity distributions at positive incidence and dash-dotted lines indicate those at negative incidence. As the front-loaded blade has highly loaded leading edge the diffusion caused by off-design incidences is much severer. The blade will have higher losses and narrower operable ranges.



Local forward sweep of the blade leading edge, and local blade bend in the end-wall regions (end-bend) can reduce the local blade loading at the leading edge to improve the front-loaded blade's incidence tolerance. [30%]

Examiner's note:

This was on compare surface friction loss generations due to viscous effect, and to compare the losses due to two typical loading distributions. Only one candidate attempted the question and failed on integration of surface friction loss for given surface velocity distributions. The [illegible] question was answered relatively well

3\. a) For small element $d\dot{m}_j$, the loss is as given: $Tds = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{d\dot{m}_j}{\dot{m}}$,

assumptions used are two streams of the same stagnation temperature, fully mixed at constant static pressure. The total mass flow rate trough the whole passage is: $\dot{m} = \rho V_2 hp\cos\alpha_2$. Assume flow to be incompressible and the Bernoulli's is valid in the inviscid core of the leakage jet, the leakage flow mass flow rate in a small chord length dz is: $d\dot{m}_j = \rho g C_d V_j dz = \rho g C_d\sqrt{2\Delta p/\rho}dz = gC_d\sqrt{2\rho\Delta p}dz$, g is the tip clearance gap and C_d the discharge coefficient.

$$Tds = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{d\dot{m}_j}{\dot{m}} = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{gC_d\sqrt{2\rho\Delta p}dz}{\rho V_2 hp\cos\alpha_2};\quad \Delta p = 0.5\rho(V_s^2 - V_p^2)$$

$$Tds = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{d\dot{m}_j}{\dot{m}} = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{gC_d\sqrt{\rho^2(V_s^2 - V_p^2)}dz}{\rho V_2 hp\cos\alpha_2}$$

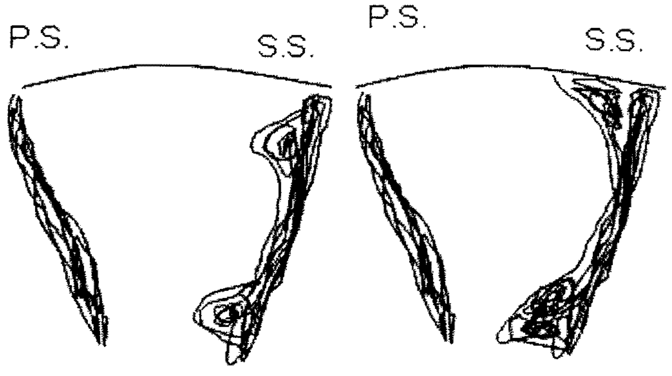
$$= \frac{gC_d C}{V_2 hp\cos\alpha_2}V_s^2\left(1-\frac{V_p}{V_s}\right)\sqrt{V_s^2(1-(\frac{V_p}{V_s})^2)}d\frac{z}{C}$$

Integrate along z/C, $T\Delta S = \int_0^1 \frac{C_d gC}{hp\cos\alpha_2}\frac{V_s^3}{V_2}\sqrt{1-(V_p/V_s)^2}\left(1-\frac{V_p}{V_s}\right)\frac{dz}{C}$

cont.....

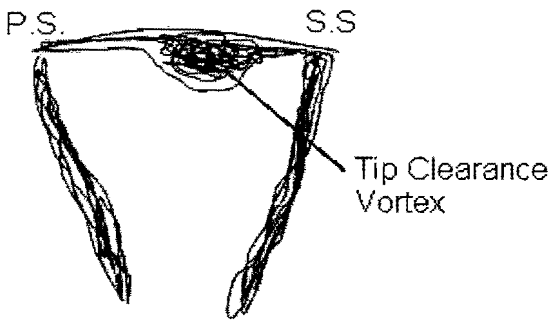
Loss coefficient: $\zeta = \frac{T\Delta S}{\frac{1}{2}V_2^2} = \frac{2C_d g C}{hp\cos\alpha_2}\int_0^1 \frac{V_s^3}{V_2^3}\sqrt{1-(V_p/V_s)^2}\left(1-\frac{V_p}{V_s}\right)\frac{dz}{C}$, as required. [30%]

b) i). A typical turbine blade row stagnation pressure contour plot features secondary flow loss cores near to the suction surface-end-wall corner, slightly lifted away from the end-walls. As sketched in (a) below. A compressor blade features a higher loss corner flow, as sketched in (b) below. This is the result of the secondary flow interacting with the suction surface boundary layer flow in the corner region. The high loss corner flow could develop into corner separations. [25%]



(a) (b)

ii) When the blade tip is unshrouded, the shroud leakage flow will interact with the main passage flow and form te tip clearance vortex which is opposite to the secondary flow vortex formed by the cross passage pressure gradient. When there is no clearance gap. The clearance flow would blow away the secondary flow, preventing it from interacting with the suction surface boundary layer. Typical downstream stagnation pressure contour plot as shown the sketch below:

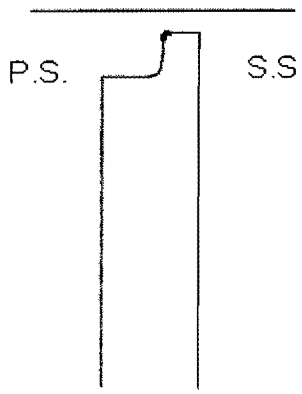


[15%]

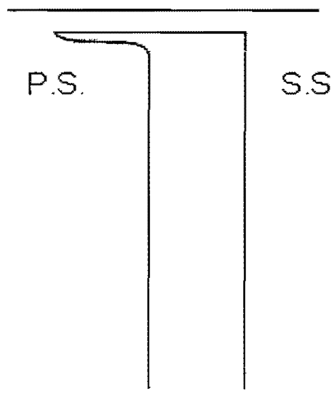
iii) The tip clearance loss is proportional to the leakage mass flow rate. For a small tip clearance the leakage flow rate is small so the direct mixing loss due to that is small. The small clearance flow can counter act with the end-wall secondary flow to prevent the corner separation from forming and to weaken the secondary flow core, thus reduces the end-wall losses. [15%]

iii) A key mechanism to reduce the unshrouded turbine rotor tip leakage loss is to reduce the leakage mass flow rate. For the fixed tip clearance gap and blade loading at the tip, the discharge coefficient C_d is a function of geometry, and reducing C_d value would reduce the loss coefficient proportionally. Any tip shape contributes to a reduction of C_d can help to reduce the tip clearance leakage loss: For example, suction side squealer, or pressure side winglet, as are sketched below. The former (sketch (a)) reduces C_d by turning the leakage flow towards the casing, forming an aerodynamic screen. (In some turbines, a full squealer, which has raised edges on the both suction side and pressure side, are also used). The driving force is reduced by high streamline curvature. The latter (sketch (b)) is functional by increasing local blade thickness, reducing the pressure gradient, as well as generating high streamline curvature around the sharp corner.

Using the blade leading edge forward sweep can reduce the fore-chord blade loading, thus reduce the leakage flow in the frontal part of the blade and this also leads to a reduction of the mixing loss. [15%]



(a)



(b)

Examiner's note:

This question was about three dimensional flows. Start from a derivation of the tip clearance leakage loss coefficient, followed by analyses of three dimensional flow patterns at the exit of turbine and compressor blade rows. The derivation itself was well done but very few were able to state the assumptions made for the model. The descriptions on the three dimensional flows at the exit of the blade row were less well presented, most of candidate failed sketch representative stagnation pressure contours as asked.