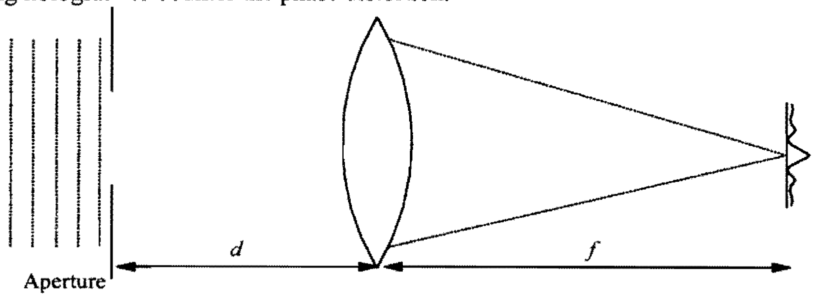
ENGINEERING TRIPOS PART IIB 2012
4B11 PHOTONIC SYSTEMS — Dr T D Wilkinson

Q1 – crib a) Far field region = Focal plane of a positive lens = FT{Hologram}
The lens introduces the quadratic phase distortion term in front of the transform. This can lead to a smearing of the Fourier transform and must be corrected for in compressed optical systems. There are several methods used to correct the phase, such as adding further lenses close to the focal plane or a compensating hologram to counter the phase distortion.



If the aperture is placed a distance d behind the lens, then there will be a corresponding change in the phase distortion term of the Fourier transform.

$$E(\alpha,\beta) = e^{\frac{jk}{2f}\left(1-\frac{d}{f}\right)\left(\alpha^2+\beta^2\right)} \iint_A A(x,y) e^{\frac{jk}{f}(\alpha x+\beta y)}\, dxdy$$

From this equation we can see another way of removing the phase distortion. If the distance is set so that $d = f$, then the phase distortion is unity and we have the full Fourier transform scaled by the factor of the focal length, f.

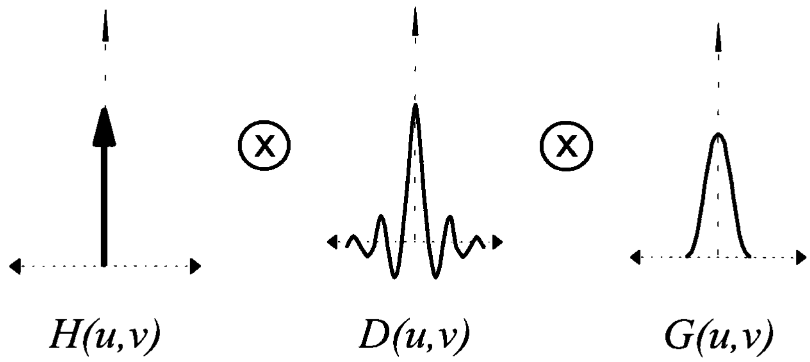
b) We normally assumed# that the illumination of the hologram is uniform and that the hologram and lens extend to infinity. This is not the case in reality, as there are a finite number of hologram pixels creating an aperture over the hologram and the light used to illuminate it will not be uniformly distributed. The illumination source is a collimated monochromatic laser which generates high quality parallel wavefronts with a wider diameter than the hologram or the lenses. Such a source will usually have a intensity distribution which can be expressed as a Gaussian beam profile or function.

$$g(x,y) = A_G e^{l\left(x^2+y^2\right)}$$

If a lens is illuminated with a collimated Gaussian source, then the focal plane will also contain a Gaussian profile which is scaled by the focal length. The actual structure of the replay field of a hologram is even more complex, as the Gaussian beam profile of the source is also apertured by the hologram and lens leading to sinc like sidelobes on the FT'ed Gaussian profile. The input illumination distribution $g(x,y)$ times the hologram aperture $d(x,y)$ times the total aperture of the FT lens (if it has a smaller diameter than the hologram) $p(x,y)$. Hence effect of the FT on these functions results in a convolution of their transforms.

$$F(u,v) = G(u,v) \otimes D(u,v) \otimes P(u,v)$$

The ideal hologram replay field $H(u,v)$ is designed as an array of delta functions in desired positions. The lens aperture $p(x,y)$ is a large circular hole, so the FT $P(u,v)$ will be a first order Bessel function (like a circular sinc function). The hologram aperture is a large square of size $N\Delta$ and its FT, $H(u,v)$ will be a sharp sinc function. The effect of the FT of the illumination $G(u,v)$ is to add a Gaussian profile. Hence, they will be delta functions convolved with a Bessel function convolved with a sinc function convolved with a Gaussian function. Spots which are placed next to one another will interfere due to the tails of the Gaussian, sinc and Bessel functions and the individual desired sharp 'spots' become ringed blobs with finite width.

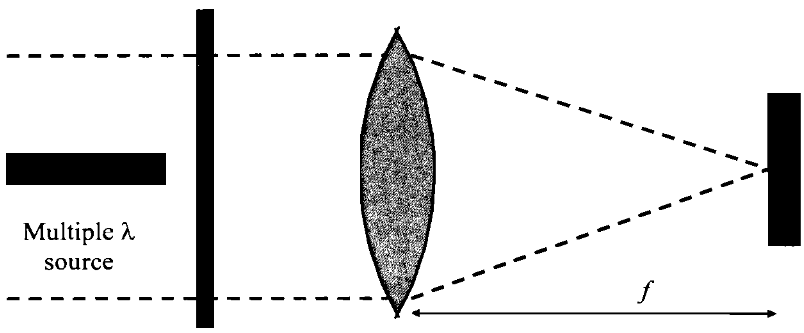


The result of this convolution process are sidelobes spread across the replay field from all of the points including the zero order. In the application of fibre to fibre switching, the number of CGH pixels is finite and forms an overall aperture, which leads to sinc or Bessel sidelobes on the individual spots. These effects lose power into the sidelobes and cause the spot to be broader than the original source fibre, leading to poor fibre launch efficiency. The sidelobes are also a source of crosstalk into neighbouring fibres and limits the spacing between channels. It is also a serious problem with the zero order as its sidelobes can be very large and means that crosstalk will be worse the closer to the centre of the replay field you go. In holographic projection this means that adjacent spots will result in interference between the spots and the neighbouring sidelobes, causing uniform areas of the image to have a non-uniform overall structure.

c) The manipulation of spatial frequencies in the replay field implies that the wavelength is held constant, however the spatial frequency is also a function of wavelength λ. Hence in Fourier holograms, spatial and wavelength properties are interchangeable.

$$u = \frac{k\alpha}{2\pi f} \qquad v = \frac{k\beta}{2\pi f}$$

If a hologram or grating is created to generate fixed positions or orders in the replay field, then if the wavelength is varied, then the positions of the orders from the hologram will vary. This is known as a wavelength sensitive or dispersive hologram or grating. If we change the pitch of the grating/hologram, then the position of the spot generated by the grating will sweep across the far field. If we have a multiple wavelength input source illuminating the grating then each wavelength has a different angle of diffraction and will lead to a different position in the far field. Hence the linewidth is converted into a physical broadening of the spots generated in the replay field, similar to that seen by apodisation. This leads to problems with launch efficiency and also to the possibility of crosstalk between the different fibres.



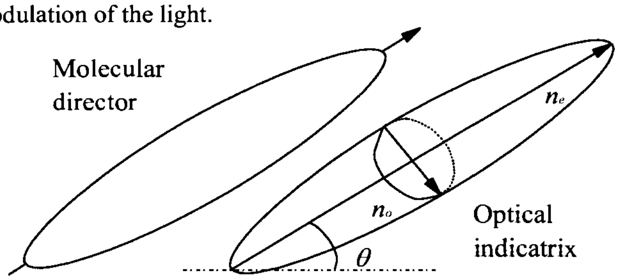
Wavelength and position will vary with the grating pitch in the far field. If we monitor a fixed point in the far field, then the wavelength will scan across that point with the changing pitch, making a wavelength selective filter. If the grating/hologram pitch, were increased as an integer, we would only have a small number of fixed wavelengths that could be tuned. However, by using one dimensional holograms, we can select any point in the output plane along the single axis which means we can select a range of angles and hence a range of wavelengths.

d) The three wavelengths can be combined to create the gamut for a single colour image time sequentially. Each laser wavelength illuminates a separate hologram and forms the corner of a triangular region on the CIE colour map. As laser sources are monochromatic, they occupy positions very close to the edges of the CIE colour map, hence the triangle generated by the RGB laser wavelengths will be larger that those generated by traditional image projectors (such as those seen with arc lamps and LEDs). As the CIE colour gamut is larger, then the number of colours generated (proportional to the area of the CIE triangle) will also be much larger. Hence the colours will be better with increased hue, chroma and saturation. The problem is that the size of each replay field generated by each hologram will be different due to the scaling being different for each wavelength. If we assume the green (532nm) wavelength is the correct size, then the image generated by the red hologram must be reduced in size and the image from the blue hologram must be increased. This is done by scaling the target images used to calculate the replay field for each hologram by the required correction factor.

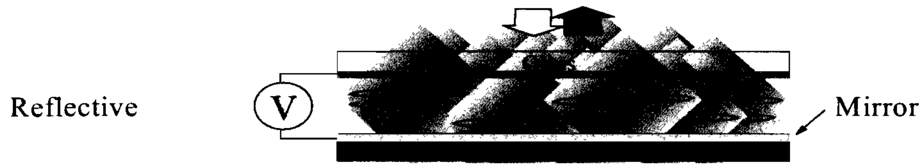
$$\alpha_{1M} = \frac{f\lambda_1}{\Delta} = \alpha_{2M} = \frac{f\lambda_2}{\Delta} \rightarrow \frac{\alpha_{1M}}{\alpha_{2M}} = \frac{\lambda_2}{\lambda_1}$$

Hence the red correction will be (532/650 = 0.818) and the blue (532/410 = 1.298)

Q2 a) The nematic phase is the least ordered mesophase before the isotropic. The molecules have only long range order and no longitudinal order. This means that the molecules retain a low viscosity, like a liquid, and are prone to flow. The anisotropy in the shape of the calimatic molecules leads to a dielectric anisotropy which is represented by a pair of dielectric constants, ε_{perp} and ε_{para}. The total dielectric anisotropy $\Delta\varepsilon$ is defined as the difference between the two. It is important to note that can be both positive as well as negative (vertically aligned nematic (VAN) devices). Molecules with a negative $\Delta\varepsilon$ behave in the opposite way electrically to those with a positive $\Delta\varepsilon$. The existence of this dielectric anisotropy means that we can move the molecules around by applying an electric field across them, combined with the flow properties means that a nematic molecule can be oriented in any direction with the use of an electric field which leads to their ability to perform greyscale modulation of the light.



The calimatic molecular shape also leads to an optical anisotropy in nematic LCs, with the two axes of the molecule appearing as the refractive index. The refractive index along the long axis of the molecules is often referred to as the extraordinary n_e (or fast n_s) and the short axis the ordinary n_o (or slow n_s) axis. The difference between the two is the birefringence. $\Delta n = n_e - n_o$

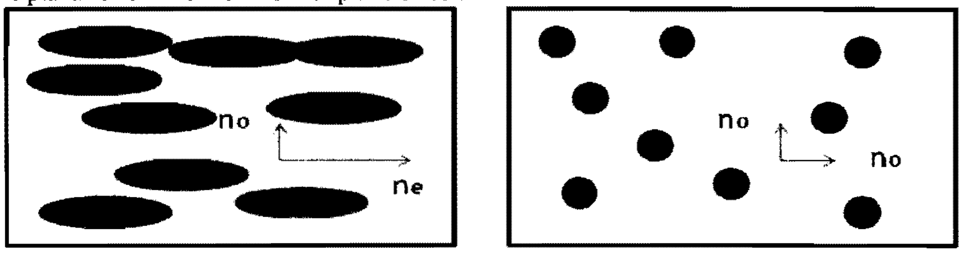


The optical anisotropy means that we can effectively rotate the axes of the indicatrix as the molecules move, creating a moveable wave plate or optical retarder. This along with polarising optics makes the basis of phase modulation. If we have an optical indicatrix oriented at an angle of θ to the plane of the cell (usually this corresponds to the plane with the glass walls and ITO electrodes), then we can calculate the refractive index seen by light passing perpendicular to the cell walls. We can then calculate the retardance Γ of the liquid crystal layer for a given cell thickness d and wavelength λ.

$$\Gamma = \frac{2\pi d\left(n(\theta) - n_o\right)}{\lambda}$$

We can now use this expression in a Jones matrix representation of the optical LC retarder to get the optical characteristics of the LC material. In the case of nematic LC materials, there is little restriction on the flow properties of the material, hence it is possible to continuously vary Γ through several rotations of π providing a thick enough cell. The main drawback of these materials is due to the fact that the flow of the material means that the speed of the modulation is often very slow (10's of msec to seconds). It is by definition an out of plane electro-optical effect.

b) One of the main limitations of nematic LCs is that they are inherently polarisation sensitive when in the planar geometry. This comes from the fact that the optical anisotropy is only in two dimensions and the planar orientation is an out of plane effect.



V - O　　　　　　V > Vth

When seen from above in the 0V state, light polarised parallel to the long axis of the molecules sees ne and light perpendicular sees no. When the LC is switched into the homeotropic state by the applied electric field, the light polarised parallel to the long axis of the molecules now sees no but the light perpendicular still sees no. Hence one polarisation is phase modulated, while the perpendicular polarisation is not.

The effects discussed above is the planar effect, where there is just the simple angle θ with respect to the cell walls. Because the nematic LC molecules are free to rotate in any position, it is possible to make more complex geometries such as twisted structures. In these devices, the molecules follow a twisted or helical path often rotating through an angle of 90° as is the case with twisted nematic or TN displays. This reduces the polarisation dependence as both polarisation states will see a change in reflective index when the LC material is switched, however it is not fully polarisation insensitive as each polarisation direction will see differing degrees of phase modulation. There is no twisting direction that will solve this inequality between the two polarisation states.

c) For a quarter waveplate, $\Gamma = 90$ degrees and $\psi = 45$ degrees.

$$W = \begin{pmatrix} e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi & -j\sin\frac{\Gamma}{2}\sin(2\psi) \\ -j\sin\frac{\Gamma}{2}\sin(2\psi) & e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi \end{pmatrix}$$

$$e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi = \frac{1}{2\sqrt{2}}(1-j)\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2\sqrt{2}}(1+j)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{\sqrt{2}}$$

$$e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi = \frac{1}{2\sqrt{2}}(1+j)\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2\sqrt{2}}(1-j)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{\sqrt{2}}$$

$$-j\sin\frac{\Gamma}{2}\sin(2\psi) = -j\frac{1}{\sqrt{2}}$$

$$QWP = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}$$

d) Multiple passes through a cell with waveplate optics is required to make them insensitive. The NLC is aligned parallel to the vertical (y) axis.

The vertical component will pass through the NLC and be fully phase modulated before passing through the QWP and being reflected back again. The two passes through the QWP are effectively the same as passing through a HWP with the same orientation of 45 degrees tot he vertical

$$QWP(two\ passes) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$$

For vertical light in the result of the QWP will be:

$$\begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}\begin{pmatrix} 0 \\ V \end{pmatrix} = \begin{pmatrix} -jV \\ 0 \end{pmatrix}$$

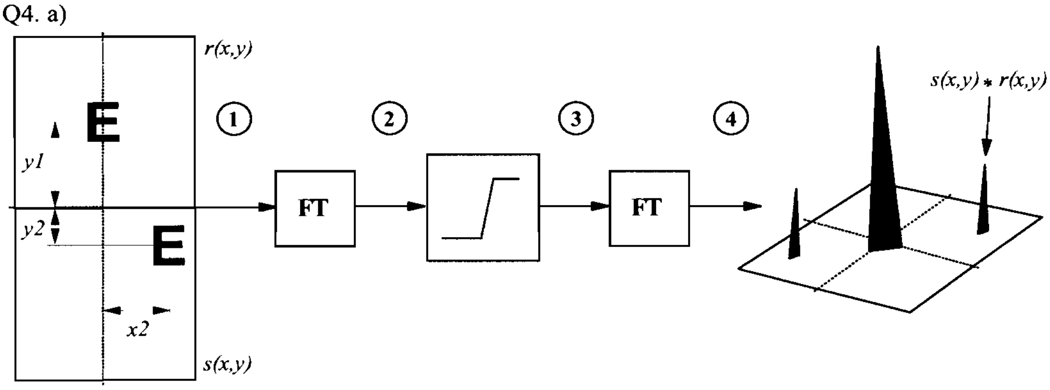
Which is horizontally polarised and so will not see the phase modulation on the return pass back through the NLC. For horizontally polarised light, process is reversed, there is no modulation from the NLC on the pass in, then the QWP:

$$\begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}\begin{pmatrix} V \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -jV \end{pmatrix}$$

Hence the light is now vertically polarised and will see the NLC phase modulation on the way back out of the cell.

Thus both states are modulated in the same way by the NLC on each pass through the cell giving a polarisation insensitive phase modulation.

Q4. a)



Plane 1 is the input plane with reference and object place side by side. This is usually done optically using a liquid crystal spatial light modulator.
Plane 2 1s where the joint power spectrum (JPS) is formed via a Fourier transform. This is done optically with coherent illumination and a lens.
Plane 3 is the non-linearly processed JPS. This can be a simple square law detector like a CCD or CMOS camera and a second LC SLM. Or it can be an optically addressed SLM (OASLM).
Plane 4 is the output plane formed by another FT lens. This image is usually grabbed by a CCD or CMOS camera.

b) In plane 1, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into plane 2.

$$S(u,v)e^{-j2\pi(x_2u-y_2v)} + R(u,v)e^{-j2\pi y_1 v}$$

The nonlinearity between planes 2 and 3 creates the correlation and in its simplest form can be modelled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

$$S^2(u,v)+R^2(u,v)+S(u,v)R(u,v)e^{-j2\pi\left(x_2u-\left(y_1+y_2\right)v\right)}+S(u,v)R(u,v)e^{-j2\pi\left(-x_2u+\left(y_1+y_2\right)v\right)}$$

The final plane 4 is after the second FT, with the central DC terms proportional to FT $[R^2 + S^2]$ and the two symmetrical correlation peaks spaced by $(x_2, -(y_1+y_2))$ and $(-x_2, y_1+y_2)$.

c) The JPS of the input now has three components

$$P(u,v)e^{-j2\pi(x_3u-y_3v)} + S(u,v)e^{-j2\pi(x_2u-y_2v)} + R(u,v)e^{-j2\pi y_1 v}$$

After the square law non-linearity we now have:

$$P^2(u,v)+S^2(u,v)+R^2(u,v)+S(u,v)R^*(u,v)e^{-j2\pi\left(x_2u-\left(y_1+y_2\right)v\right)}+S^*(u,v)R(u,v)e^{-j2\pi\left(-x_2u+\left(y_1+y_2\right)v\right)}$$
$$+P(u,v)R^*(u,v)e^{-j2\pi\left(x_3u-\left(y_1+y_3\right)v\right)}+P^*(u,v)R(u,v)e^{-j2\pi\left(-x_3u+\left(y_1+y_3\right)v\right)}$$
$$+S(u,v)P^*(u,v)e^{-j2\pi\left((x_2-x_3)u-\left(y_2-y_3\right)v\right)}+S^*(u,v)P(u,v)e^{-j2\pi\left(-(x_2-x_3)u+\left(y_2-y_3\right)v\right)}$$

This plane not only contains the desired RS and RP correlations, it also contains the unwanted cross correlations of P and S which could be as strong or even stronger than the other pairs, leading to incorrect identification of possible correlation peaks.

If P and S match R, then all peaks will be present, hence the extra PS peaks can be eliminated, however if P matches S, then the PS cross correlation will be incorrectly recognised as an SR correlation.

If the unknown input image is displayed and the reference image removed, then the PS correlation will be the only peaks present. If this plane is stored and then R is added to the input, then the PS error peaks can be removed by subtracting the stored plane from the one with R included.

Q3 a) First consider the electric field amplitudes of the two arms, E_1 and E_2:

$$E_1(z) = A_1 \exp j(\beta z + \Delta\phi)$$
$$E_2(z) = A_2 \exp j(\beta z)$$

where E_1 and E_2 differ from each other only by the phase shift $\Delta\phi$ induced in arm 1. The total electric field amplitude E_{tot} when the two arms recombine is simply the sum of the upper and lower values:

$$E_{tot} = E_1 + E_2$$

The power output per unit area (P_{out}) is the same as the intensity. To calculate this, we multiply E by its complex conjugate:

$$P_{out} = E_{tot} \cdot E_{tot}^*$$

As the recombining Y-junction has symmetric geometry for a 3dB splitter, then the field amplitudes in the two arms are equal:

$$A_1 = A_2 = A$$

Making the appropriate substitutions above, we can therefore express the total optical power output as:

$$P_{out} = [A\exp(j\beta z)\exp(j\Delta\phi) + A\exp(j\beta z)] \times [A\exp(-j\beta z)\exp(-j\Delta\phi) + A\exp(-j\beta z)]$$
$$P_{out} = [A\exp(j\beta z)\exp(j\Delta\phi) + A\exp(j\beta z)] \times [A\exp(-j\beta z)\exp(-j\Delta\phi) + A\exp(-j\beta z)]$$

Using the following identity: $\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

and simplifying, then gives the required result: $P_{out} = \underline{2A^2(1+\cos\Delta\phi)}$

(Note that A is not defined in the question, and in this solution is the amplitude of each of the two arms. Students may instead define A as the amplitude of the combined input beam, in which case their final answer may differ by a factor of 4).

b) The cleaved interferometer now acts as a source of Young's fringes. Before the induced index change, maxima are therefore located where the formula $d \sin\theta = n\lambda$ is satisfied (the zeroth order being found at 0°). After the induced index change, the position of the maxima can be found where:

$$d\sin\theta - \Delta n \cdot D = n\lambda \qquad \text{(note the additional retardance caused by the index change)}$$

Rearranging for the index change, and using $n = 0$ for zeroth order, gives:

$$\Delta n = \frac{d\sin\theta}{D}$$

Substituting values, $\theta_2 = 1°$, $D = 100$ μm, $d = 4a = 20$ μm, gives us: $\Delta n = 3.49 \times 10^{-3}$
Expressed as a percentage of the original index (1.500), this gives us a change of **0.233 %**.

c) The optical power per unit area (intensity) in the core is given by:

$$P_{core} = E_{core} \cdot E^*_{core}$$
$$P_{core} = A\sin(\kappa_1 x)\exp(-j\beta z) \times A\sin(\kappa_1 x)\exp(j\beta z)$$
$$P_{core} = A^2 \sin^2(\kappa_1 x)$$

The total power per unit area contained within the core is therefore given by:

$$\begin{aligned}
\Sigma P_{core} &= \int_{-a}^{a} A^2 \sin^2(\kappa_1 x) dx \\
&= \frac{A^2}{2}\int_{-a}^{a}(1-\cos(2\kappa_1 x))dx \\
&= \frac{A^2}{2}\left[x - \frac{\sin(2\kappa_1 x)}{2\kappa_1}\right]_{-a}^{a} \\
&= \frac{A^2}{2}\left[\left(a - \frac{\sin(2\kappa_1 a)}{2\kappa_1}\right) - \left(-a - \frac{\sin(-2\kappa_1 a)}{2\kappa_1}\right)\right] \\
&= \underline{A^2\left(a - \frac{\sin(2\kappa_1 a)}{2\kappa_1}\right)}
\end{aligned}$$

A similar expression can be obtained for the total power per unit area in the cladding:

$$\begin{aligned}
P_{cladding} &= E_{cladding} \cdot E^*_{cladding} \\
&= B\exp(-\kappa_2 x)\exp(-j\beta z) \times B\sin(-\kappa_2 x)\exp(j\beta z) \\
&= B^2 \exp(-2\kappa_2 x)
\end{aligned}$$

$$\begin{aligned}
\Sigma P_{cladding} &= \int_{a}^{\infty} B^2 \exp(-2\kappa_2 x) dx + \int_{-\infty}^{-a} B^2 \exp(-2\kappa_2 x) dx \\
&= 2B^2\left[\frac{\exp(-2\kappa_2 a)}{-2\kappa_2}\right]_a^{\infty}
\end{aligned}$$

$$= \frac{B^2}{\kappa_2} \exp(-2\kappa_2 a)$$

We now have our 2 expressions, and for the final part we need to find the following ratio:

$$\frac{\Sigma P_{cladding}}{\Sigma P_{core}} = \frac{B^2/\kappa_2 \exp(-2\kappa_2 a)}{A^2\left(a - \frac{\sin(2\kappa_1 a)}{2\kappa_1}\right)}$$

We know every term in this expression except A and B, which we must now eliminate. We do this by considering that the electric field amplitude must be continuous at the core-cladding interface. We can therefore write down the following expression:

$$E_{core}(x = a) = E_{cladding}(x = a)$$
$$A \sin(\kappa_1 a) = B \exp(-\kappa_2 a)$$
$$A = \frac{B \exp(-\kappa_2 a)}{\sin(\kappa_1 a)}$$

Substitute this expression for A in the power ratio expression above to give:

$$\frac{\Sigma P_{cladding}}{\Sigma P_{core}} = \frac{B^2 \exp(-2\kappa_2 a).\sin^2(\kappa_1 a)}{\kappa_2\left(a - \frac{\sin(2\kappa_1 a)}{2\kappa_1}\right) B^2 \exp(-2\kappa_2 a)}$$

$$= \frac{\sin^2(\kappa_1 a)}{\kappa_2\left(a - \frac{\sin(2\kappa_1 a)}{2\kappa_1}\right)}$$

κ_1 and κ_2 are defined in the question as: $\kappa_1 = k\sqrt{n_1^2 - n_{eff}^2}$ and $\kappa_2 = k\sqrt{n_{eff}^2 - n_2^2}$, and $k = 2\pi/\lambda$.

Substitute the values: $a = 5$ µm, $n_1 = 1.500$, $n_2 = 1.498$, $n_{eff} = 1.499$, $\lambda = 1330$ nm, gives us:

$$\frac{\Sigma P_{cladding}}{\Sigma P_{core}} = 4.01 \times 10^{-4}$$

which when expressed as a percentage is **0.040 %**

Examiner's comments

Q1: *Well answered. Basic book work with a few twists. All coped with diffraction and linewidth. Quite a few got the colour corrections wrong. Long question.*

Q2: *A liquid crystal question, some of the basic theory was a bit lacking. Surprisingly few could do the complex numbers for the Jones matrix but quite a few got the final part on QWP..*

Q3: *Disappointingly avoided question as it was a very good one, well structured and quite doable. This section was easy to 'bin' and so suffered this year .*

Q4: *This question was still a little too easy as it resembled past questions. It was well answered and overall the understanding was very good.*