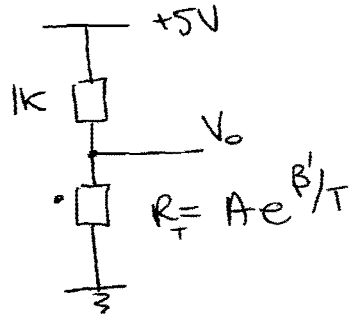
ENGINEERING TRIPOS PART IIB 2012
4B13 - ELECTRONIC SENSORS AND INSTRUMENTATION

CRIB 4B13 2012 DR. P.A. ROBERTSON

1 (a)



$$V_o = \frac{R_T \cdot 5}{(1000 + R_T)} \text{ V}$$

$R_T = 1000$ @ $T = 293$K

$\beta' = 3300$

$$\therefore \ln 1000 = \ln A + \frac{3300}{293}$$

$$6.91 = \ln A + 11.26$$

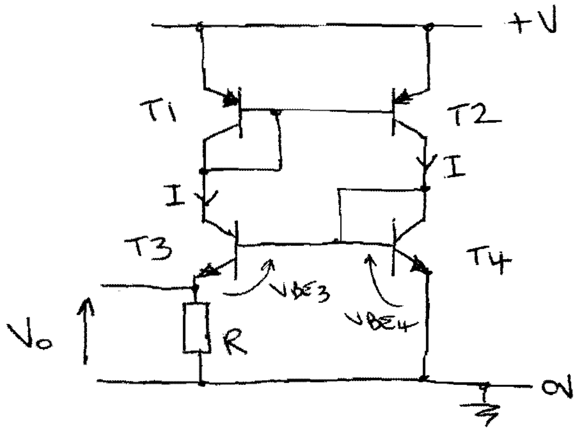
$\therefore A = 0.0129$

So, at $55°C = 328K$, $R_T = 0.0129\, e^{\frac{3300}{328}} = 302\ \Omega$

$$\therefore V_o = \frac{302 \times 5}{1302} = \underline{1.16 \text{ V}}$$

[20%]

(b)



Ebers-Moll eqn. for bipolar transistors:

$$I_C = I_S e^{V_{BE}/(kT/q)}$$

with $V_t = kT/q$

T1 and T2 are matched and connected as current mirrors to T3 and T4. T3 is scaled to have an area of r× that of T4 – so the current density is ÷r for T3.

Ebers-Moll for current density:

$$J_{C3} = J_{S3}\, e^{V_{BE3}/V_t} \quad \text{and} \quad J_{C4} = J_{S4}\, e^{V_{BE4}/V_t}$$

now $r J_{C3} = J_{C4}$ and $J_{S3} = J_{S4} = J_S$ as this is a Material property

so $r \cancel{J_S} e^{V_{BE3}/V_t} = \cancel{J_S} e^{V_{BE4}/V_t} \Rightarrow V_t \ln r = V_{BE4} - V_{BE3} = V_o$

[20%] $\therefore V_o = \frac{kT}{q} \ln r = \text{const.} \times \underline{T}$ typ. range −50°C – 150°C

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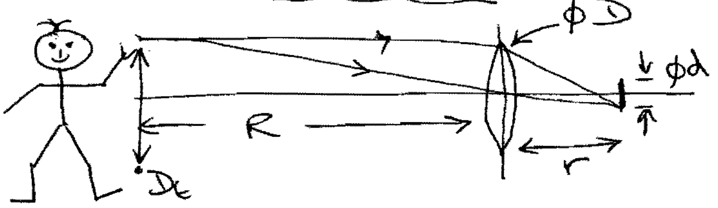
1(c)



D = 30 mm
r = 25 mm
d = 5 mm

Lambert's Law : $\delta W = \frac{W \cos\theta}{\pi} A \, \delta\omega$ = power emitted in solid angle $\delta\omega$

with W = total power emitted per unit area
θ = angle to surface normal
A = area of emitting surface

Stephan's Law : $W = \epsilon \sigma_{SB} T^4$ = power emitted per unit area
ϵ = emissivity = 0·90
T = absolute temperature
σ_{SB} = Stephan - Boltzman constant = $5.6 \times 10^{-8}\ W\,m^{-2}K^{-1}$

$$\Rightarrow \quad \delta W = \frac{\epsilon \sigma_{SB} T^4}{\pi} \cdot \frac{\pi D_t^2}{4} \cdot \frac{\frac{\pi D^2}{4}}{4\pi R^2} \cdot 4\pi = \frac{\epsilon \sigma_{SB} T^4 \pi d^2 D^2}{16 r^2}$$

(since $D_t/R = d/r$ by similar Δ's) $\Rightarrow$ $\delta W = 3{\cdot}2\ mW$

[30%]

(d) Metal strain gauge has G.F. = 2
Power dissipation in s.g. = $0{\cdot}01^2 \times 120$ = 12 mW = P
with 0.1 mm thick adhesive and area of 5 mm²

$$P = \frac{kA}{t} \Delta T = \frac{0{\cdot}25 \times 5 \times 10^{-6}}{0{\cdot}1 \times 10^{-3}} \Delta T = 0{\cdot}012$$

$\therefore \Delta T = 0{\cdot}96\ ^\circ C \Rightarrow$ 0·096% change in resistance

with a G.F. of 2, this is equivalent to strain of 0·048%
(which is quite a lot compared to strain max of eg: 0.5%).

[30%]

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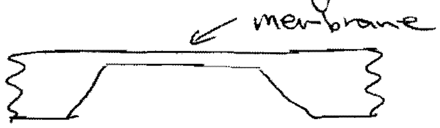
2 (a) Pressure sensor usually bulk micromachined from Si wafer



- membrane / diaphragm is fabricated with Boron doping and anisotropic etching with hot KOH soln. - the doped section of Si forms the thin section

For surface and bulk micromachining the processes involved are
- photolithography:
  spin-on resist, expose, develop, etch, remove
- sacrificial layers, etch masks + etch stops
- deposition of layers for surface structures
  poly-Si, SiOx, Si_3N_4, metals
- etching RIE dry anisotropic, directional
  KOH wet aniostrop + isotrop for non-crystalline matls.
- bonding wafer sections

- fabrication of integrated electronics + strain gauge sensors by IC process steps (CMOS).
- readout methods : piezo-resistive using deposited or doped strain gauges in stress areas + reference devices for temperature compensation
  : capacitive using electrodes with variable air gaps to monitor displacements

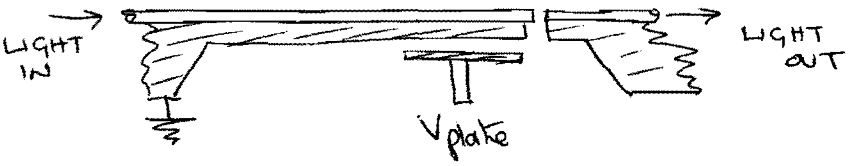
piezo-resistive	capacitive
+ low impedance, robust signal	- high imp., local electronics
- higher power consumption	+ low power consumption
- thermal drift	+ low temperature coeff.
- not so well suited to surface MEMS	+ good for surface MEMS

2(a) contd

Accelerometers - usually planar devices made with surface micromachining with capacitive readout technique. Proof mass and inter-digital capacitor fingers and suspension beams are patterned and sacrificial layer etched away to leave a suspended structure.

Pressure sensor - uses a membrane left from bulk etching - readout can be via strain gauges diffused & patterned into the diaphragm. [40%]

(b)



(i) $C = \frac{A\epsilon_0}{d} = \frac{(15\times10^{-6})^2\ 8.854\times10^{-12}}{5\times10^{-6}} \approx 3.98\times10^{-16}\ F$

(ii) In order to deflect the beam by 2μm, the capacitor force

$F \Rightarrow \qquad \delta = \frac{FL^3}{3EI} = 2\times10^{-6}$ with $L = 10^{-3}\ m$

$E = 145\ GN\ m^{-2}$ (data book 140-155)

$I = \frac{1}{12}bd^3$ with $\left.\begin{array}{l} b = 15\times10^{-6}\ m \\ d = 5\times10^{-6}\ m \end{array}\right\}$ $I = 1.56\times10^{-22}\ m^4$

$\therefore F = 1.36\times10^{-7}\ N = \frac{1}{2}V_p^2\frac{\partial C}{\partial x}$ with $C = \frac{A\epsilon_0}{x}$, $\frac{\partial C}{\partial x} = \frac{-C}{x}$

$\therefore F = \frac{1}{2}V_p^2\frac{C}{d} = \frac{1}{2}V^2\frac{3.98\times10^{-16}}{5\times10^{-6}}$ $\therefore$ $\underline{V_p = 58.5\ V}$ [40%]

(c) m, Beam mass = $\rho_{Si}\times L\times b\times d = 1.74\times10^{-13}\ kg$, $\rho_{Si} = 2.32\ kg\ m^{-3}$

Guesstimate centre of vibrating mass as 2/3 along beam = 0.66 mm, then spring constant, s, = $F/\delta = \frac{3EI}{L^3} = 0.236\ Nm^{-1}$ and $f_{res} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$

$\therefore f_{res} = 185\ kHz$ so $t_{settle} = \frac{Q}{\pi} = \frac{50}{\quad} = 0.27\ ms$ [20%]

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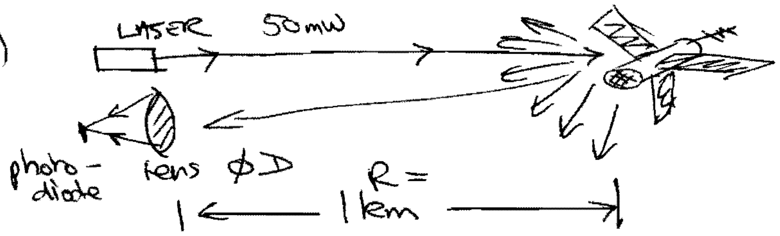
3 (a)



$D = 0.1\,m$
$R = 1000\,m$

$$\text{Collected light} = \frac{\frac{\pi D^2}{4}}{2\pi R^2} \cdot 500\times10^{-3} = 625\ pW$$

@ 0.36 A/W, the photo-current = $\underline{225\ pA}$

[25%]

(b) with amplitude modulation at 120MHz, the wavelength of the modulation $= \frac{3\times10^8}{120\times10^6} = 2.5\,m$

For a path length change of $2\times50\,m$, we get 40 beats at the mixer $\therefore$ 50 m/s ⇒ $\underline{40\ Hz}$ Doppler.

[15%]

(c) For a closing speed of 500 m/s, the required bandwidth is 400Hz. Hence noise sources are:–

$\underline{V_{rms}}$

Amp. current noise: $0.06\times10^{-12}\times\sqrt{400}\times10^4 = 1.2\times10^{-8}$
$i_{noise}\times R$

Amp voltage noise: $1.5\times10^{-9}\times\sqrt{400} = 3.0\times10^{-8}$

Resistor thermal noise: $\sqrt{4kTRB} = (4 . 1.38\times10^{-23} . 300 . 10^4 . 400)^{1/2}$
$= 25.7\times10^{-8}$

$$\therefore\ V_{noise} = \sqrt{\Sigma V_n^2} = 259\ nV_{rms}$$

$$V_{sig} = 225\times10^{-12}\,A\times10^4\,\Omega = 2.25\,\mu V$$

$\therefore$ $\underline{S/N \text{ ratio} \simeq 8.7}$ (ok but not very good!)

[35%]

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3 (d) Photon energy at 635 nm given by $E = \frac{hc}{\lambda}$

∴ $E = \frac{6.626\times10^{-34} \cdot 3\times10^{8}}{635\times10^{-9}} = 3.13\times10^{-19}$ J

∴ 1W of optical power $= \frac{1}{3.13\times10^{-19}}$ photons/sec

So, for a Q.E. of unity, the current would be $\frac{1.6\times10^{-19}}{3.13\times10^{-19}} = 0.51$ A/W

Hence for 0.36 A/W the Q.E. = 0.71 at this wavelength

[10%]

(e) To measure range, the flight time of the laser beam must be determined. This can be done by pulsing the laser and measuring the delay for the reflected beam to return or the laser can be continuously modulated at a pair of frequencies and the phase shift determined for each – this allows the range ambiguity to be resolved for repeated wavelengths of a single modulation frequency.

For 1000m range, the max. pulse repeat rate $= \frac{3\times10^{8}}{2000} = 150$ kHz before there is pulse-echo overlap.

For twin frequency modulation, the range ambiguity appears when $n\lambda_1 = (n+1)\lambda_2 = 2000$m

with $\lambda_1 = \frac{3\times10^{8}}{120\times10^{6}} = 25$m then $n = 80$, so $\lambda_2 = 24.69$m

so for λ_2, $f_2 = 121.5$ MHz

[15%]

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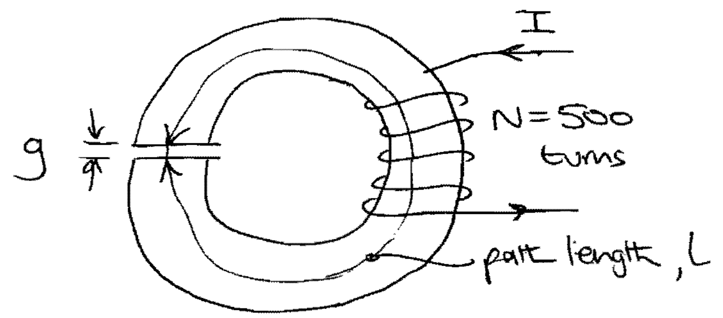
4(a)



$g = 1\,\text{mm}$ or $0.5\,\text{mm}$
$l = 105\pi$ mm
$A = 100\ \text{mm}^2$

Under zero load $g = 1\,\text{mm}$ $\qquad \int H.dl = \oint J.ds$

$NI = H_m l + H_0 g$ and $B_m = B_0 = B$ for flux conservation

with $B_m = \mu_0\mu_r H_m$, $B_0 = \mu_0 H_0$

$NI = H_0\left(g + {}^{l}/_{\mu_r}\right) \Rightarrow B = \dfrac{\mu_0 NI}{\left(g + {}^{l}/_{\mu_r}\right)}$ and $\phi = BA$

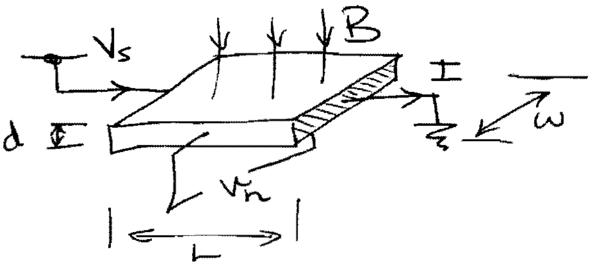
$$L = \frac{N\phi}{I} = \frac{\mu_0 N^2 A}{\underbrace{\left(g + {}^{l}/_{\mu_r}\right)}_{\substack{1.22\text{ mm}\\ \text{or}\\ 0.72\text{ mm}}}}$$

so with $N = 500$, $A = 100\times10^{-6}\ \text{m}^2$
$g = 10^{-3}\,\text{m}$, $l = 0.33\,\text{m}$
$\mu_r = 1500$, $\mu_0 = 4\pi\times10^{-7}\ \text{Hm}^{-1}$

$L_{\text{zero load}} = 0.0257\ \text{H}$ (1mm gap)
$L_{500N} = 0.0435\ \text{H}$ (0.5mm gap) $\qquad$ [25%]

(b) $B_{ON} = \dfrac{\overset{4\pi\times10^{-7}}{\mu_0}\ \overset{500}{N}\ \overset{1}{I}}{1.22\times10^{-3}} = 0.515\ \text{T}$, $\quad B_{500N} = 0.873\ \text{T}$

(c)



$Bqv_d = \dfrac{q v_h}{w}$ for force bal.

$v_d = \dfrac{\mu V_s}{L}$

$\therefore \dfrac{B\mu V_s w}{L} = v_h = 0.75B$

So, @ zero load, $v_h = 0.386\,\text{V}$
@ 500N, $v_h = 0.655\,\text{V}$
$\Big\}\ \Delta V = 0.269\,\text{V}$

[35%]

[illegible]

4(d) If we consider the energy stored in each case, $\frac{1}{2}LI^2$ for 1mm gap $= 0.0129$ J

" " 0.5mm $= 0.0218$ J

$\therefore$ a 0.5mm displacement causes a change in stored energy of 8.9×10^{-3} J in the magnetic circuit. So, by virtual work, the magnetic force is $\frac{8.9\times10^{-3}\ \text{J}}{0.5\times10^{-3}\ \text{m}} = F_m = 17.8$ N

This is a small, but not negligible force compared to the 500N applied force to deflect the toroid gap if no current were applied.

Hence, the load cell over-reads by about 3.5% at this load. Note, the effect is not linear – it gets worse as the gap is closed, but then magnetic saturation will happen in the core tending to reduce it as the gap gets very small.

Or we can consider $F\,\overset{\text{gap}}{\delta x} = \overset{\text{energy}}{\delta E}$ $\therefore$ $F = \frac{dE}{dx} = \frac{dE}{dL}\cdot\frac{dL}{dx}$

where $E = \frac{1}{2}LI^2$ and $L = \frac{\mu_0 N^2 A}{(g + \ell/\mu_r)}$ $\therefore \frac{dL}{dx} = \frac{-L}{(g+\ell/\mu_r)}$

$\therefore \frac{dE}{dL} = \frac{1}{2}I^2$

$\therefore F = \frac{-\frac{1}{2}I^2L}{(g+\ell/\mu_r)}$ = 10.5N for g=1mm
15.1N for g=0.5mm

So, if we compensate by setting the zero reading, the magnetic force error is only ≃1% @ 500N.

[25%]

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5 (a) $V = N\frac{d\phi}{dt} = 2\pi f N A B$

So for $f = 40$ kHz, $B = 10^{-3}$ T, $N = 200$, $A = \frac{\pi}{4}(4\times10^{-2})^2$

$\therefore V = 63$ V amplitude. $= \underline{44.6\ V_{rms}}$ [15%]

(b) Acoustic impedance of air $= \rho v = 408$ kg $m^{-2}s^{-1}$

Transducer acoustic impedance $= 10^5$ kg $m^{-2}s^{-1}$

$\therefore$ Coupling coeff $= \frac{4Z_tZ_a}{(Z_t+Z_a)^2} = 0.0162$

Power in transducer $= \frac{V^2}{R} = \frac{44.6^2}{5000} = 0.4$ W

Power converted to ultrasound $= 0.4 \times 0.15 = 0.06$ W

Ultrasound coupled to air $= 006 \times 0.0162 = 0.96$ mW

At a range of 2m, with isotropic radiator (no atten.)

u/sonic power density $= \frac{9.6\times10^{-4}}{4\pi 2^2} = 1.91\times10^{-5}$ W m^{-2}

With attenuation of 2dB m^{-1} × 2m $= -4$dB $= \times 10^{-4/10}$

$= \times 0.4$

$P_{u/s} \Rightarrow \underline{7.64\times10^{-6}\ W\ m^{-2}}$

[35%]

(c) Power intercepted by receiving transducer $= \frac{\pi d^2}{4} \times P_{u/s}$

$= \frac{\pi}{4}(4\times10^{-2})^2 \times 7.64\times10^{-6} = 9.60\times10^{-9}$ W

Of this, the power coupled into the transducer $= \times 0.0162$

$= 1.55\times10^{-10}$ W

Of this, the power converted to electrical signal $= \times 0.15$

$= 2.33\times10^{-11}$ W

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5(c) cont'd.

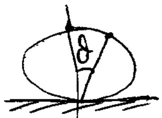
Thus the power delivered to a 500Ω matched load

$$= 2.33\times10^{-11} = \frac{V_r^2}{R} \quad \therefore V_r = 0.34\text{ mV into load}$$

Hence, open circuit voltage = 0.68 mV rms ≈ 2mV pp

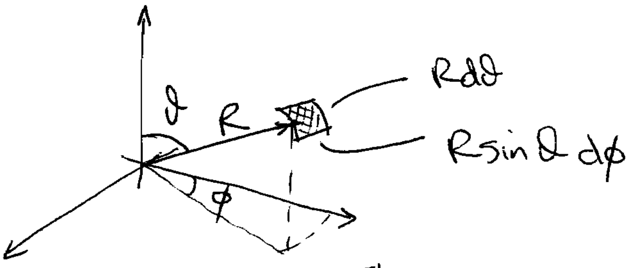
[30%]

(d) A Lambertian beam profile has a $\cos\theta$ dependence on emitted intensity, where θ is the angle to the normal.



polar plot.

$$I = I_0\cos\theta$$



$$\text{Total power emitted, } P = \int_0^{\pi/2}\int_0^{2\pi} I_0\cos\theta\, R^2\sin\theta\, d\phi\, d\theta$$

$$\therefore P = I_0R^2\,2\pi\int_0^{\pi/2}\underbrace{\sin\theta\cos\theta}_{=\frac{1}{2}\sin2\theta}\,d\theta$$

$$\therefore P = I_0R^2\,2\pi\left[-\tfrac{1}{2}\underline{\cos2\theta/2}\right]_0^{\pi/2} = I_0R^2\pi$$

$$\therefore I_0 = \frac{P}{\pi R^2} \quad \text{compared to} \quad \frac{P}{4\pi R^2}$$

Lambertian — isotropic (full sphere)

Hence received signal power will be 16x greater (gain both ways) = x4 for signal voltage. **But**, the system is now directional, so this is only achieved for optimal alignments. In some cases the signal could be worse.

[20%]

4B13 comments

Q1 Temperature & strain sensing

Quite a popular question which was well answered by most candidates. Most knew how to calculate the thermistor resistance correctly although there were a number of errors in the transistor circuit. The pyrometer and strain gauge sections were mainly correctly answered.

Q2 MEMS optical switch

A rather unpopular question only attempted by about 10% of the candidates. The MEMs process description generally lacked detail, although all who attempted the question knew the structure of accelerometers and pressure sensors. The second half of the question on the cantilever beam was quite well answered, but the settling time was poorly estimated in many cases.

Q3 Optical velocity / range sensing

A popular and well attempted question, which was well answered. The signal magnitude and Doppler frequency were correctly estimated in most cases and the noise calculation was quite well done – some chose to use a wider bandwidth than the Doppler frequency; this was equally valid. The quantum efficiency of the photodiode was also correctly determined by most people.

Q4 Electromagnetic load cell

Another fairly popular question with a good range of answers. Most could evaluate the sensor inductance correctly although there were minor errors concerning the toroid cross-sectional area. The Hall sensor section was also well answered but only a few candidates correctly ascertained the magnetic force offset in the last part.

Q5 Inductive / ultrasonic detection system

This was the most popular question, being answered by almost all candidates. The induced voltage in the coil was simple, but stumped a few candidates. The ultrasonic calculations were generally well done although a number assumed 'isotropic' to mean only a hemisphere. The final part on a Lambertian beam profile was correctly interpreted by most candidates, but they tended to get the details wrong.