

**ENGINEERING TRIPOS PART IIB 2012  
4B19 RENEWABLE ELECTRICAL POWER**

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1 (a) (i) Resource:

Land based wind turbines:- extensive (many GW) but constrained by planning.

Tidal currents: substantial but there are few easy sites.

Biomass: substantial (wood, straw, elephant grass) but competes with food crops.

(ii) Technology

Wind turbines: mature but can always be improved.

Tidal currents: developing (e.g. Strangford Lough) but reliability and survivability still issues, especially for more exposed sites.

Biomass: mature but handling of large quantities of biomass is tricky.

(iii) Electrical equipment

Wind turbines: permanent magnet or induction generators with 100% conversion or doubly fed slip ring machines.

Tidal currents: as wind turbines.

Biomass: conventional synchro machines.

(iv) Environmental Impact

Wind turbines: visual, especially in areas of natural beauty, some noise, some birds killed.

Tidal currents: possible effects on marine life, shipping.

Biomass: usual effects of intensive agriculture, displacement of other crops.

(b) (i) Reliability

Offshore environment is harsher – more failures so reliability is poorer.

(ii) Maintenance

Offshore is more difficult and expensive.

(iii) Capacity factor

Offshore generally has higher winds increasing capacity factor but reliability issues will reduce it.

(iv) Choice of turbine type

Reliability is critical but although gear boxes have a bad reputation it's unclear that direct drive machines are obviously more reliable.

(c) (i) DC links to turbine clusters avoid excessive charging currents in AC cables although converters are required. DC is attractive at say 20 miles +.

(ii) Long distance DC links can bypass national bottle necks (e.g. North Wales to Scotland link which bypasses the current England – Scotland interconnector). Also, they can link countries e.g. UK and the Republic of Ireland to transfer surplus wind power.

### Examiner's comment:

This first part of this question was on various means of renewable generation. Most candidates were able to answer this part although some candidates were confused about the various types electrical generator that could be used. The second part of the question on specific aspects of wind turbines was again generally well answered but the final part on the role of DC links was not. Very few candidates knew that charging currents associated with cable capacitance was the limiting factor in undersea AC cables.

2 (a) Wind turbines have an optimal tip-speed ratio (ratio of the speed of the blade tip,  $\omega R$ , to the wind speed,  $v$ ) at which they operate with the maximum power coefficient, thereby maximizing their power output. If the angular speed of the turbine is allowed to remain in proportion to the wind speed, then optimal tip-speed ratio is attained at all wind speeds, thereby maximizing power output.

(b) (i) The wind speed at which the wind turbine develops rated power is  $12 \text{ ms}^{-1}$ . Assuming that the turbine is operated so that its power coefficient is maximized, then the power coefficient  $C_p$  will be 0.4. Thus, the rated power is given by:

$$P = 0.5C_p\rho Av^3 \text{ with } C_p = 0.4, \rho = 1.23 \text{ kgm}^{-3}, A = \pi \times 40^2 = 5027 \text{ m}^2 \text{ and } v = 12 \text{ ms}^{-1} \text{ giving } P = 2.14 \text{ MW.}$$

(ii) No power is developed at the wind speeds of  $2 \text{ ms}^{-1}$  (below cut-in) and  $22 \text{ ms}^{-1}$  (above stall). The power output at wind speeds of  $3.5 \text{ ms}^{-1}$ ,  $7 \text{ ms}^{-1}$  and  $10 \text{ ms}^{-1}$  may be found by scaling the rated power by  $(v/12)^3$ . At the wind speed of  $14 \text{ ms}^{-1}$ , the output power is capped at the rated power of 2.14 MW. This information is entered into the table below, from which the output energy is found at the 4 wind speeds by multiplying power by the number of hours. The sum of these energies gives the output annual electrical energy.

Wind speed ( $\text{ms}^{-1}$ )	Power (MW)	Days	Hours	Energy (MWhr)
3.5	0.053	65	1560	82.7
7	0.425	180	4320	1836
10	1.24	55	1320	1637
14	2.14	30	720	1541

giving a total annual energy output of 5096 MWhr or 5.096 GWhr.

$$\text{Capacity factor} = 5096 / (24 \times 365 \times 2.14) = 27.2\%$$

(iii) The most common wind speed is  $7 \text{ ms}^{-1}$ , and so the tip-speed ratio is 8 with the corresponding power coefficient of 0.4 only at this wind speed. At the other wind speeds the tip-speed ratio will be different because the turbine angular speed is fixed. Thus, the tip-speed ratio,  $\lambda$ , is given by:

$\lambda = 8 \times 7/v$  at other wind speeds giving values of 16, 8, 5.6 and 4 at the wind speeds of 3.5, 7, 10 and  $14 \text{ ms}^{-1}$  respectively. The corresponding power coefficients can be read directly from the table except for  $\lambda = 5.6$ . For this case, linear interpolation is used:

$$C_{p(\lambda=5.6)} = 0.22 + (5.6 - 4) \times (0.4 - 0.22) / (8 - 4) = 0.292$$

Thus, the output power at the 4 wind speeds may be calculated using  $P = 0.5C_p\rho Av^3$  with  $C_p$  now known at the wind speeds. This information is given in the table below, from which the annual energy output is found as before.

Wind speed ( $\text{ms}^{-1}$ )	Power (MW)	Days	Hours	Energy (MWhr)
3.5	0.033	65	1560	51.5
7	0.425	180	4320	1836
10	0.903	55	1320	1192
14	1.87	30	720	1346

giving a total annual energy output of 4426 MWhr or 4.426 GWhr.

$$\text{Capacity factor} = 4426 / (24 \times 365 \times 2.14) = 23.6\%$$

(c) Discounted cash flow analysis is a method of determining the economic viability of a scheme which takes account of the cost of borrowing money to implement the scheme. The basic idea is that all cash flow is re-based to a common point in time (usually the start of the project) by taking into account the real interest rate (monetary interest rate – inflation). In its simplest form, it is assumed that the construction time of the project is negligibly small and that ongoing maintenance costs as well as revenue income rises at the real interest rate.

(d) Consider first the low interest rate economic climate.

For the fixed speed scheme, the annual costs are  $2000000 \times \text{£}40/1000 + 0.01 \times 2000000 = \text{£}100000$  and the annual income is  $4426 \times \text{£}70 = \text{£}309820$ . This gives a profit of  $\text{£}209820$ .

For the variable speed scheme the costs are  $2200000 \times \text{£}40/1000 + 0.015 \times 2200000 = \text{£}121000$  and the annual income is  $5096 \times \text{£}70 = \text{£}356720$ . This gives a profit of  $\text{£}235720$ .

Now consider the high interest rate economic climate.

For the fixed speed scheme, the annual costs are  $2000000 \times \text{£}71/1000 + 0.01 \times 2000000 = \text{£}162000$  and the annual income is  $4426 \times \text{£}70 = \text{£}309820$ . This gives a profit of  $\text{£}147820$ .

For the variable speed scheme the costs are  $2200000 \times \text{£}71/1000 + 0.015 \times 2200000 = \text{£}189200$  and the annual income is  $5096 \times \text{£}70 = \text{£}356720$ . This gives a profit of  $\text{£}167520$ .

Assumptions: the construction time is negligibly small; the real interest rate remains constant over the 25 year lifetime of the project; operations and maintenance costs as well as electricity income received increases at the real interest rate, so that their values remain constant when rebased to the start of the project.

Comment: The variable speed scheme will always produce more income than the fixed speed scheme, but has higher capital and operations and maintenance costs. It produces more profit during both economic climates, however it is possible that if the capital costs were much higher that this would not be the case if the real interest rate was high.

### Examiner's comment:

This question concerned the analysis of variable vs fixed speed large scale wind power, both in terms of the annual energy output and economics through a discounted cash flow analysis. There were many excellent attempts at this question, showing a good understanding of the main concepts. The most common mistake was failing to determine the correct power coefficient at wind speeds other than the most probably wind speed when analysing the power output of the fixed speed system.

3 (a) Water of mass  $M$  stored at height  $H$  with respect to the point where the energy is extracted has potential energy given by

$$PE = MgH$$

Power is the rate of change of potential energy. Supposing that water is allowed to flow from height  $H$  to zero, giving up its energy to a turbine-generator. Here we assume that the head  $H$  remains constant ie inflows to the store of water = outflows. Thus the power is given by

$$P = gHdM/dt = gH\rho Q$$

where  $\rho$  is the density of water ( $1000 \text{ kgm}^{-3}$ ) and  $Q$  is the volumetric flow rate in  $\text{m}^3 \text{ s}^{-1}$ . There will be power losses owing to frictional drag and turbulence in the water, turbine losses and generator losses. This is accounted for by the system efficiency,  $\eta$ . Thus, the available power is given by:

$$P = \eta\rho gHQ$$

The turbine choice is made by considering the head of water and the flow rate for a particular scheme. A high head, low flow rate will favour an impulse turbine. A low head, high flow rate will favour a propeller turbine. Medium head, medium flow rate schemes favour Francis turbines.

These turbines have an optimum specific speed at which they operate. Specific speed is rather like tip-speed ratio for wind turbines, and relates the system rpm, head of water and rated power through the optimum specific speed. Thus, for a given value of  $P$  and  $H$ , and knowing the optimum specific speed of the turbine, the optimum rotational speed for the system may be found.

The optimum rotational speed of hydroelectric schemes are typically of the order of several hundred rpm. Ideally the generator will require no gearbox, and be able to connect directly to the grid. Because of the relatively low rotational speed, salient-pole synchronous generators fit both of these requirements. They are also favoured because their excitation emf can be controlled so that they contribute to the reactive power demand of the power system.

(b) (i)  $H = 30 \text{ m}$ ,  $Q = 10 \text{ m}^3 \text{ s}^{-1}$ ,  $\rho = 1000 \text{ kgm}^{-3}$ ,  $g = 9.81 \text{ ms}^{-2}$  and  $\eta = 75\%$ . Putting these numbers into the power expression gives  $P = 2207 \text{ kW}$ .

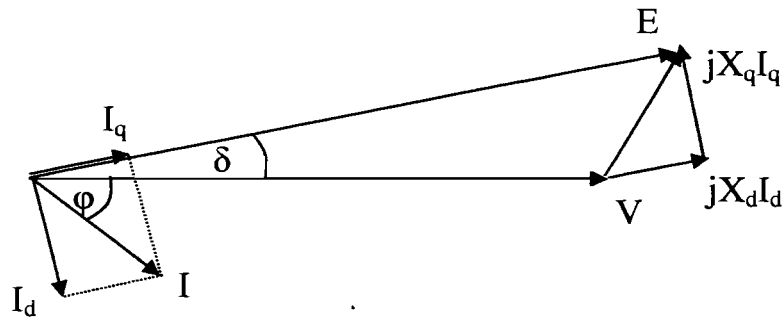
$$(ii) N_s = 200.7 = n.2207^{1/2}.30^{-5/4} = 0.669n \text{ giving } n = 300 \text{ rpm} = 31.4 \text{ rads}^{-1}.$$

$$\text{Using } P = T\omega \text{ gives } T = 2207/31.4 = 70.3 \text{ kNm}$$

(iii) Synchronous speed in rpm is  $60f/p$  where  $f$  is the bus frequency,  $50 \text{ Hz}$ , and  $p$  is the generator pole-pairs giving  $300 = 3000/p$  and so  $p = 10$  pole-pairs.

Worst case scenario for the generator is operating at rated power with a  $0.6$  lagging power factor, giving a generator rating of  $2207/0.6 = 3.68 \text{ MVA}$ .

(c) (i) Phase voltage  $= 11 \text{ kV}/\sqrt{3} = 6.35 \text{ kV}$ ,  $P = 2.207 \text{ MW}$  and using  $P = 3VI\cos\phi$  gives  $I = 193 \text{ A}$  lagging the phase voltage by  $\cos^{-1}(0.6) = 53.1^\circ$ . Phasor diagram for this is shown below.



From phasor diagram:

$$I_q = I \cos(\varphi + \delta) = I(\cos\varphi \cos\delta - \sin\varphi \sin\delta)$$

$$V \sin\delta = X_q I_q = X_q I(\cos\varphi \cos\delta - \sin\varphi \sin\delta)$$

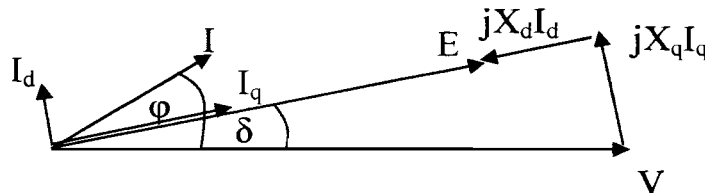
Divide through by  $\cos\delta$  and make  $\tan\delta$  the subject of the equation:

$$\tan\delta = X_q I \cos\varphi / (V + X_d I \sin\varphi) = (10 \times 193 \times 0.6) / (6350 + 10 \times 193 \times 0.8) = 0.147 \text{ giving } \delta = 8.35^\circ.$$

$$I_d = I \sin(\varphi + \delta) = 170 \text{ A}$$

$$E = V \cos\delta + I_d X_d = 8825 \text{ V} = 15.3 \text{ kV line}$$

(ii) Current now leads voltage by  $\cos^{-1}(0.8) = 36.9^\circ$  and the new current is found using  $P = 3VI \cos\varphi$  giving  $I = 145 \text{ A}$ . The phasor diagram for this situation is shown below.



From phasor diagram:

$$I_q = I \cos(\varphi - \delta) = I(\cos\varphi \cos\delta + \sin\varphi \sin\delta)$$

$$V \sin\delta = X_q I_q = X_q I(\cos\varphi \cos\delta + \sin\varphi \sin\delta)$$

Divide through by  $\cos\delta$  and make  $\tan\delta$  the subject of the equation:

$$\tan\delta = X_q I \cos\varphi / (V - X_d I \sin\varphi) = (10 \times 145 \times 0.8) / (6350 - 10 \times 145 \times 0.6) = 0.212 \text{ giving } \delta = 12.0^\circ.$$

$$I_d = I \sin(\varphi + \delta) = 109 \text{ A}$$

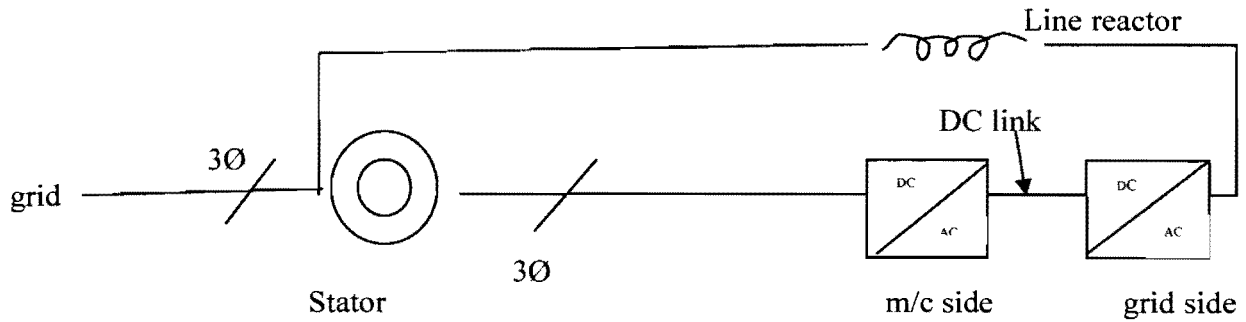
$$E = V \cos\delta + I_d X_d = 7849 \text{ V} = 13.6 \text{ kV line}$$

### Examiner's comment:

This question was on principles of hydroelectric generation followed by some salient pole synchronous generator calculations. There were a good number of excellent answers, with most students scored heavily on the first sections concerning general principles and simple calculations. Predictably, the salient pole synchronous generator calculations proved more challenging, although many students who got the phasor diagrams correct went on to achieve high marks.

4 (a) Doubly fed systems have a reduced converter rating and so are economical.

(b)



(i) machine side – supplies rotor with variable voltage, variable frequency to allow speed to vary. Power flow can be in either direction.

(ii) grid side converter – transfer power between the 3Ø grid and the dc link. Output is fixed frequency and manually fixed voltage.

(iii) The line side reactor is necessary for proper converter action – the mains is sinusoidal but the converter o/p is pwm so the reactor takes the difference – effectively averaging, and it also allows phase angle control of the power flow into the grid.

(c) (i) Total power = 1.5MW = Pstator + Protor

$$Protor = \left( \frac{N_r - N_s}{N_s} \right) Pstator$$

$$1.5 \text{ MW} = Pstator + \left( \frac{1360 - 1000}{1000} \right) Pstator$$

$$Pstator = 1.1 \text{ MW}$$

$$Protor = 1.1 \times \left( \frac{1360 - 1000}{1000} \right) = 0.397 \text{ MW}$$

$$\text{The VA rating is } \frac{P}{\cos\phi} = \frac{0.397}{0.9} = 0.441 \text{ MVA}$$

Machine side needs to be at least 441 KVA

Grid side needs to be at least 397 KVA ( $\phi f = 1$ )

(ii) Speed deviation is proportional to frequency

$$\text{So } \frac{360}{1000} = \frac{fv}{50} \text{ so } fv = 18Hz$$

+ 18Hz for increase in speed,

- 18Hz for reduction in speed.

Effectively a sequence change.

(iii) Using the formula, and  $m=1$ ,

$V_{DC} = \frac{2\sqrt{2}}{\sqrt{3}} V_{ac}$  with  $V_{ac} = 690$  V so  $V_{DC} = 1127$  V. This is a minimum value – add say 10% so 1250 V nominal would be a good figure.

(iv) 690 V would be readily obtainable from the machine side converter with the chosen dc link and this would be at maximum frequency deviation.

(d) BDFM offers (a) no brushes so lower maintenance and (b) more robust construction.

### Examiner's comment:

This question on the doubly-fed arrangement for wind turbines was not popular although three of the candidates obtained a good mark (average 14), the fourth only completing part of the question. Most candidates did not realize that 1.5 MW was the total output, not the output from the stator and that the rotor voltage could be set as desired by choosing the rotor turns – in other words the stator-rotor turns ratio is not fixed at 1:1. As a result low rotor voltages were specified which waste inverter capacity.