Engineering Tripos Part II B

Module 4C2: Designing with Composites

CRIB 2011/12 (M SUTCLIFFE)

1 (a) Extreme ν along and transverse to fibre Eqn $\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$

$\downarrow -\nu_{12}\varepsilon_1$

$\leftarrow \equiv \xrightarrow{\varepsilon_1}$ Relatively easy to strain transversely due to compliance of matrix $\Rightarrow \nu \sim 0.3$

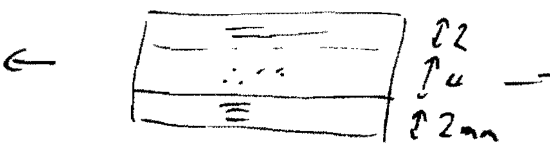
$\uparrow$

$\downarrow -\nu_{21}\varepsilon_2$

$\leftarrow ||| \rightarrow \varepsilon_2$ Now fibres resist the strain transverse to applied load $\Rightarrow$ small ν_{21}

[15%]

(b)



Well answered, though (a) tended to have vague comments not addressing the question

To find σ in 0s

1) Find A matrix
2) $\varepsilon = A^{-1}N$
3) $\sigma = Q\varepsilon$

} No shear so only $A_{11} \to A_{22}$ needed

$\nu_{21} = \nu_{12}\frac{E_2}{E_1} = 0.06$

$Q_0 = \frac{1}{0.982}\begin{pmatrix} 40 & 2.4 \\ 2.4 & 8 \end{pmatrix}$ MPa, $Q_{90} = \begin{pmatrix} 8 & 2.4 \\ 2.4 & 40 \end{pmatrix}\frac{1}{0.982}$ MPa

$A = \frac{4}{0.982}\begin{pmatrix} 48 & 2.4 \\ 2.4 & 48 \end{pmatrix}$ mm.MPa, $A^{-1} = \frac{0.982}{\Delta \times 4}\begin{pmatrix} 48 & -4.8 \\ -4.8 & 48 \end{pmatrix}\frac{1}{\text{mm MPa}}$

$\Delta = 48^2 - 4.8^2$

$N = \begin{pmatrix} 50 \times 8 \\ 0 \\ 0 \end{pmatrix}$ MPa.mm , $\sigma_0 = Q_0\varepsilon = Q_0 A^{-1} N$

Need to keep track of units

$= \frac{1}{0.982}\begin{pmatrix} 40 & 2.4 \\ 2.4 & 8 \end{pmatrix}\frac{0.982}{4\Delta}\begin{pmatrix} 48 & -4.8 \\ -4.8 & 48 \end{pmatrix}\begin{pmatrix} 400 \\ 0 \end{pmatrix}$ MPa

$\uparrow$

This step caught people out

$= \begin{pmatrix} 83.6 \\ 3.37 \end{pmatrix}$ MPa

$\sigma_{12} = 0$

[60%]

1 (c) Now the lay-up is unsymmetrical. This will cause curvature associated with in-plane loading. The boundary conditions of the problem will affect whether there is any curvature induced due to Poisson's ratio effects. To solve these we need to follow the same method as (b), but now the full ABD matrix needs to be calculated. If a stress boundary condition is applied, then the strains and curvatures can be found directly from inverting the ABD matrix. If the boundary conditions include some displacement constraint (for example if the ends are clamped or restrained in some way to prevent curvature), the mixed problem will need to be solved. Once the global strains are found the stresses in the individual plies again follows from the Q matrix.

2. (a) CFRP - good for lightweight applications where stiffness and strength is important. For larger structures there is a premium on performance (and self-weight becomes important) so that the cost implications are less critical. Pultruded material is relatively easy to make into prismatic shapes giving a cheap manufacturing route with a good production rate. For the spar the unidirectional nature of the pultruded composite (with fibres being well-aligned) can give good stiffness and strength properties.

(b) Joints have a complex stress pattern, with the anisotropic nature of composites meaning that it is difficult to get load into the fibres. This tends to produce mixed-mode loading. This combination of factors, together with the various failure modes in composites, means that it is difficult to predict failure. For this reason there is little by way of codes and standards for joining. This means that to design a joint reliably there needs to be substantial testing. This would not just be on smaller typical coupons but also on larger sub-components or structures, which might change the loading or manufacturing details. Moreover airworthiness certification will require strict testing.

(c) Although specific strength and stiffness will always work in the favour of composites, they also possess a range of other properties which can tip the balance either in favour or against them. Often material selection requires a portfolio of properties. For example
Cost - always important. The raw material cost of composites tends to be high, but a good manufacturing route might bring the component cost down.
Corrosion resistance can be good - e.g pipes, boats
Surface finish - good finish using smooth moulds
Radar resistance for stealth
Thermal expansion tends to be small
Aesthetics - e.g. helmets
Others examples ...

(d) The issue here is that out-of-autoclave manufacturing will tend to produce lower mechanical properties, but it is a cheaper route which can allow much faster cycle times than autoclaving (both the installation and running costs of autoclaves are high). Moreover there is not a limitation on size. As commercial manufacturers of aircraft extend both the size and extent of composites parts in their aircraft the cost considerations mean that an out-of-autoclave route giving decent mechanical properties becomes attractive. Moreover making lots of smaller parts will introduce joints, always a source of concern and added weight.

(e) Transverse tensile strength of composites is affected either by failure in the matrix or by failure in the matrix-fibre interface. Sizing, a chemical treatment of the fibres, can prevent premature failure of the interface and hence increase the transverse tensile strength.

[This question was well answered, with lots of good comments backed up by details. To get full marks most of the points mentioned in the cribs were required.]

3. (a) Fundamentally splitting (cracks running in the fibre direction within plies) is caused by the low fracture toughness of the matrix. Stresses driving splitting can arise from transverse loading but also a range of other local effects such as holes, notches, kinking from cracks and across fibres. [15%]

(b) Need to measure toughness. Note that mixed mode loading occurs. Could estimate splitting toughness from interlaminar toughness

DCB (mode I)

Short beam shear (mode II)

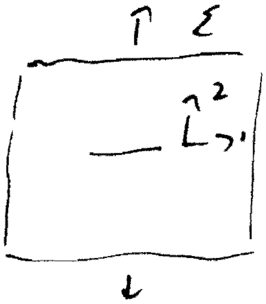
[10%]

(c) (i) $S_{11} = 1/39$, $S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.26}{39}$, $S_{22} = \frac{1}{E_2} = \frac{1}{8.3}$, $S_{66} = \frac{1}{G_{12}} = \frac{1}{4.1}$ $\frac{1}{GPa}$

$$\frac{1}{E_A'} = \left(\frac{S_{11} S_{22}}{2}\right)^{\frac{1}{2}} \left[\left(\frac{S_{22}}{S_{11}}\right)^{\frac{1}{2}} \left(1 + \frac{2 \cdot S_{12} + S_{66}}{2\sqrt{S_{11} S_{22}}}\right)\right]^{\frac{1}{2}} = \frac{1}{9.86} \frac{1}{GPa}$$

$$E_B' = E_A' \left(\frac{S_{22}}{S_{11}}\right)^{\frac{1}{2}} = 21.6 \text{ GPa}$$

Here



Here: $K_I = \Sigma\sqrt{\pi a}$

$K_{II} = 0$

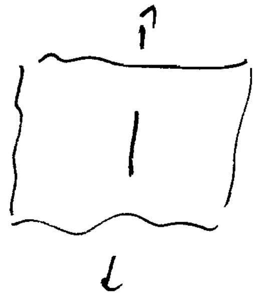
$G = K_I^2 / E_A'$

Put $G = G_c$

$\Rightarrow \Sigma^2 \pi a = G_c E_A'$ at failure

$$\Sigma = \sqrt{\frac{10^4 \cdot 9.86 \times 10^9}{\pi \cdot 5 \times 10^{-3}}} = 79.2 \text{ MPa}$$

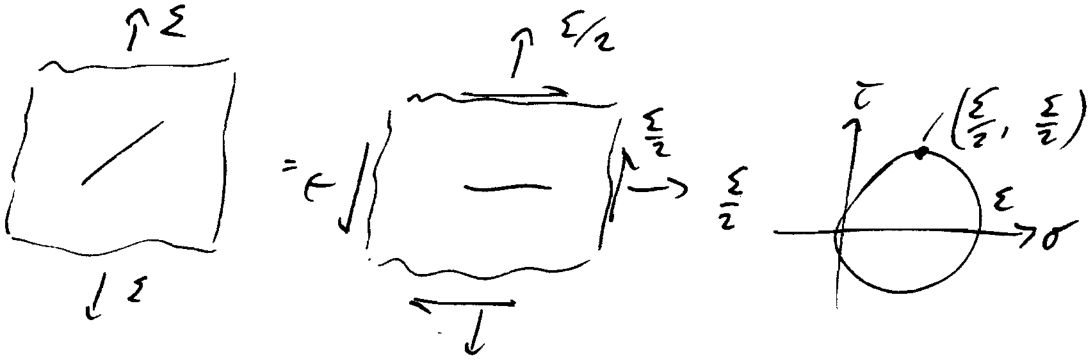
3 (c) (ii)



$K_I = K_{II} = 0$, $\sigma \to \infty$

No cracking, in theory.

(iii)



$K_I = \frac{\Sigma}{2}\sqrt{\pi a}$, $K_{II} = \frac{\Sigma}{2}\sqrt{\pi a}$

$G = G_I + G_{II} = \frac{\Sigma^2}{4}\pi a\left(\frac{1}{E_A'} + \frac{1}{E_B'}\right)$

$G = G_c \Rightarrow \Sigma = \frac{2}{\sqrt{\pi \cdot 5 \times 10^{-3}}}\sqrt{10^4 \cdot 6.76 \times 10^9} = 131\ MPa$

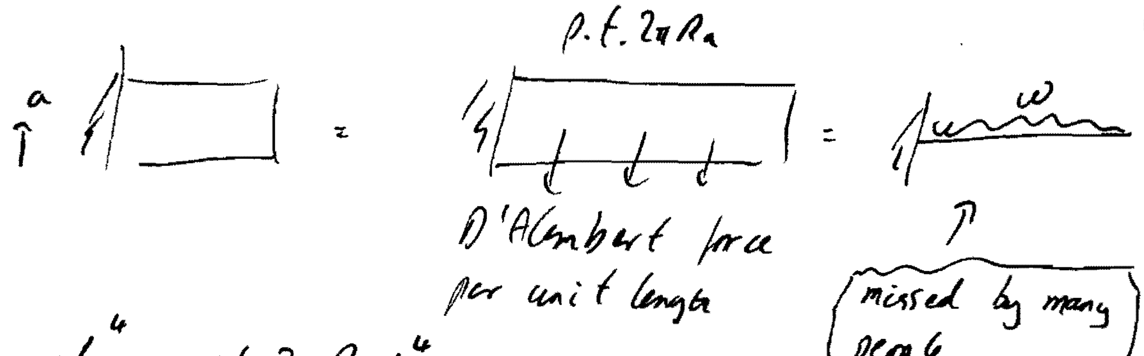
(d) Expect toughness to depend on mode mixity.
LEFM may not be appropriate, need to consider large scale bridging.

[15 %]

~~Ex from~~

Most problems arose from calculating values for K_I and K_{II} for the various loadings, and for (c)(iii) of not adding G_I and G_{II}

4 (a)



$$\delta = \frac{wL^4}{8EI} = \frac{\rho t \cdot 2\pi R a L^4}{8E\pi R^3 t}$$ where E is for beam bending

$$= \frac{\rho a L^4}{4ER^2}$$ – independent of wall thickness

Need $\frac{E}{\rho} > \frac{aL^4}{4\delta R^2} = 4 \times 10^7 \ \frac{N m^{-2}}{kg\, m^{-3}}$

	CFRP	GFRP	Kevlar	
E	140	45	80	
ρ	1500	1900	1400	
E/ρ	9.3×10^7	2.6×10^7	5.7×10^7	$\frac{N m^{-2}}{kg\, m^{-3}}$

GFRP not adequate
CFRP & Kevlar both OK.
More scope for off-axis plies in CFRP. [35%]

{not asked to consider strength here}

4(b) – Choose only ±65 and 0, assume enough off-axis plies to prevent splitting (also 90s difficult for tubes)

– for a tube unsymmetrical should be OK but keep balanced

– no safety factor

Need $E_x > 4\times10^7\times1500 = 60$ GPa $\Rightarrow$ %0 > 30 from carpet plot

For shear strength, use ε allowable

$$\varepsilon_{LT} = 0.005 = \frac{N_{xy}}{tG} = \frac{Q}{2\pi R^2 t G}$$ ← (this step caused problems)

$$\Rightarrow tG > \frac{4\times10^3}{2\pi(0.05)^2\times0.005} = 5.08\times10^7 \text{ Nm}^{-1}$$

Try 0:65 = 30:70 $\Rightarrow$ G = 27.5 GPa, t = 1.85 mm $\Rightarrow$ 14.8 plies

$\Rightarrow$ Try 5 plies of 0, 10 plies of ±65

→ 33% 0 so ok on stiffness

Check strength: G = 26.5 GPa, t = 1.875 mm,

$$\gamma = \frac{4\times10^3}{2\pi\times0.05^2\times1.875\times10^{-3}\times26.5\times10^9} = 0.51\%$$ ok ✓ (close enough)

Alternative layups, eg $(0_6(\pm65)_6)$ or $[0_3\ \pm65_3]_s$

Suitable layup $0_5(\pm65)_5$ putting ±65 on outside for impact. [45%]

(c) Check impact, joints, fatigue

Test structure carefully

Manufacture – autoclave to maximise performance

May be better to use eg ±30° plies.

Thinner plies to optimise better.

Temperature / thermal expansion effects. [20%]

This part not answered well for 20%