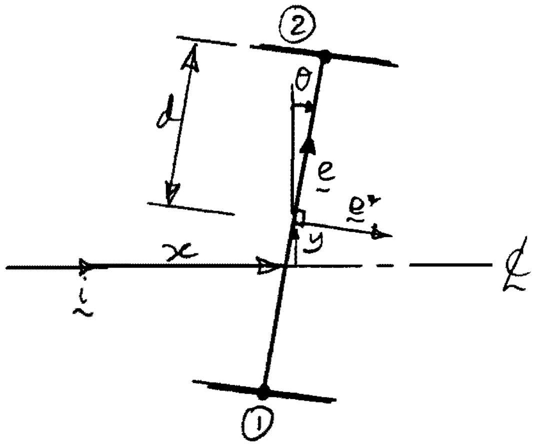
Q1 ...



Position vector of wheel (2):

$$\underset{\sim}{r}_2 = x\underset{\sim}{i} + y\underset{\sim}{j} + d\underset{\sim}{e}$$

$$\dot{\underset{\sim}{r}}_2 = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j} + d\dot{\theta}\underset{\sim}{e}^* \quad \text{— (1)}$$

Now $\dot{x} = u$

& $\underset{\sim}{i} = \sin\theta\,\underset{\sim}{e} + \cos\theta\,\underset{\sim}{e}^* \approx \theta\underset{\sim}{e} + \underset{\sim}{e}^*$

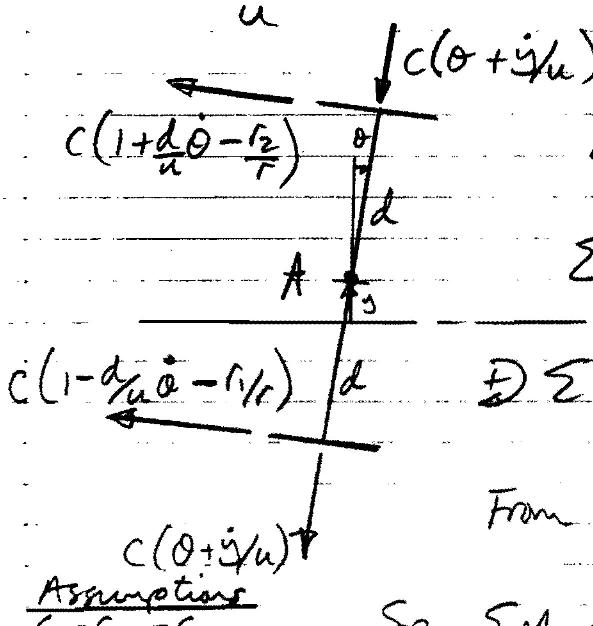
$\underset{\sim}{j} = \cos\theta\,\underset{\sim}{e} - \sin\theta\,\underset{\sim}{e}^* \approx \underset{\sim}{e} - \theta\underset{\sim}{e}^*$

so (1) becomes: $\dot{\underset{\sim}{r}}_2 = (u + d\dot{\theta} - \underbrace{\dot{y}\theta}_{\text{neglect (small term)}})\underset{\sim}{e}^* + (\dot{y} + u\theta)\underset{\sim}{e}$ — (2)

If the wheelset rotates with ang. vel $\omega = u/r$, then the longitudinal creep velocity is $v_2 = u + d\dot{\theta} - \frac{r_2 u}{r}$ — (3)

Assuming the lateral and long'l creep forces are

$X = -C\frac{v_x}{u}$ & $Y = -C\frac{v_y}{u}$ gives — (4)



Σ lateral forces $= 2C(\theta + \dot{y}/u)$ — (5)

Σ longitudinal forces $= 0$

Σ Moments at $A = dC\left[\left(\frac{r_2 - r_1}{r}\right) - \frac{2d\dot{\theta}}{u}\right]$

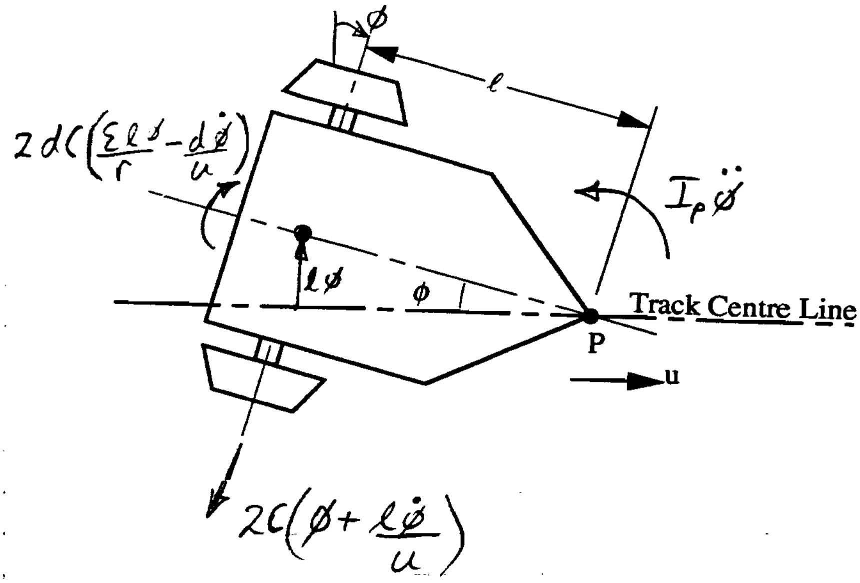
From wheel geometry $r_2 - r_1 = 2\varepsilon y$

So $\Sigma M_A = 2dC\left[\frac{\varepsilon y}{r} - \frac{d\dot{\theta}}{u}\right]$ — (6)

(40%)

Assumptions
- $C_{11} = C_{22} = C$
- Linear creep
- No spin creep
- Small y, θ (10%)

Q1 (Cont)



$\theta \to \phi$

$y = l\phi$

$\dot y = l\dot\phi$

$\circlearrowleft \Sigma M_P: \quad l.2c\left(\phi+\frac{l\dot\phi}{u}\right)+I_p\ddot\phi-2dc\left(\frac{\varepsilon l\phi}{r}-\frac{d\dot\phi}{u}\right)=0$

$\Rightarrow I_p\ddot\phi+\frac{2c}{u}\left(l^2+d^2\right)\dot\phi+2cl\left(1-\frac{d\varepsilon}{r}\right)\theta=0$

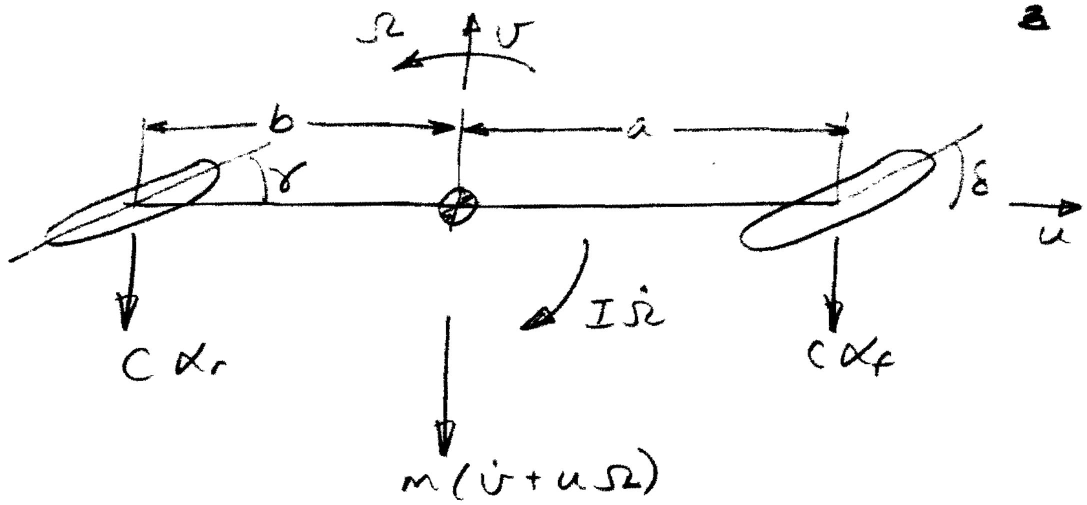
Motion is stable if all coefficients are positive

ie $u>0$ and $1-\frac{d\varepsilon}{r}>0 \Rightarrow \underline{\underline{\varepsilon<r/d}}$ (30%)

This implies an upper limit on the allowable conicity and no lower limit - the conicity can ever be negative. This is quite different to the free wheelset which requires $\varepsilon>0$.

Above the upper limit, $\varepsilon=r/d$, the yawing moment of the long'l forces: $2dc\left(\frac{\varepsilon l\phi}{r}\right)$ exceeds $l.2c\phi$ and therefore destabilises the system. However r/d is a very large conicity — much larger than practical values. (20%)

Q2.



<u>Slip angles</u>:

$$\left.\begin{aligned} \alpha_f &= \frac{v + a\Omega}{u} - \delta \\ \alpha_r &= \frac{v - b\Omega}{u} - \gamma \end{aligned}\right\} \quad (1)$$

(a) <u>Force and moment eqn</u>:

$$\left.\begin{aligned} & m(\dot{v}+u\Omega) + C\alpha_f + C\alpha_r = 0 \\ & I\dot{\Omega} + aC\alpha_f - bC\alpha_r = 0 \end{aligned}\right\} \quad (2)$$

(1) into (2) gives

$$\left.\begin{aligned} & m(\dot{v}+u\Omega) + 2C\frac{v}{u} + (a-b)C\frac{\Omega}{u} = C(\delta+\gamma) \\ \& \quad & I\dot{\Omega} + C(a-b)\frac{v}{u} + C(a^2+b^2)\frac{\Omega}{u} = C(a\delta - b\gamma) \end{aligned}\right\} \quad (3)$$

Steady state turn: $\dot{v} = \dot{\Omega} = 0$, $\gamma = K\delta$

$\& \; u = R\Omega$

(3) becomes

$$\begin{bmatrix} mu^2 + (a-b)C & 2C \\ C(a^2+b^2) & C(a-b) \end{bmatrix} \begin{Bmatrix} \Omega/u \\ v/u \end{Bmatrix} = C\delta \begin{Bmatrix} 1+K \\ a-bK \end{Bmatrix}$$

Q2 (cont)

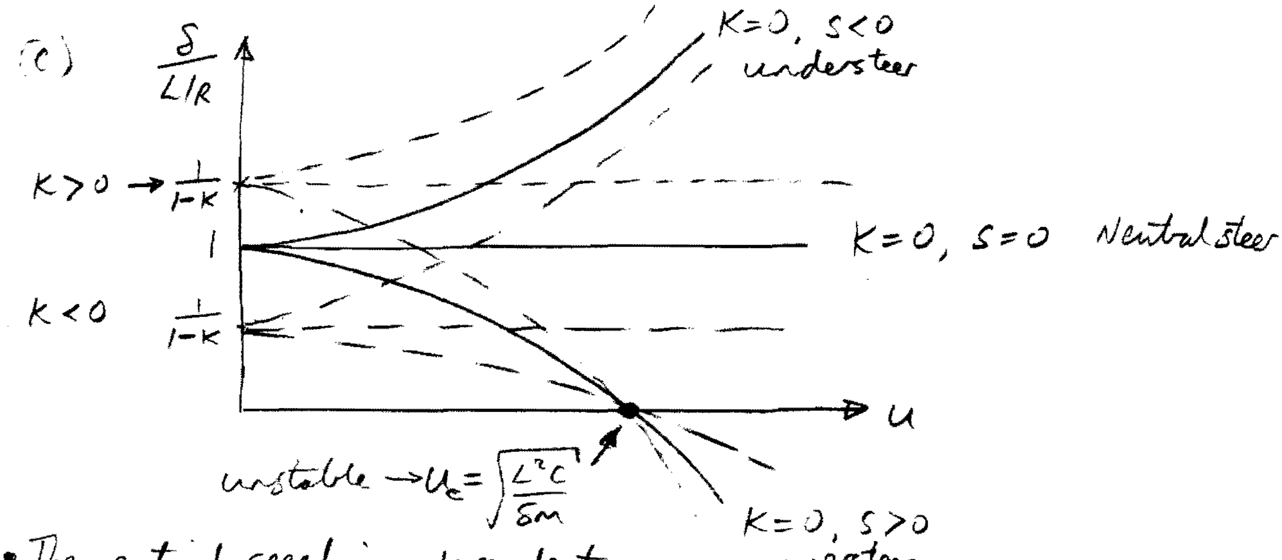
So
$$\begin{Bmatrix} \frac{\Omega}{u} \\ v/u \end{Bmatrix} = \frac{\begin{bmatrix} c(a-b) & -2c \\ -c(a^2+b^2) & mu^2+(a-b)c \end{bmatrix} C\delta \begin{Bmatrix} 1+K \\ a-bK \end{Bmatrix}}{\left[mu^2+(a-b)c\right]\left[c(a-b)\right] - 2c^2(a^2+b^2)}$$

$$\frac{\Omega}{u\delta} = \frac{1/R}{\delta} = \frac{c^2(a-b)(1+K) - 2c^2(a-bK)}{mu^2 \cancel{c}(a-b) + c^2(a-b)^2 - 2c^2(a^2+b^2)}$$

$$= \frac{cL(K-1)}{mu^2(a-b) - cL^2}$$ where $L = a+b$ and $S = a-b$

So $$\frac{\delta}{L/R} = \frac{1 - \frac{Sm}{L^2C}u^2}{1-K}$$

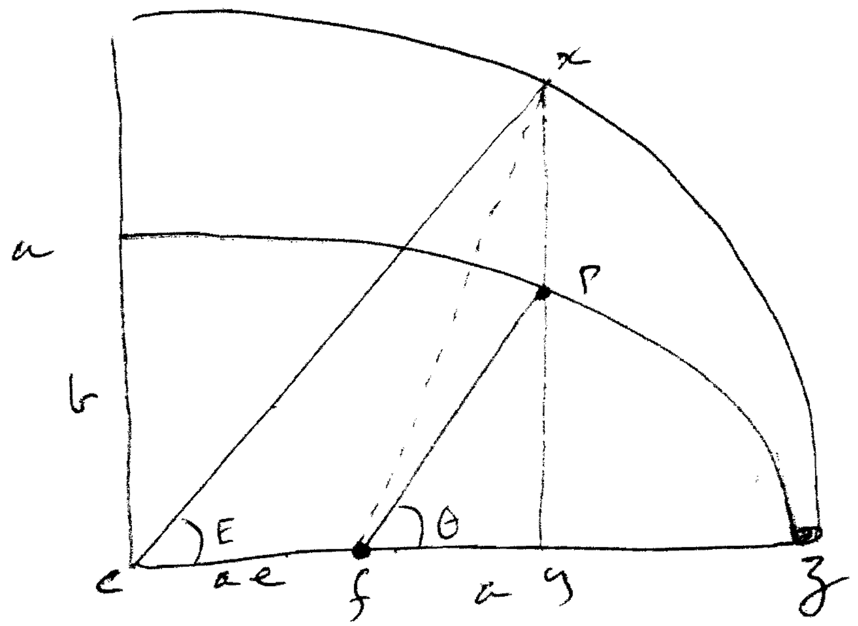
(c)



- The critical speed is independent of K so stability is not altered by this 4WS control strategy
- Understeer – more steer angle needed for higher speed turn $\frac{d\delta}{du} > 0$ ($S<0$)
- Oversteer – $d\delta/du < 0$ ($S>0$)
- If $K<0$, then $d\delta/du$ is decreased for a given S making the vehicle more Neutral steering, which is desirable
- If $K>0$, $d\delta/du$ increases – so best design has $K<0$

3a) Mean anomaly = $\dfrac{2\pi \times \text{area swept since periapsis}}{\text{total area}}$

Mean Motion = rate of change of Mean anomaly



$$M = \frac{2\pi \times \text{area } fPz}{\pi a b} \rightarrow fPz = \frac{abM}{2}$$

$$\therefore fxz = fPz \times \frac{a}{b} = \frac{Ma^2}{2}$$

$$fxz + fxc = \text{[sector of radius } a\text{, angle } E]$$

$$\frac{Ma^2}{2} + \frac{ae\, a \sin E}{2} = \frac{a^2 E}{2}$$

$$\rightarrow M = E - e \sin E$$

3(b) i) Signal time < reference time, so weekly roll-over has occurred

$t = 3001 + 604800 - 560{,}670 = 47131$ s.

ii) Mean motion $= \sqrt{\dfrac{\mu}{a^3}} = \sqrt{\dfrac{398603}{26562^3}}$

$\underline{n = 145.84 \times 10^{-6} \text{ rad/s}}$

So mean anomaly

$= 0.1703 + 47131 \times 145.84 \times 10^{-6}$

$= 7.0439$

$> 2\pi$, so subtract 2π to give

$\underline{M = 0.7607}$ radians

iii) Let $E = 0.7754$ radians

$M = E - e \sin E$

$= 0.7754 - 0.021 \times 0.700$

$= \underline{0.7607}$

$\therefore \underline{E = 0.7754}$

Now $\cos\theta = \dfrac{\cos E - e}{1 - e\cos E} = \dfrac{0.6935}{0.9850}$

$= 0.704$

$\rightarrow \underline{\theta = 45.25°}$ or $\underline{0.7898 \text{ radians}}$

3b) iv) $r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{26550}{1.01478}$

$= \underline{26,163.5 \text{ km}}$

Argument of Latitude $= 0.2574 + 0.7898$

$= 1.0472$ radians

(60.0°)

So distance from equatorial plane is $26163.5 \sin 60 \sin 55$

$= \underline{18,561 \text{ km}}$

v) Right ascension has changed by

$7.83 \times 10^{-9} \times 47131 = 0.0004$ radians

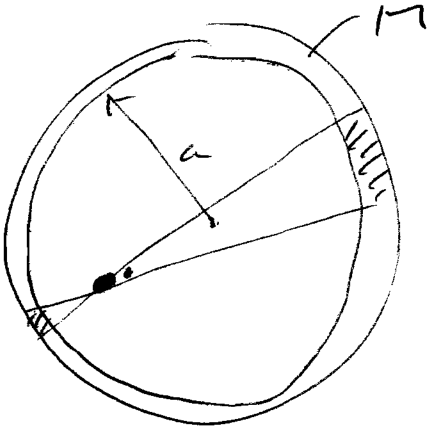
So current R.A. is $\underline{0.2879}$

Distance of satellite from Earth's polar axis is $\sqrt{26163^2 - 18561^2}$

$= 18439$ km

$\therefore$ error is $18439 \times 0.0004 = \underline{7.4 \text{ km}}$

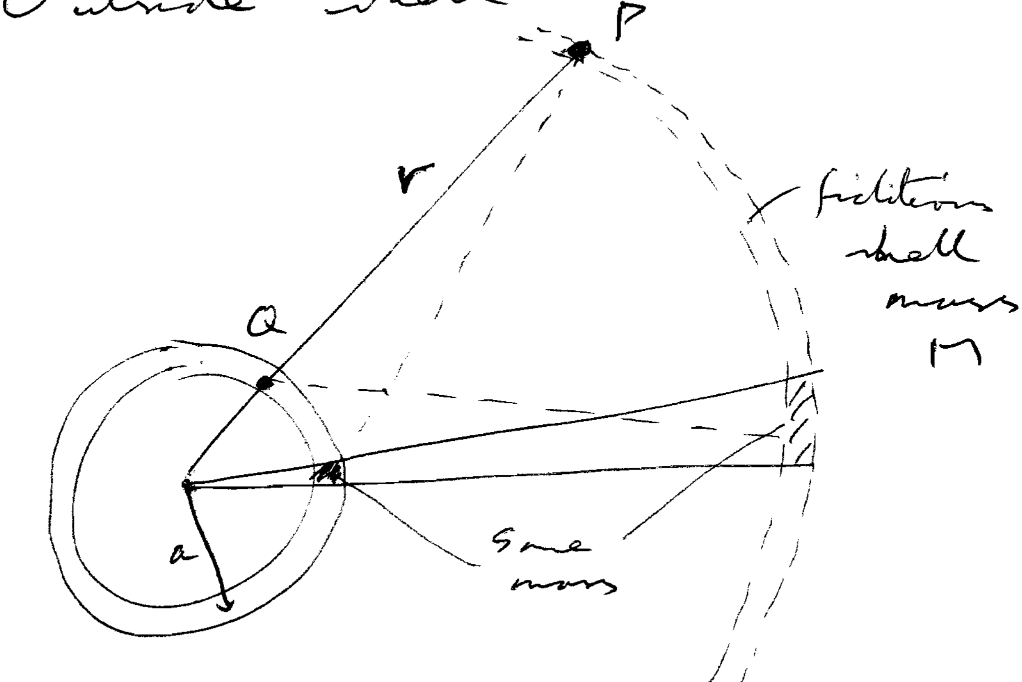
4a) Inside shell



Effects cancel, so potential is constant

$\therefore$ calculate at centre $= \frac{M}{a}$

Outside shell



Potential at P from real piece of shell is same as potential at Q from fictitious piece

$\therefore$ Potential from whole shell $= \frac{M}{r}$

i.e. if M was at origin.

4b) i) If $r > a$, potential from core is

$$u(r) = \tfrac{4}{3}\pi a^3 \rho_0 G / r = 4\pi \rho_0 G a^2 \left(\frac{a}{3r}\right)$$

For area between a & r, potential is

$$u(r) = \int_{x=a}^{r} G \, 4\pi x^2 \rho_0 \left(\frac{a}{x}\right)^4 / r \; dx$$

$$= \frac{4\pi \rho_0 a^4 G}{r} \left[-\frac{1}{x}\right]_{x=a}^{x=r}$$

$$= 4\pi \rho_0 G a^2 \left(\frac{a}{r} - \frac{a^2}{r^2}\right)$$

For area outside r, potential is

$$u(r) = \int_{x=r}^{\infty} 4\pi x^2 \rho_0 \left(\frac{a}{x}\right)^4 \frac{G}{x} \; dx$$

$$= 4\pi \rho_0 G a^4 \left[-\frac{1}{2x^2}\right]_{x=r}^{x=\infty}$$

$$= 4\pi \rho_0 G a^2 \left(\frac{a^2}{2r^2}\right)$$

Adding gives $u(r) = 4\pi \rho_0 G a^2 \left(\frac{4a}{3r} - \frac{a^2}{2r^2}\right)$

4b) ii) Central force $= f(u) = \nabla u = \frac{\partial u}{\partial r}\underline{e}_r$

$\therefore$ force $\to$ centre $= -4\pi\rho_0 G a^2\left(\frac{a^2}{r^3} - \frac{4a}{3r^2}\right)$

$= 4\pi\rho_0 G a^2\left(a^2u^3 - \frac{4au^2}{3}\right)$

From formula in D.B.,

$$\frac{\partial^2 u}{\partial \theta^2} + u = -\frac{4\pi\rho_0 G a^2}{h^2}\left(a^2 u - \frac{4a}{3}\right)$$

$$\frac{\partial^2 u}{\partial \theta^2} + u\left(1 + \frac{4\pi\rho_0 G a^4}{h^2}\right) = \frac{16\pi\rho_0 G a^3}{3h^2}$$

For which the general solution is

$$u = \frac{1}{r} = A\cos\left(\sqrt{1 + \frac{4\pi\rho_0 G a^4}{h^2}}\,\theta + \theta_0\right) + \frac{16\pi\rho G a^3}{3h^2 + 12\pi\rho_0 G a^4}$$