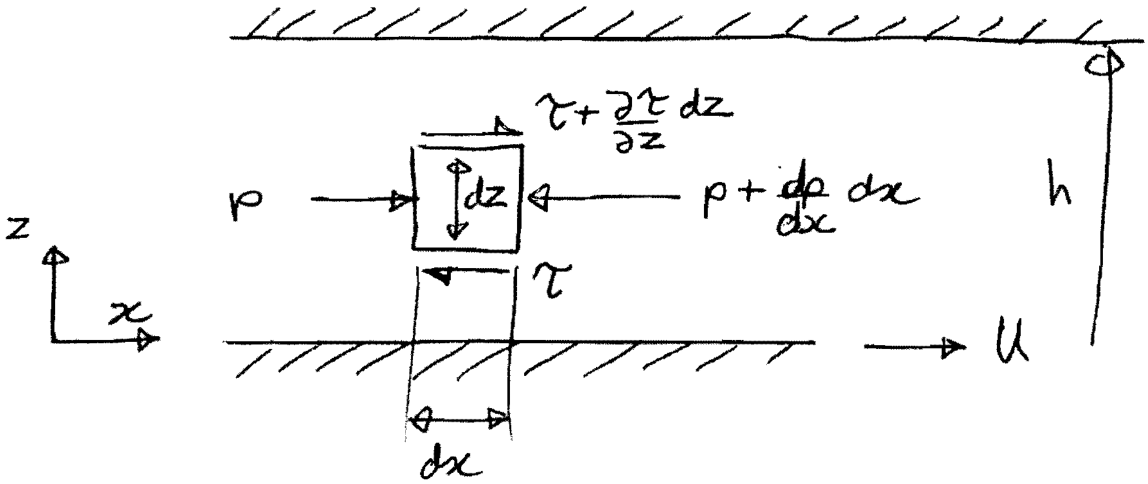
Engineering Tripos Part IIB 2012

Module 4C16

Advanced Machine Design

Solutions by D J Cole

1. (a) (i)



consider equilibrium in x-direction

$$p.dz + \left(\tau + \frac{\partial \tau}{\partial z} dz\right) dx = \tau dx + \left(p + \frac{dp}{dx} dx\right) dz$$

$$\frac{\partial \tau}{\partial z} dz\, dx = \frac{dp}{dx} dx\, dz$$

$$\frac{\partial \tau}{\partial z} = \frac{dp}{dx}$$

ii) integrating w.r.t. z.

$$\tau = \left(\frac{dp}{dx}\right) z + A$$

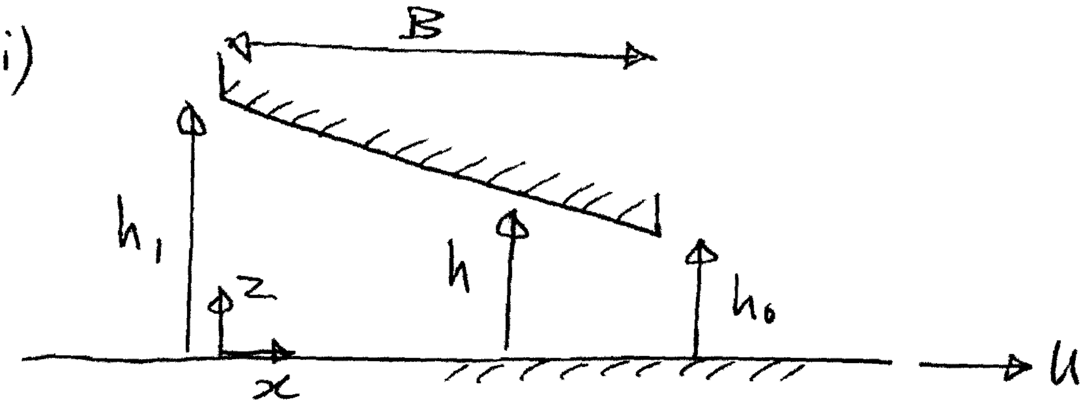
zero resistance at top surface

$\therefore$ $\tau = 0$ at $z = h$

$$\therefore \quad 0 = \frac{dp}{dx} \cdot h + A$$

$$\therefore \quad \tau = \frac{dp}{dx}(z - h)$$

b) i)



starting from $\tau = \frac{dp}{dx}(z-h)$

Newtonian fluid, $\tau = \eta \frac{du}{dz}$

$$\therefore \quad \eta \frac{du}{dz} = \frac{dp}{dx}(z-h)$$

Integrate $\quad \eta u = \frac{dp}{dx}\left(\frac{z^2}{2} - zh\right) + A$

conditions $\quad u = U$ at $z = 0 \quad \therefore A = \eta U$

$$\therefore \quad u = U + \frac{1}{\eta}\left(\frac{dp}{dx}\right) z \left(\frac{z}{2} - h\right)$$

volumetric flow per unit width q is constant

$$q = \int_0^h u \, dz$$

$$= Uh + \frac{1}{\eta}\left(\frac{dp}{dx}\right)\left[\frac{z^3}{6} - \frac{z^2 h}{2}\right]_0^h$$

$$q = Uh - \frac{1}{3\eta}\left(\frac{dp}{dx}\right) h^3$$

flow is constant with x so $\frac{dq}{dx} = 0$

$$0 = U\frac{dh}{dx} - \frac{1}{3\eta}\frac{d}{dx}\left(h^3 \frac{dp}{dx}\right)$$

$$\therefore \quad 3\eta U \frac{dh}{dx} = \frac{d}{dx}\left(h^3 \frac{dp}{dx}\right)$$

ii) $$F = \int_0^B \tau\Big|_{z=0} dx \quad , \quad \text{but } \tau\Big|_{z=0} = -h\frac{dp}{dx}$$

from (a)(ii)

$$\therefore \quad F = \int_0^B h \frac{dp}{dx}\, dx$$

integrate by parts $$F = [h p]_0^B - \int_0^B p \frac{dh}{dx}\, dx$$

but $p = 0$ at entry and exit, so $[\ \ ] = 0$

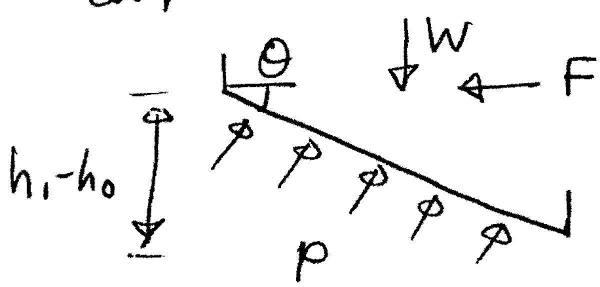
$$F = -\int_0^B p \frac{dh}{dx}\, dx$$

but $\frac{dh}{dx} = -\frac{K h_0}{B}$ $\left(\text{from } \frac{h}{h_0} = 1 + K - K\frac{x}{B}\right)$

$$\therefore \quad F = \frac{K h_0}{B} \underbrace{\int_0^B p\, dx}_{= W}$$

$$\therefore \quad \frac{F}{W} = \frac{K h_0}{B} = \frac{h_1 - h_0}{B}$$

Confirm :



no shear force on upper surface,

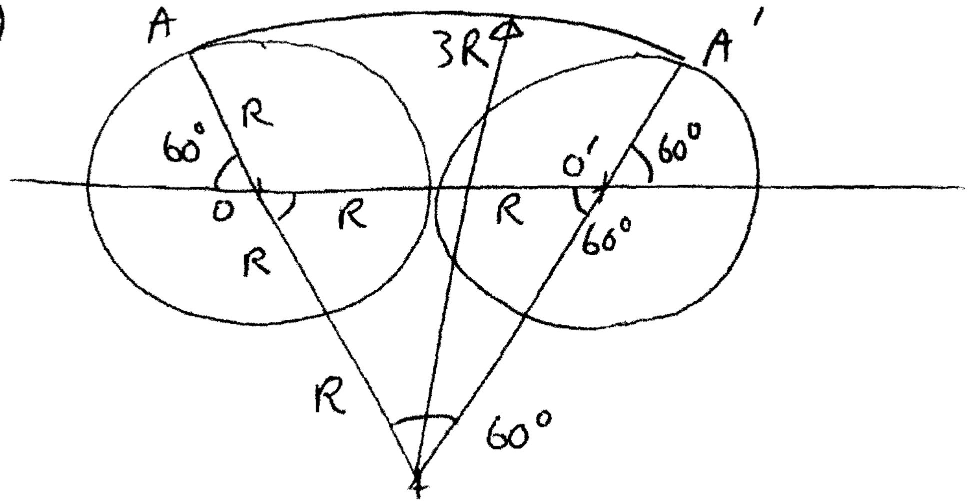
$$\frac{F}{W} = \tan\theta$$

so coef. of friction $= \frac{h_1 - h_0}{B}$

2. a) • Good at high speed
• Reliable
• Easy to tailor the motion
• Low cost

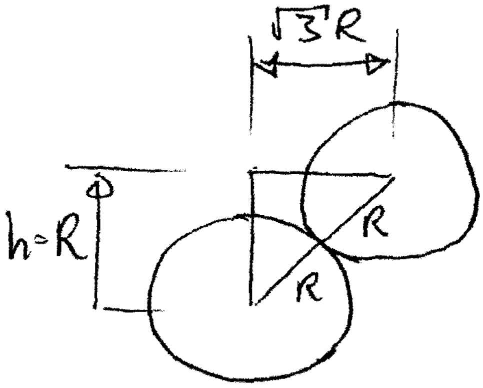
Design issues: avoid loss of contact
limit the contact stresses and wear

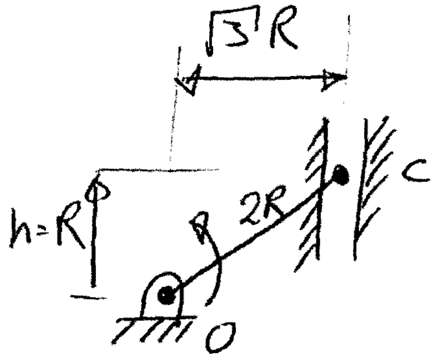
b) i)



by inspection, flank radius is 3R.

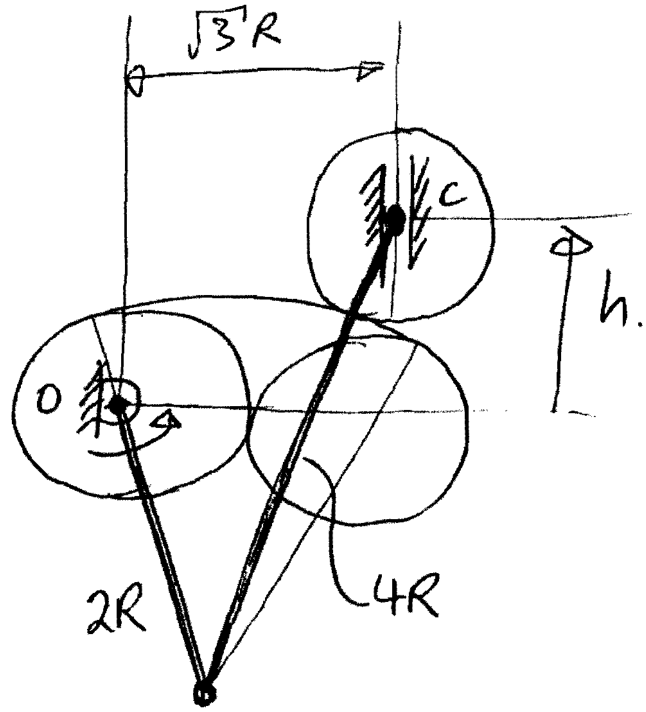
ii) contact on base circle.



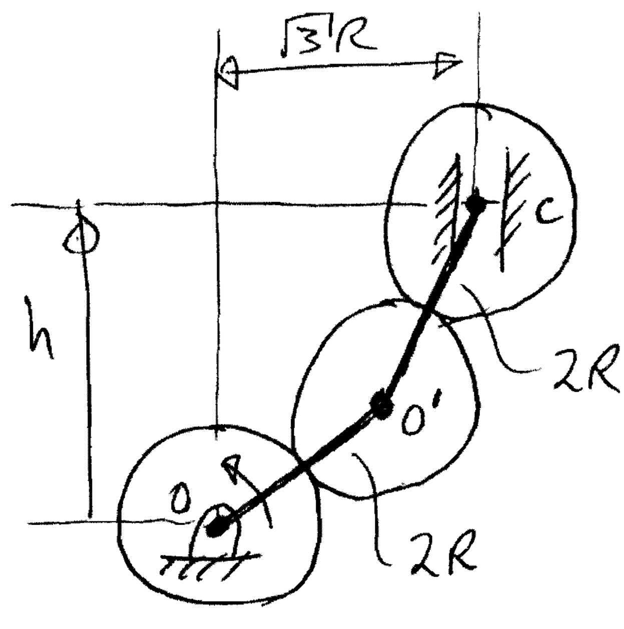


mechanism is 'locked'!

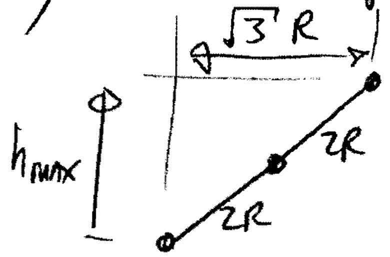
Contact on flank:



contact on tip circle:

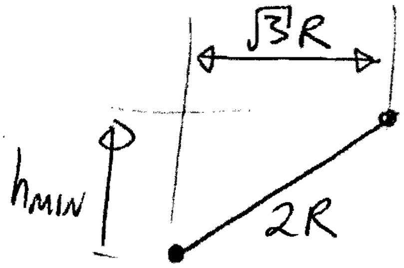


iii) Max height is when contact is on tip circle:



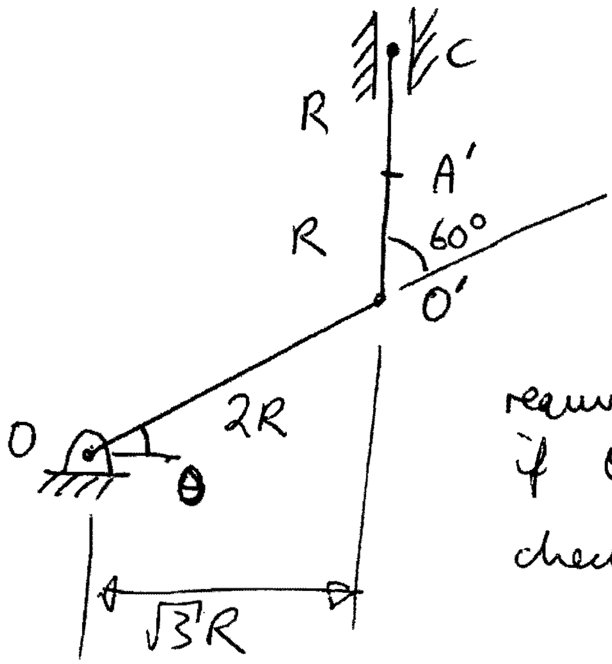
$$h_{max} = \sqrt{16R^2 - 3R^2} = \sqrt{13}\,R$$

min height when contact is on base circle:



$$h_{MIN} = \sqrt{4R^2 - 3R^2} = R$$

contact at A':

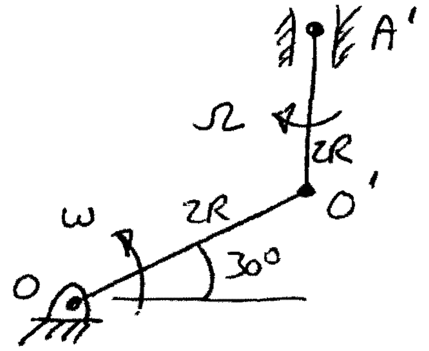


required conditions achieved if $\theta = 30°$.

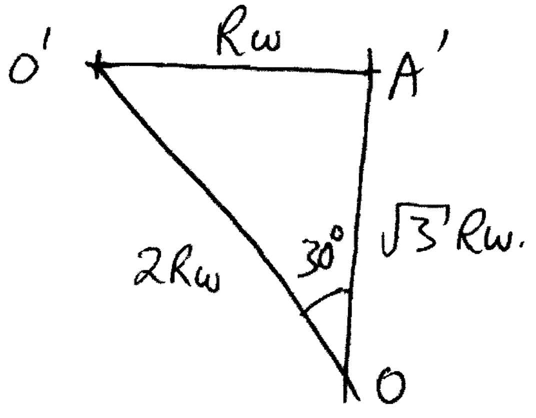
check: $\cos\theta = \dfrac{\sqrt{3}R}{2R}$

$\theta = 30°$ ✓.

iv) contact on tip circle at A':



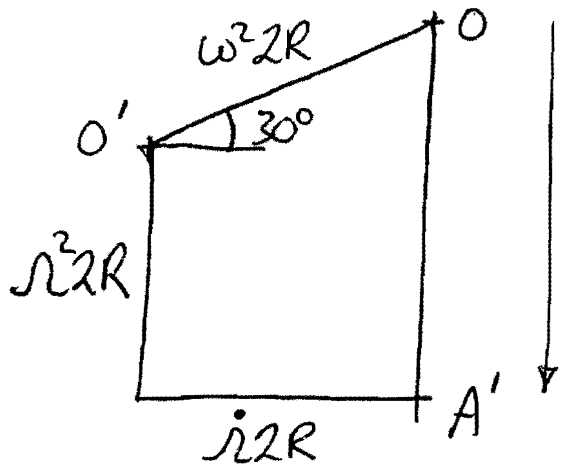
velocity diagram:



$$\therefore \Omega = \frac{R\omega}{2R} = \frac{\omega}{2}$$

hence follower velocity is $\sqrt{3}R\omega$ ↑

acceleration diagram:



$$\text{accn of follower} = \omega^2 2R \sin 30^\circ + \Omega^2 2R$$

$$= \omega^2 R + \frac{\omega^2}{4} 2R$$

$$= \omega^2 R \cdot \frac{3}{2} \downarrow$$

v) discontinuity in equivalent mechanisms is likely to give discontinuity in acceleration, leading to large forces, wear, fatigue and noise.

3 a)

Performance criteria of energy storage devices:

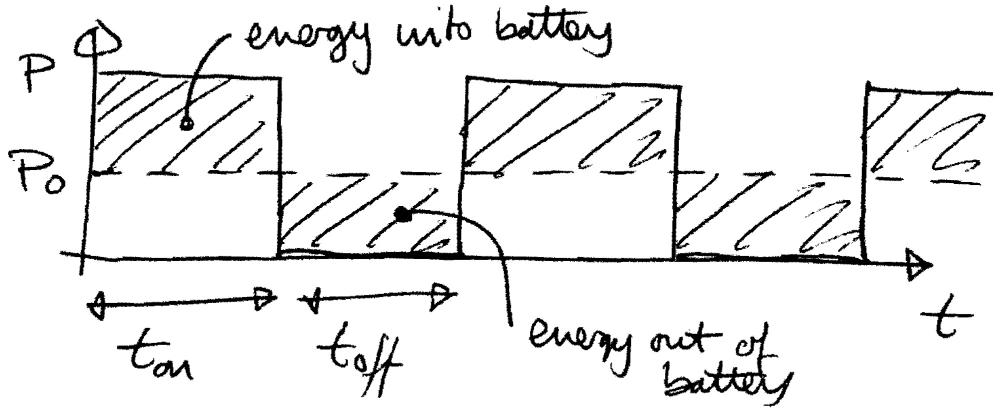
- specific energy kJ/kg
- specific power W/kg
- energy density kJ/m^3
- power density W/m^3
- cycle life
- cost £/kWh
- efficiency
- self discharge rate

Chemical batteries in automotive drives:

Strengths - ease of controlling power
low self-discharge rate
specific energy and energy density
(although not compared to liquid fuel)

Weaknesses - temperature controlled environment required.
finite cycle life
high cost.
specific power and power density are low
(particularly for charging)

b) plot power as a function of time



no acceleration hence $P_0 = (CV_0^2 + a)V_0$

assume energy in = energy out

$$t_{on}(P - P_0) = t_{off}(P_0)$$

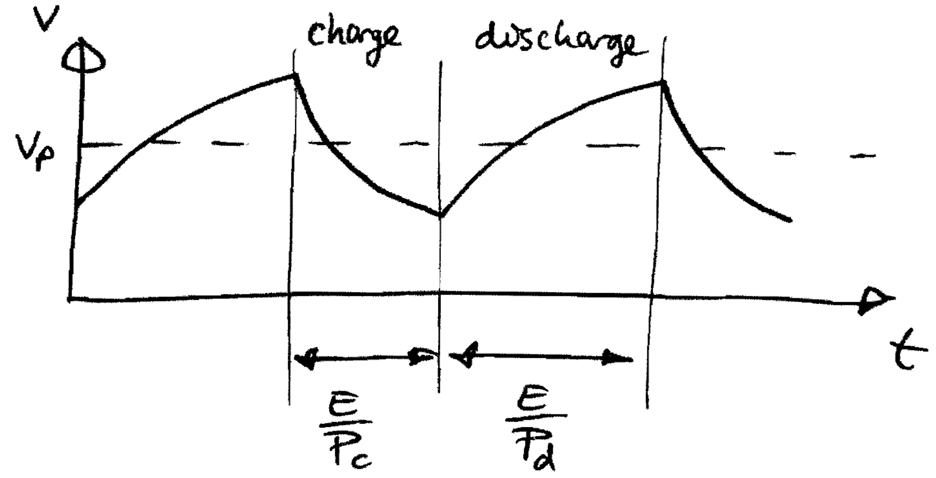
$$\frac{t_{off}}{t_{on}} = \frac{P - P_0}{P_0} = \frac{P}{P_0} - 1$$

$$= \frac{P}{(CV_0^2 + a)V_0} - 1$$

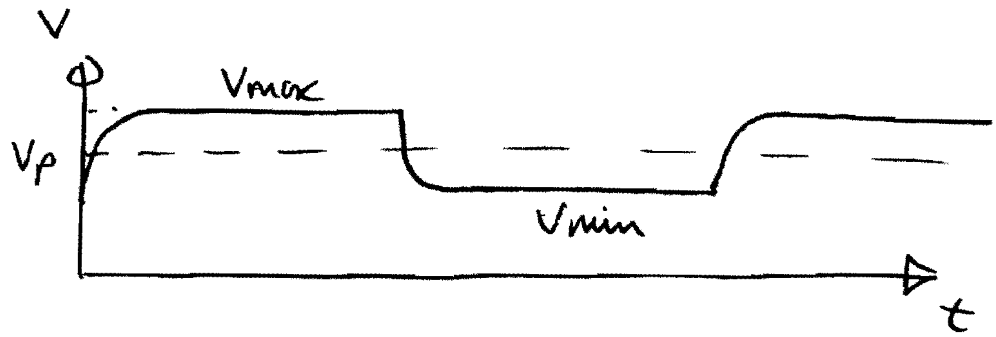
energy storage $E = (P - P_0)\, t_{on}$

$$E = \left(P - (CV_0^2 + a)V_0\right) t_{on}$$

c) i)



ii)



steady state speed condition, $\frac{dv}{dt} = 0$

$\therefore \quad f = cv^2 + a$

power $= (cv^2 + a)v$

at max speed $\quad P + P_d = cV_{max}^3 + aV_{max}$

if a small $\quad V_{max} \approx \sqrt[3]{\frac{P + P_d}{c}}$

at min speed $\quad V_{min} \approx \sqrt[3]{\frac{P - P_c}{c}}$

iii) $f = m\frac{dv}{dt} + cv^2 + a$

constant speed v_p for constant power P

$\frac{dv}{dt} = 0 \quad \therefore \quad P = (cv_p^2 + a)v_p$

$P = cv_p^3 + av_p$

discharging phase at speed v_p:

$$P + P_d = m\frac{dv}{dt}v_p + cv_p^3 + av_p$$

but $P = cv_p^3 + av_p$

$$\therefore \quad P_d = m\frac{dv}{dt} \cdot v_p$$

$$\therefore \frac{dv}{dt} = \frac{P_d}{mv_p}$$

similarly for discharging phase:

$$\frac{dv}{dt} = -\frac{P_c}{mv_p}$$

4C16 2012 Examiner's comments

Q1 Hydrodynamic bearing

Part (a) was generally answered well. There were many good solutions to part (b)(i), but many other solutions involved vain attempts at manipulating formulae from the data sheet, with little account taken of the specified slip conditions. Most candidates failed to make much progress with part (b)(ii).

Q2 Cam mechanism

The discussion parts of the question were answered well. The equivalent mechanisms sketched in part (b)(ii) were often wrong or lacked sufficient detail. In part (b)(iv) many candidates were able to find the velocity, but solutions to the acceleration were poor. The most common approach was to derive an expression for the height of the follower in terms of the cam angle and then to differentiate twice to find the velocity and acceleration. The velocity was usually found correctly, but failure to account correctly for the motion of the upper link of the equivalent mechanism led to wrong answers for the acceleration. Only a handful of candidates used unit vectors or velocity/acceleration diagrams.

Q3 Hybrid drive

The discussion part of the question was answered well. The derivations in part (b) were also answered well by most candidates. There was a range of answers to part (c), with no particular error predominating. Many candidates sketched an appropriate graph of power first, but then sketched a graph of vehicle speed that was inconsistent with this.

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