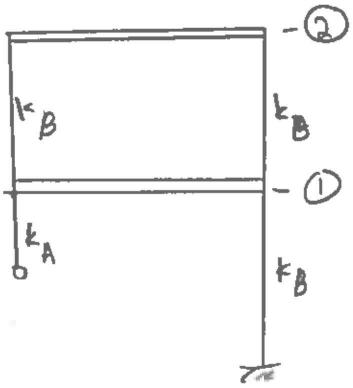
Module 4D6

① (a)



$$k_A = \frac{3EI}{L^3} = \frac{6EI}{16}$$

$$k_B = \frac{12EI}{L^3} = \frac{3EI}{16}$$

$$k_1 = \frac{6EI}{16} + \frac{3EI}{16} = \frac{9EI}{16}$$

$$k_2 = \frac{6EI}{16}$$

Mode shapes: $[k - w^2 m][\phi] = 0$

$$\left(\frac{EI}{16}\begin{bmatrix} 15 & -6 \\ -6 & 6 \end{bmatrix} - w^2 m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0$$

$$\lambda = \frac{16 w^2 m}{EI} = 0.32\, w^2 \rightarrow \begin{bmatrix} 15-\lambda & -6 \\ -6 & 6-\lambda \end{bmatrix}\begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = 0$$

$$w = 3.06\ \tfrac{rad}{s} \rightarrow \lambda = 3.0 \rightarrow 12\phi_{11} - 6\phi_{21} = 0 \rightarrow \Phi_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$w = 7.5\ \tfrac{rad}{s} \rightarrow \lambda = 18.0 \rightarrow -3\phi_{12} - 6\phi_{22} = 0 \rightarrow \Phi_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

(b) (i) From Responce Spectra: Ground Displacement ≈ 8 inches

Ground Acceleration ≈ 0.32 g

① (b) (ii) From Plot: $\omega_1 = 3.06\ \text{rad/s} \rightarrow S_{a_1} \approx 0.15g$

$\omega_2 = 7.5\ \text{rad/s} \rightarrow S_{a_2} \approx 0.6g$

$$\Gamma_1 = \frac{1+0.5}{1^2+0.5^2} = 1.2$$

$$\Gamma_2 = \frac{1-2}{1^2+2^2} = 0.2$$

Mode 1: $S_{d_1} = \frac{S_{a_1}}{\omega_1^2} = 0.158\ m \rightarrow u_1 = \Gamma_1 S_{d_1} \Phi_1 = \begin{bmatrix} 0.095 \\ 0.19 \end{bmatrix} m$

Mode 2: $S_{d_1} = \frac{S_{a_2}}{\omega_2^2} = 0.105\ m \rightarrow u_2 = \Gamma_2 S_{d_2} \Phi_2 = \begin{bmatrix} -0.042 \\ 0.021 \end{bmatrix} m$

Bottom Floor: $u_{total} = \left[(0.095^2 + (0.042)^2\right]^{1/2} = (0.104\ m)$

Max column shear $= 0.104\left(\frac{6EI}{16}\right) = \boxed{3900\ N}$

Max > 3000

Yields!

Top Floor: $u_2 - u_1 = \left[(0.095)^2 + (0.063)^2\right]^{1/2} = 0.114\ m$

Max column shear $= 0.114\left(\frac{3EI}{16}\right) = 2140\ N$

(c)(i) $T_1 = \frac{2\pi}{\omega_1} = 2.05\ s.$

$$\delta_A = \frac{V_y}{k_A} = \frac{3\ kN}{\frac{6}{16}EI} = \frac{8}{EI} = 0.08\ m$$

$\delta_1 = \Gamma_1 \frac{S_{a_1}}{\omega_1^2}\phi_{11} \rightarrow S_{a_1} = \frac{\delta_1 \omega_1^2}{\Gamma_1 \phi_{11}} = 1.25\ m/s^2 \approx 0.13g$

PGA = 0.5 → S_{a_1} on plot = 0.13 (2) = 0.26g.

From plot → $T_1 = 2.05\ s.$ & $S_{a_1} = 0.26g$ → $\boxed{\mu \approx 5}$

(c)(ii)

1. Higher modes
2. Design spectra are averages
3. Localization of damage

4D6 - Dynamics in Civil Engineering.

Q2) a) Equal angle section 200 mm × 200 mm

Steel Young's Modulus from Data book = 210 GPa $\quad L = 3\text{m}$

I for this section from data book = 2627 cm^4. $\quad m = 54.2\ \text{kg/m}$

$$\therefore EI = 210\times10^9 \times 2627 \times \left(\frac{1}{100}\right)^4 = 5.5167\times10^6\ \text{Nm}^2$$

$$\bar{U}(x) = \frac{1}{3L^4}\left[x^4 - 4x^3L + 6x^2L^2\right]$$

$$M_{eq} = \int_0^L m\,\bar{U}^2\,dx = \frac{m}{9L^8}\int_0^L \left(x^4 - 4x^3L + 6x^2L^2\right)^2 dx$$

$$= \frac{m}{9L^8}\int_0^L \left[x^8 + 16x^6L^2 + 36x^4L^4 - 8x^7L - 48x^5L^3 + 12x^6L^2\right] dx$$

$$= \frac{m}{9L^8}\left[\frac{x^9}{9} + 16L^2\frac{x^7}{7} + 36L^4\frac{x^5}{5} - 8L\frac{x^8}{8} - 48L^3\frac{x^6}{6} + 12L^2\frac{x^7}{7}\right]_0^L$$

$$= \frac{m}{9L^8}\left[\frac{L^9}{9} + \frac{16}{7}L^9 + \frac{36}{5}L^9 - L^9 - 8L^9 + \frac{12}{7}L^9\right] = \underline{\underline{0.2568\ mL}}$$

$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{U}}{dx^2}\right)^2 dx \qquad \frac{d\bar{U}}{dx} = \frac{1}{3L^4}\left[4x^3 - 12x^2L + 12xL^2\right]$$

$$\frac{d^2\bar{U}}{dx^2} = \frac{1}{3L^4}\left[12x^2 - 24xL + 12L^2\right]$$

$$\therefore K_{eq} = \frac{EI}{9L^8}\int_0^L \left(12x^2 - 24xL + 12L^2\right)^2 dx$$

$$= \frac{EI}{9L^8}\int_0^L \left[144x^4 + 576x^2L^2 + 144L^4\right] \cancel{dx} - 576x^3L - 576xL^3 + 288x^2L^2\Big]\, dx$$

$$= \frac{EI}{9L^8}\left[144\frac{x^5}{5} + \frac{576}{3}x^3L^2 + 144xL^4\right]_0^L - 576\frac{x^4}{4}L - 576\frac{x^2}{2}L^3 + \frac{288}{3}x^3L^2\Big]_0^L$$

$$= \frac{EI}{9L^8}\left[\frac{144}{5}L^5 + \frac{576}{3}L^5 + 144L^5 - \frac{576}{4}L^5 - \frac{576}{2}L^5 + \frac{288}{3}L^5\right]$$

$$= \frac{EI}{9L^3}\left[\frac{144}{5} + \frac{576}{3} + 144 - \frac{576}{4} - \frac{576}{2} + \frac{288}{3}\right] = \underline{\underline{3.2\frac{EI}{L^3}}}$$

$$\therefore \omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{\frac{3.2\,EI}{L^3}}{0.2568\,mL}} = \sqrt{\frac{3.2\,EI}{0.2568\,mL^4}}.$$

$$= \sqrt{\frac{3.2 \times 5.5167 \times 10^6}{0.2568 \times 54.2 \times 3^4}} = 125.13 \text{ rad/s}$$

or $\underline{19.92 \text{ Hz}}$

$\therefore$ Time period $= \frac{1}{19.92} = \underline{0.0502 \text{ sec}}$ [30 %]

2 b) Using the data sheets

$t_d = 0.05$ sec

$$\frac{t_d}{T} = \frac{0.05}{0.05} = 1 \Rightarrow DAF = 1.5$$

$F_{eq} = f_1 \times \bar{U}(x)$ at P $= 5 kN \times 1 = 5 kN$

$$\therefore U_{static} = \frac{F_{eq}}{K_{eq}} = \frac{5 \times 10^3}{\frac{3.2 \times 5.5167 \times 10^6}{3^3}} = 7.647 \times 10^{-3} \text{ m}$$

$= \underline{7.65 \text{ mm}}$

$\therefore$ Max dynamic deflection $= DAF \times U_{static}$

$= \underline{11.475 \text{ mm}}$ [20 %]

2 c) For point Q $x = 1.5$m

$$\therefore \bar{U}(x)_{x=1.5m} = \frac{1}{3 \times 3^4}\left[1.5^4 - 4 \times 1.5^3 \times 3 + 6 \times 1.5^2 \times 3^2\right]$$

$= 0.3542$

$F_{eq} = F \cdot \bar{U}(x) = 5 \times 10^3 \times 0.3542 = 1.771 \times 10^3$ N.

$$\therefore U_{static}|_Q = \frac{F_{eq}}{K_{eq}} = \frac{1.771 \times 10^3}{\frac{3.2 \times 5.5167 \times 10^6}{3^3}} = 2.708 \times 10^{-3} \text{ m}$$

$\therefore$ Max dynamic deflection at Q $= DAF \times U_{st} = 1.5 \times 2.708 \times 10^{-3}$

$= 4.063 \times 10^{-3}$ m or $\underline{4.063 \text{ mm}}$

[30%]

2 d) If the fixity condition of the cantilever to the column deteriates then

i) The natural frequency of the cantilever will <u>reduce</u> as the fixity now offers lower stiffness. [10%]

ii) The reduction in fixity may cause the static deflection to increase (with lower stiffness of the whole system). However the dynamic deflection will depend on the actual natural frequency and hence on the DAF. (which is a functn of t_d/T)

[10%]

3 a) When using FE analyses for a multi phase medium like soil, we must consider both solid and fluid phases of soil. Typically a Biot's formulation is used. This formulation will couple both the solid and fluid phases with an additional coupling term.

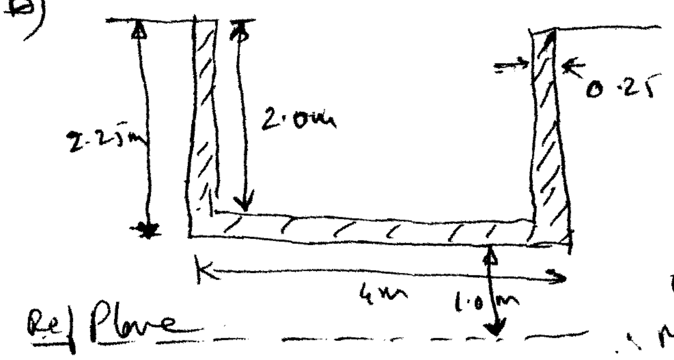
1) For the solid phase, we solve for solid nodes displacements under a given loading, assuming some pore pressure distribution.

2) Then fluid pressures are recalculated using the solid node displacements (hence soil strains) from above,

3) The fluid pressures are substituted back in step 1) and the process is repeated until overall convergence is achieved.

[15%]

3 b)



Volume of concrete in side walls $= (4^2-3.5^2)\times 2 = 7.5\ m^3$

Base volume $= 4\times 4\times 0.25 = 4\ m^3$

Total volume $= 11.5\ m^3$

Total weight $= 24\times 11.5 = 276\ kN$.

$\therefore$ Mass $= 28.135$ tons

3 b) Cont

vertical stress due to tank = $\frac{28.13\times10^3\times9.81}{4^2} = \frac{276}{4^2} = 17.25$ kPa

vertical stress due to soil = $\gamma_{sat}\times z = 19.5\times1 = 19.5$ kPa.

water pressure on ref plane = $\gamma_w \cdot z = 10\times3.25 = 32.5$ kPa.

$\therefore \sigma_v' = 17.25+19.5-32.5 = \underline{4.25 \text{ kPa}}$

$K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.428$; $p' = \sigma_v'\left[\frac{1+2K_0}{3}\right] = 0.62\sigma_v' = \underline{\underline{2.635 \text{ kPa}}}$

Void ratio $e = 0.9$ $\therefore G_{max} = 100\frac{[3-e]^2}{1+e}\sqrt{p'}$

$= 100\frac{[3-0.9]^2}{1.9}\sqrt{2.635/100}$

$= \underline{\underline{11.91 \text{ MPa}}}$.

$2l = 4m$ $2b = 4m$ $e = 2.25$ m $\Rightarrow l/b = 1$; $e/b = 1.125$m.

$L = 2$ m $b = 2$m

Using Data Sheets

$K_h = \frac{Gb}{2-\nu}\left[6.8\,(l/b)^{0.65}+2.4\right]\left[1+\left(0.33+\frac{1.34}{1+l/b}\right)\left(\frac{e}{b}\right)^{0.8}\right]$

$= \frac{Gb}{2-\nu}\,[9.2]\,[2.1] = 22.73\,G_{max}$

$= \underline{\underline{270.71}}$ MN/m.

$K_v = \frac{Gb}{2-\nu}\left[3.1\,(l/b)^{0.75}+1.6\right]\left[1+\left(0.25+0.25\frac{b}{l}\right)(e/b)^{0.8}\right]$

$= \frac{Gb}{2-\nu}\,[4.7]\,[1.55] = 8.57\,G_{max}$

$= \underline{\underline{102.07 \text{ MN/m}}}$

$K_{rot} = \frac{Gb^3}{1-\nu}\left[3.2\,l/b+0.8\right]\left[1+e/b+\frac{1.6}{0.35+l/b}(e/b)^2\right]$

$= \frac{Gb^3}{1-\nu}\,[4]\,[3.625] = 165.71\,G_{max}$

$= \underline{\underline{1973.61}}$ MN-m/rad

[40%]

3 c) Horizontal $f_h = \frac{1}{2\pi}\sqrt{\frac{K_h}{m}}$

$$= \frac{1}{2\pi}\sqrt{\frac{270.71 \times 10^6}{28.135 \times 10^3}} = \underline{\underline{15.6 \text{ Hz}}}$$

vertical $f_v = \frac{1}{2\pi}\sqrt{\frac{K_v}{m}}$

$$= \frac{1}{2\pi}\sqrt{\frac{102.07 \times 10^6}{28.135 \times 10^3}} = \underline{\underline{9.6 \text{ Hz}}}$$

rocking $f_{rock} = \frac{1}{2\pi}\sqrt{\frac{K_{rx}}{I}}$.

$$I = mr^2 = 28.135 \times [2]^2 \times 1000$$
$$= 112540 \text{ kg-m}^2$$

$$\therefore f_{rock} = \frac{1}{2\pi}\sqrt{\frac{1973.61 \times 10^6}{112540}} = \underline{\underline{21.1 \text{ Hz}}}$$ [15%]

3 d) When full liquefaction occurs $\sigma_v' = 0$ on the reference plane.
Prior to liquefaction from (a) $\sigma_v' = 4.25$ kPa $= u_{excess}$
$\therefore$ If the earthquake generates an excess pore pressure of 4.25 kPa $\sigma_v' = 4.25 - 4.25 = 0$ = full liquefaction will result. [15%]

3 e) If the tank is empty, following liquefaction it will float due to very high buoyancy. Such floatation of underground structures was observed in many recent earthquakes.

If the water tank is full, the effective stress will be much higher (due to weight of water). This will mean that full liquefaction won't occur and therefore the water tank will not float.

[15%]

4D6 Q4, 2012

a) The finite element method proceeds by inputting the geometry, mass and stiffness of all elements. These are then assembled into the total mass (or inertia) $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices. These may have many thousands of degrees of freedom (the size of the nodal displacement vector $\mathbf{X}$). The matrices are diagonalised by seeking harmonic solutions $\mathbf{X}(\mathbf{r}, t) = \mathbf{X}_0(\mathbf{r})e^{i\omega t}$ such that the free vibration problem

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{0}$$

leads to the eigenproblem

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X}_0 = \mathbf{0}$$

which has eigenvalues ω_i^2 and associated eigenvectors ϕ_i.

A general displacement can then be expressed as

$$\mathbf{X} = \sum_i a_i(t)\phi_i(\mathbf{r})$$

where the a_i are modal amplitudes. This leads, via orthogonality of the modes, to the equations of motion in the new modal coordinates:

$$m_i\ddot{a}_i + c_i\dot{a}_i + k_i a_i = f_i(t)$$

where mode-generalised damping and forcing terms have been added, and $m_i = \phi_i^T\mathbf{M}\phi_i$ is the mode generalised mass, etc. These equations describe a set of single degree of freedom uncoupled oscillators (assuming the damping is Rayleigh) which may be solved independently. Typically one would solve for the behaviour in only the first few lowest frequency modes.

For a suspension bridge, one should use a two-step process. Dead loads should be applied first in a nonlinear static step to obtain the nonlinear equilibrium, thereby incorporating tension-stiffening effects in cables, and gravity-stiffening pendulum effects in the lateral bridge motions. The dynamic modes should then be obtained as the linearisation around this nonlinear equilibrium.

b) The local rotational inertia terms are $I\ddot{\theta}$, where I is the mass moment of inertia per unit length, and θ is the local rotation (or pitch). But

$$\theta = \theta_{central}\sin(\pi x/L)$$

The mode generalisation of the torsional mode leads to

$$m_i = \phi_i^T\mathbf{M}\phi_i = \int \phi^2 I \; dL = I\int_0^L \sin^2(\frac{\pi x}{L}) \; dL = \frac{IL}{2}$$

giving a mode-generalised mass moment of inertia of

$$I_i = \frac{(320 \times 10^3)400}{2} = 64 \times 10^6 \text{ kg m}^2$$

From $\omega = \sqrt{K/M}$ we obtain the mode generalised stiffness

$$K_i = I_i\omega_i^2 = (64 \times 10^6)(0.5 \times 2\pi)^2 = 640 \times 10^6 \ \text{ Nm/radian}$$

c) Static divergence.

Locally, the destabilising moment is $M = 3.3\rho U^2 B^2 \alpha$ Nm/m.

But $\alpha(x) = \theta_{central} \sin(\pi x/L)$, thus the mode-generalisation of the applied moment is

$$\begin{aligned} T_i &= 3.3\rho U^2 B^2 \theta_{central} \int_0^L \sin(\frac{\pi x}{L}) \ dL \\ &= 3.3\rho U^2 B^2 \theta_{central} \frac{2L}{\pi} \end{aligned}$$

Static divergence will occur when this torque exceeds the static stabilising torque. Thus when

$$3.3\rho U^2 B^2 \theta_{central} \frac{2L}{\pi} > K_i \theta_{central}$$

That is, when

$$\begin{aligned} U^2 &> \frac{K_i \pi}{3.3\rho B^2 (2L)} \\ U &> \sqrt{\frac{(640 \times 10^6)\pi}{3.3(1.25)(15^2)(2 \times 400)}} = 52 \text{ m/s} \end{aligned}$$

d) Mode-generalising the harmonic torque as above leads to a mode-generalised harmonic moment

$$T_i = \frac{2L}{\pi} 0.2 \rho U B^3 \; \dot{\theta}_{central}$$

Single degree of freedom torsional flutter will occur when this exceeds the damping term on the left-hand side of the modal equation of motion:

$$C\dot{\theta}_{central} = 2\xi\omega_i I_i \dot{\theta}_{central}$$

This occurs when

$$U > \frac{\pi C}{2L(0.2\rho B^3)} = \frac{\pi(2)(0.005)(0.5 \times 2\pi)(64 \times 10^6)}{2(400)(0.2)(1.25)(15^3)} = 9.3 \text{ m/s}$$

e) The creation of positive pitching moment flutter derivatives by the H-shaped cross-section of Tacoma led to its collapse in a single degree of freedom torsional flutter mode at low wind speeds, exactly as will happen to this bridge if the windshields are fitted.