4D7 Concrete & Masonry Structures CRIB

Q1 (a) OPC – Production process & main ingredients

Limestone (chalk) 75% blended with clay (25%) to form a slurry which is heated in a long rotary kiln to approx. 1400–1500°C to form grey pellets, referred to as clinker, which are ground into fine powder to which a small amount (4%) of gypsum ($CaSO_4$) is added (to retard the set when mixed with water). The final product is OPC.

OPC – main anhydrous compounds in cement.

C_3S – tricalcium silicate
C_2S – dicalcium silicate
C_3A – tricalcium aluminate
C_4AF – tetracalcium aluminoferrite
+ Alkalis (K_2O, Na_2O)
+ MgO, TiO, Mn_2O_3

Hydration process
Anhydrous compounds + gypsum mixed with water to form unstable supersaturated solution from which calcium silicate & aluminate hydrates precipitate. This is a exothermic reaction.

Two main groups of reactions during hydration.

1. Fast reactions (1st 4 hours) – causes cement to set.
   This is due to hydration of C_3A – very rapid & generates considerable heat hence unwanted as it causes the cement to set too quickly. Gypsum is added to react with C_3A to reduce heat & rate of set. This forms ettringite which is expansive & unwanted.
2. Slower reactions – cause cement to harden. C_2S & C_3S hydrate to form tobermorite, which is hardened cement paste, the main bonding material which forms 70% of the structure.

Q1(b) Consider 4C's of concrete durability. + H_2O.

1. Cement content 2. Cover 3. Compaction 4. Curing + W/c.

↓ permeability
↓ pore/capillary size
↑ alkalinity

Cement content – wish to ensure adequate cement content to ensure strength & also so that sufficient binding matrix is available to hydrate, react & fill voids within composite aggregate/sand/cement mixture.
- also promotes alkaline environment & hence passivation of reinforcing steel to reduce likelihood of corrosion.
- Want cement to be available to promote 'autogeneous healing' of fine cracks.

protects r/f from corrosion

Cover – provides protective barrier to ingress of deleterious materials (e.g. chlorides, carbonation)
- ensures adequate bond to r/f bars.
(should not be too large or may get spalling).

↓ voids/air gaps
↓ permeability

Compaction – provides dense, impermeable concrete by removing air pockets/bubbles. This again assists in preventing ingress of corrosive materials toward r/f.
(Must avoid segregation from over-compaction).

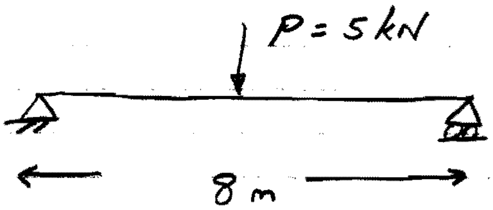
↓ Permeability
↓ rate of initiation of corrosion

Curing – essential to prevent drying out of exposed surface of concrete which can result in incomplete hydration reaction. This may inhibit the closing of capillary pores within the matrix, again allowing greater permeability to corrosive materials. Similarly you may get surface cracking from drying shrinkage.

LOW

WATER CEMENT RATIO – perhaps the most important parameter to control is W/c ratio. It should be kept to the minimum necessary to permit full hydration & adequate workability (as measured by a slump test) The theoretical minimum (~0.3) ensures the full hydration reaction can occur & hence capillary pores are closed up & high interaction between cement grains promotes strength. This in turn decreases permeability. Too much water results in permeable, low strength concrete e.g. $W/c \gtrsim 0.7$.

Q1(c)



$M_p = 5$ kN

<u>Strength</u> (Resistance)

$M_R = 40$ kNm.

<u>Load effect</u> (Stress resultant)

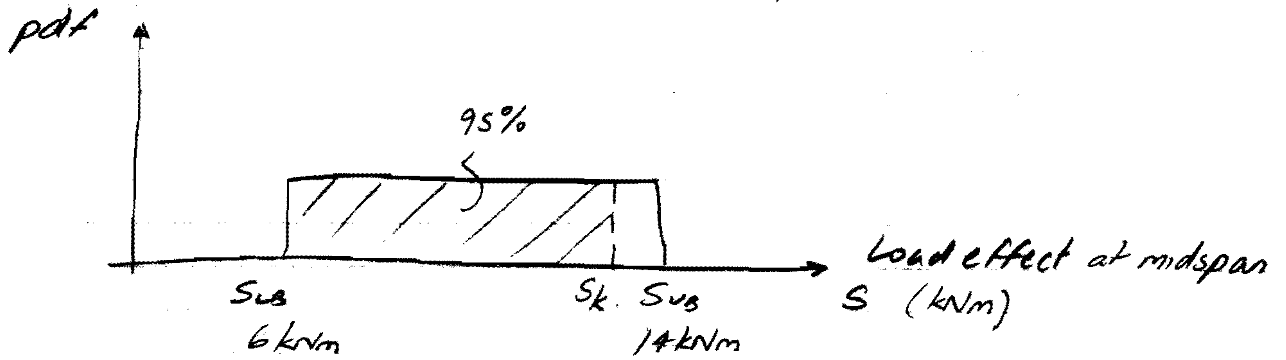
$$S = \frac{PL}{4} = \frac{5 \times 8}{4} = 10 \text{ kNm}.$$

Range for load effect. (Lower + Upper bound)

$$S_{LB} = \frac{3 \times 8}{4} = 6 \text{ kNm}$$

$$S_{UB} = \frac{7 \times 8}{4} = 14 \text{ kNm}$$

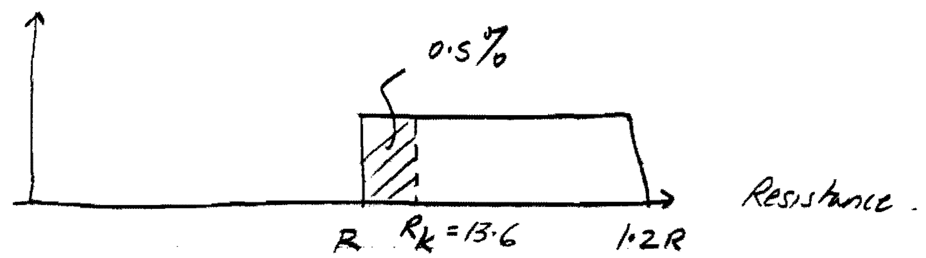
(i)



Characteristic Load Effect

$$S_k = 6 + 0.95(14-6) = \underline{13.6 \text{ kNm}}$$

(ii) <u>Resistance</u> (Strength).



$$R_k = 0.05 \times (1.2R - R) + R = 13.6$$

$$1.01R = 13.6.$$

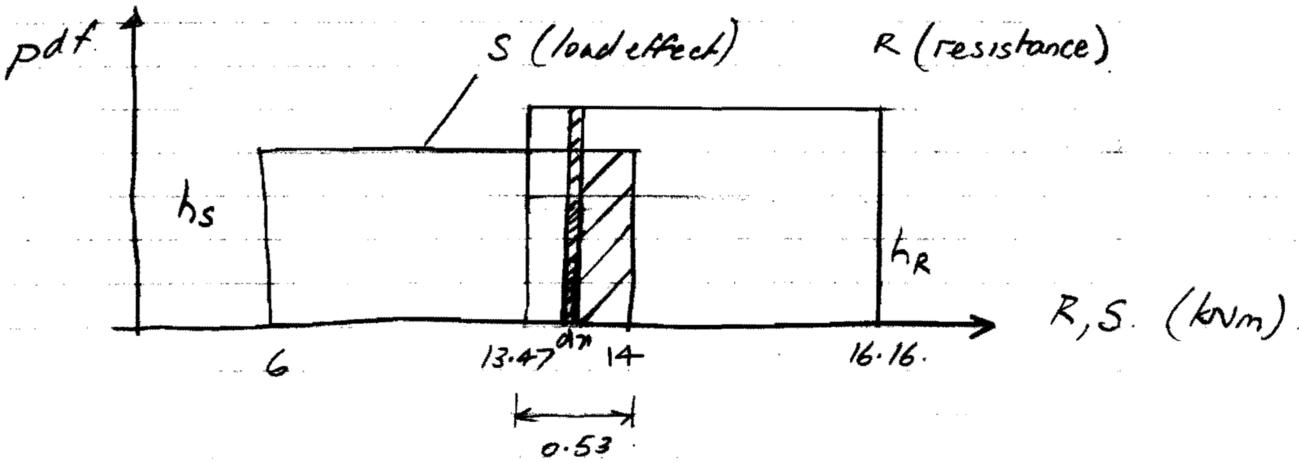
Lower limit $R = \frac{13.6}{1.01} = 13.47$ kNm.

Upper limit $1.2R = 1.2 \times 13.47 = 16.16$ kNm.

Q 1(c)(ii) (cont.)

Probability failure

$$P_f = \Sigma \, \text{Prob}(R \text{ in range } dx) \times \text{Prob}(S > R_{dx})$$



Height of R pdf. Area $= h_R \times (16.16 - 13.47) = 1.0$

$$h_R = \frac{1}{2.69} = 0.3717$$

Height of S pdf. Area $= h_S \times (14 - 6) = 1.0$

$$h_S = \frac{1}{8} = 0.125$$

Prob. resistance in range x to $x+dx$ is area under pdf of R.

$$P(x \le R \le x+dx) = h_R . dx = 0.3717 \, dx$$

Prob load is greater than x is area above x under pdf of S.

$$P(S \ge x) = (14 - x) h_S = 0.125(14 - x)$$

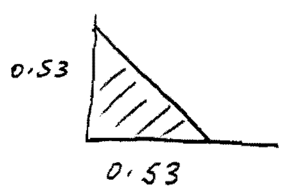
$\therefore$ Prob. fail $P_f = \int_{13.47}^{14} (14 - x) 0.125 \times 0.3717 \, dx$

$$= 0.125 \times 0.3717 \int_0^{0.53} (0.53 - x) \, dx$$

$$= 0.125 \times 0.3717 \times \frac{0.53^2}{2}$$

$$P_f = 6.53 \times 10^{-3}$$

Area under graph
At $x = 14$, $I = 0$
At $x = 13.74$, $I =$



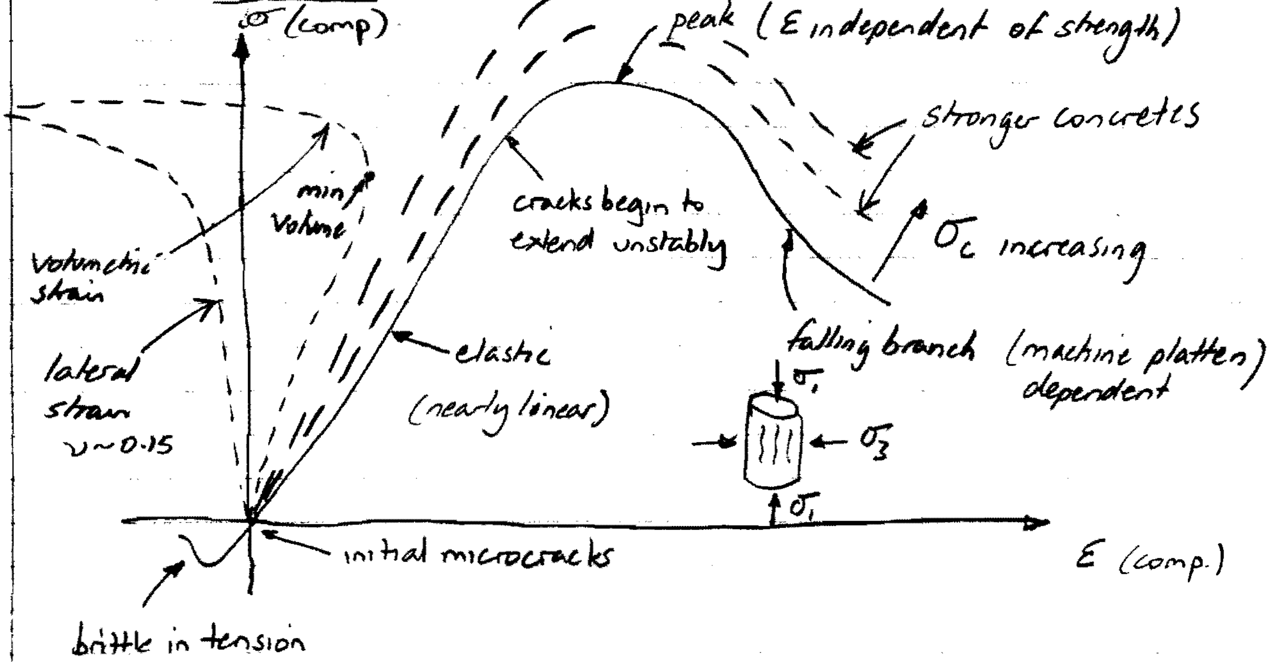
Area $= \frac{1}{2} \times 0.53^2$

Examiner's comment:

This was the most popular question, attempted by all but one student. Part (a) was straightforward bookwork although a number dropped marks by omitting to refer to the hydration process and the effects of the different anhydrous products. The clear message to students is that they must carefully read the question and answer what is asked for rather than regurgitate everything they can remember on the general topic area. Part (b) should also have been very straightforward, with the majority able to list the 4C's + w/c ratio; however a number were less clear on why these parameters were important.

The reliability question in part c) required an integration of the convolution integral. The most common error was to incorrectly set the limits of integration. A few failed to recognize that you cannot use methods derived for normally distribution parameters to cases like this with non-normal pdfs. Others failed to recognize the difference between load and load effect i.e. moment, despite the use of bold type in the question to highlight this issue.

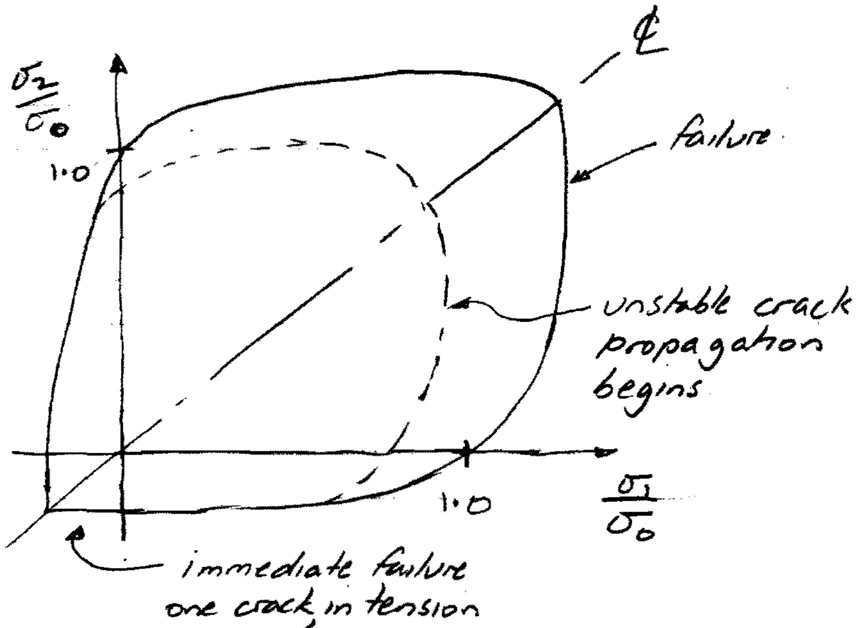
Q 2 (a) Uniaxial



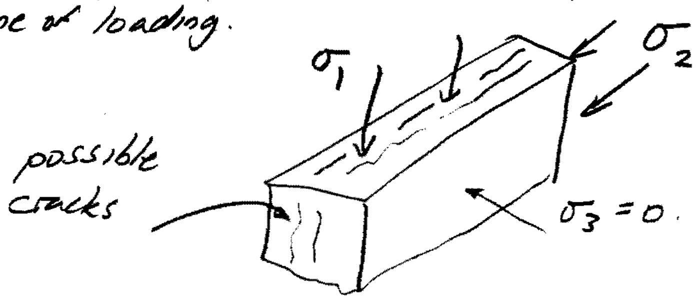
Increasing lateral stress (σ_3) has the effect of increasing ductility (shows as a rise in falling branch of $\sigma - \varepsilon$ graphs)

Biaxial

σ_0 = uniaxial compressive strength



Magnitude of biaxial strength not much higher than uniaxial (15 to 20%) because the specimens can crack in the plane of stress + bulk out of the plane of loading.



Q2 (a) (cont.)

Triaxial — Crushing is inhibited in compression $\therefore$ very strong — also more ductile $\sigma - \varepsilon$ curves

Can provide confinement in columns, members using links wrapped around members.

Examples where triaxial properties are exploited:

1. Lateral binding of columns e.g. in earthquake regions
2. Concrete hinges in columns.

Q 2(b) Whole life costing

Add to the initial capital cost the discounted "present value" of estimated future repair costs + also potential future income streams (e.g. from tolls). Discounting based on the idea of investing now at compound interest a sum of money which would grow to pay for the repairs in time.

So a future cost C_i after i years of interest rates r would have a present value

$$\frac{C_i}{(1+r)^{i-1}}$$

The $(i-1)$ term is used because one might need money at the start of the year for repairs to be paid for near the end.

Discount rate is vital. At 3% $\frac{1}{(1+0.03)^{10}} = 0.744$ for example, whilst at 6% $\frac{1}{(1+0.06)^{10}} = 0.558$. The rate used should be the long-term rates, working in "real" terms, i.e. after allowing for inflation. Whole idea is that inflation is not knowable, so work in "real" costs, using present-day prices for cost estimates.

Out-of-use (disruption) costs can be much more than direct repair costs but are often not included in calculations. Issue is who pays for these costs — often not the person incurring the repair costs. Ideally this should still be taken into account.

Obsolescence also important — if structure is not longer needed why include the repair costs?

Q 2(b) (cont)

(i) $WLC_1 = 40 + \frac{10}{(1.03)^9} + \frac{10}{(1.03)^{19}} + \frac{10}{(1.03)^{29}} = £57.61k.$

(use i-1 since assuming funding for repairs is allocated at the beginning of year).
(no need to repair at 40 years since that is end of design life)
* (could use i or i-1 for index - state reason for choice)

(ii) Use continuous discounting $A_0 = A \exp(-r_c.t)$ (from notes)

where $1+r = 1.03 = \exp.(r_c)$ so $r_c = 0.02956$.

$dA_0 = k \exp(-r_c t)dt$ where k is rate of spend (£500/year)

So for cathodic protection

$$A_0 = k \int_0^T \exp(-r_c t)dt$$

$$= \frac{k}{r_c}\left[1 - \exp(-r_c.T)\right] \qquad k = 0.5$$

Some question over when to switch off cathodic protection - one could in theory allow structure to deteriorate over the last few years of life. So could say T = 35 years for cathodic protection.

$$WLC_2 = 50 + \frac{0.5}{0.02956}\left[1 - \exp(-35 \times 0.02956)\right]$$

$$= £60.9k$$

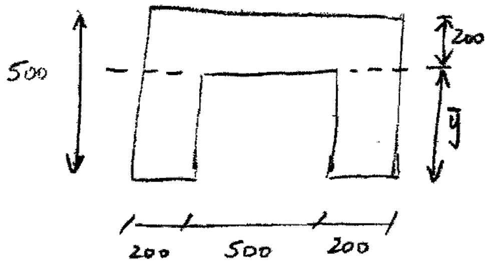
So the traditional reinforcement option seems cheaper

(If assume cathodic protection left on for full 40 years then $WLC_2 = £61.73k$)

Examiner's comment:

Part (a) on mechanical properties of concrete proved to be the most problematic part of this question despite only requiring a straightforward recounting of the lecture notes. Most students could discuss uniaxial behaviour but many struggled to describe key features of biaxial and triaxial behaviour or cite examples of where such effects were exploited. The whole life costing section in part (b) was very straightforward and well answered by most. Perhaps the most common mistake here was to include the maintenance costs in the last year of the design life when no maintenance expenditure would be expected if the structure was life-expired.

Q3 (a) 1st cracking



$\bar{y}$ & I_{UN} given in question

Centroid. $\bar{y} = \dfrac{500 \times 400 \times 250 + 500 \times 200 \times 400}{500 \times (400 + 200)}$

$= \dfrac{1800}{6} = 300 \text{ mm}$

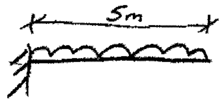
$I_{un} = \dfrac{900 \times 200^3}{3} + \dfrac{400 \times 300^3}{3} = 2.4E9 + 3.6E9$

$= 6E9 \text{ mm}^4$

$\sigma_t = \dfrac{My}{I} \Rightarrow M_{cr} = \dfrac{\sigma_t I_{un}}{y} = \dfrac{4 \times 6E9}{200} = 120E6 \text{ Nmm} \ (120 \text{kNm})$

$M_{cr} = 120 \text{kNm}$

Maximum moment near support



$M = \dfrac{wL^2}{2}$ $\quad L = 5 \text{ m}$. $\quad W_{LL} = 6 \text{kN/m}$

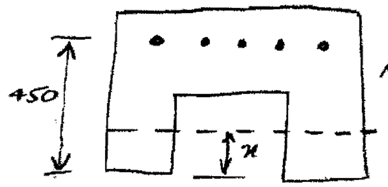
$A_g = 500 \times (400 + 200) = 300E3 \text{ mm}^2$

$\therefore w_{cr} = \dfrac{2M}{L^2} = \dfrac{2 \times 120}{25} = 9.6 \text{ kN/m}$. $\quad \gamma_{conc} = 24 \text{ kN/m}^3$

$\therefore W_{DL} = 24 \times 0.3 = 7.2 \text{ kN/m}$

$W_{DL} = 7.2 \Rightarrow W_{LL} = 2.4 \text{ kN/m}$ for 1st cracking

(b) Require I_{cr}.



$m = \dfrac{E_s}{E_{ce}} = \dfrac{210}{30} = 7$.

$A_s = 1608 \text{ mm}^2$

Transformed steel area $A_s' = mA_s = 7 \times 1608 = 11.26 \times 10^3 \text{ mm}^2$

To find n.a. $\quad 200 \times 2 \times x \times \dfrac{x}{2} = 11.26 \times 10^3 \times (450 - x)$

$200x^2 + 11.26 \times 10^3 x - 11.26 \times 10^3 \times 450 = 0$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = \dfrac{-11.26 \times 10^3 + \sqrt{(11.26 \times 10^3)^2 + 4 \cdot 200 \cdot 11.26 \times 10^3}}{2 \times 200}$

$x = 133.5 \text{ mm}$ $\quad$ & $d - x = 450 - 133.5 = 316.5 \text{ mm}$

from parallel axes formula.

$\therefore I_{cr} = \dfrac{400 \times 133.5^3}{3} + 11.26 \times 10^3 \times 316.5^2 = 1.449 \times 10^9 \text{ mm}^4$

$\sigma_c = \dfrac{My}{I_{cr}}$ $\quad$ where $M = \dfrac{wL^2}{2}$ $\quad \gamma_{f_{SLS}} = 1.0$ for DL & LL

$W_{LL} = 6 \text{ kN/m}$.

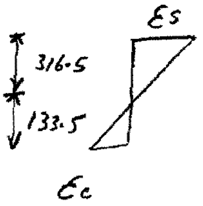
$W_{DL} = 7.2 \text{ kN/m}$ from part (a)

Q3(b) cont. ∴ At SLS $M = 13.2 \times \frac{25}{2} = 165\ kNm$. since $W = 13.2\ kN/m$

$$\therefore\ \sigma_c = \frac{165\times10^3 \times 133.5\times10^{-3}}{1.449\times10^{-3}} = 15.2\times10^6\ N/m^2 \text{ i.e. } 15.2\ MPa$$

compression at bottom flange of beam.

Steel strain



$$\varepsilon_c = \frac{\sigma_c}{E_{c}} = \frac{15.2}{30\times10^3} = 506.7\times10^{-6}.$$

$$\varepsilon_s = \frac{316.5}{133.5}\times\varepsilon_c = \underline{0.0012} \quad < \varepsilon_y \quad \text{NOT YIELDED}$$

where $\varepsilon_y = \frac{f_{yk}}{E} = \frac{460}{210\times10^3}$

$= 0.00219$

(c) (i) Tip deflection - uncracked beam (SLS)

From data book $\delta_{tip} = \frac{WL^3}{8EI}$ where W = total load = wL.

$$= \frac{wL^4}{8EI}$$

$$\delta_{UN} = \frac{13.2\times10^3 \times 5^4}{8\times30\times10^9\times6\times10^{-3}} = 5.7\times10^{-3}\ m \quad \text{i.e. } \underline{5.7mm}$$

(ii) Tip deflection - cracked beam (SLS)

$$\delta_{cr} = 5.7\times\frac{6}{1.449} = \underline{23.7mm}$$

(d) $\delta = \zeta\delta_{cr} + (1-\zeta)\delta_{UN}$ (formula from Data sheet)

N.B. $\delta_{cr} = \alpha_{II}$; $\delta_{UN} = \alpha_I$ in formula.

$$\zeta = 1 - \beta\cdot\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

Also $\frac{M_{cr}}{M_{SLS}} = \frac{120}{165} = 0.727(3)\ \left(\equiv \frac{\sigma_{sr}}{\sigma_s}\right)$

$\beta = 1.0$ short-term loading.

σ_{sr} is stress in steel for loading to cause cracking on cracked section.

$w_{cr} = 9.6\ kN/m$ from (a) $\quad I_{cr} = 1.449E9\ mm^4 \quad M_{cr} = 120\ kNm.$

$$\sigma_c = \frac{120}{165}\times15.2 = 11.05\ MPa \text{ at cracking}$$

Q3 (d) cont.

$$\sigma_{sr} = \frac{316.5}{133.5} \times \overset{\sigma_c}{11.05} \times \overset{m}{7} = \underline{183.5\ MPa}$$

σ_s is stress in steel at full load on cracked section.

$$\sigma_s = E_s \varepsilon_s = 210 \times 10^3 \times 0.0012 = \underline{252\ MPa}$$

$$\therefore \quad \xi = 1 - \left(\frac{183.5}{252}\right)^2 = 0.47$$

$$\therefore \quad \delta = \underbrace{0.53 \times 5.7}_{3.02} + \underbrace{0.47 \times 23.7}_{11.14} = 14.2\ mm$$

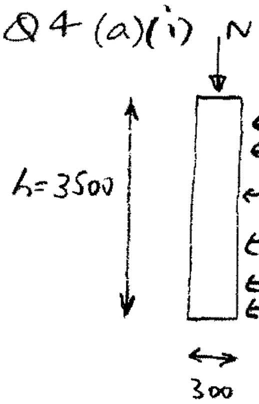
$\therefore$ $\underline{\delta_{tip} = 14.2mm}$ with allowance for tension stiffening.

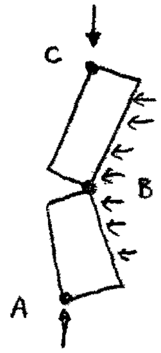
Actual tip deflection will be less since beam will not be cracked all the way along to the free end & hence curvature in this end region will be less than that assumed above.

Examiner's comment:

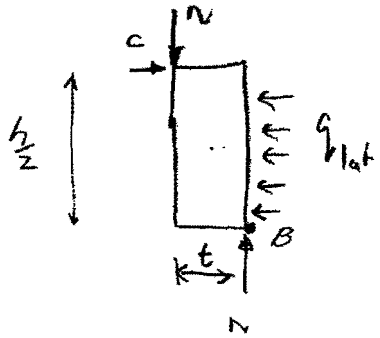
This question was purely analytical and on reflection probably the hardest on the paper, requiring a number of calculations to derive section properties, strains and deflections. On the whole it was quite well answered with several excellent and near complete solutions from students who clearly fully understood the procedure. Inevitably under exam conditions, several became tangled when trying to derive the beam's cracked section property although this should have been relatively straight forward.

Q4 (a)(i)





Mechanism



FBD of top half BC

Moments about C. $N.t = q_{lat} \cdot \frac{h}{2} \cdot \frac{h}{4}$

$$\therefore q_{lat} = \frac{8Nt}{h^2} = \frac{8 \times 900 \times 0.3}{3.5^2} = 176.3 \text{ kPa}.$$

Adopt Factor of Safety of 2 overall $\Rightarrow q_{lat} = \frac{4Nt}{h^2} = 88.2$ kPa.

(Cannot use normal γ_m since material strength is not involved so effectively have a factor on applied loading.)

(ii) Vertical force = 900 kN

Wall area (per meter length) $A = 0.3 m^2$.

$$\sigma = \frac{900 \times 10^3}{0.3} = 3.0 \times 10^6 \text{ N/m}^2 \quad 3.00 \text{ MPa}.$$

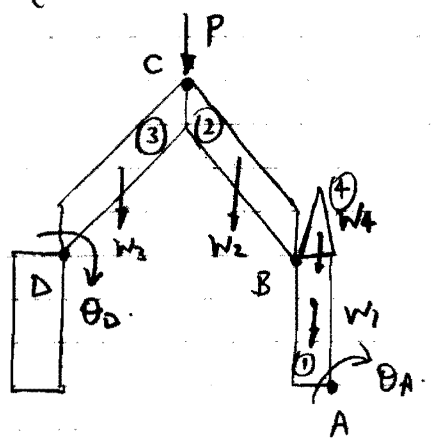
This does not allow for factors of safety & is based on crushing strength of masonry. In practice would wish to apply partial factors on loads & compressive strengths

Apply $\gamma_f = 1.5$ & $\gamma_m = 3$

$$\Rightarrow \frac{\sigma}{3} = 1.5 \times 3.00 \quad \Rightarrow \sigma = 13.5 \text{ MPa} \quad \text{for masonry}.$$

Bricks to be Class 7 with mortar (i)
or Class 10 " " (ii)

Q 4(b). Find P.



Assume rotation ϕ at instantaneous centre for mechanism with hinges at A, B, C + D.

Scale on diagram : $L \equiv 40\,mm$ (Fig 3)

$$\delta_C = \vec{IC}.\phi = \left(\frac{22}{40}L\right)\phi \qquad (S = r\theta) \quad \theta = \frac{S}{r}$$

$$\delta_C = \vec{DC}.\theta_D = \left(\frac{69}{40}L\right)\theta_D$$

$$\therefore \theta_D = \frac{\delta_C}{\vec{DC}} = \frac{\delta_C}{\left(\frac{69}{40}L\right)} = \frac{\left(\frac{22}{40}L\right)\phi}{\left(\frac{69}{40}L\right)} = \frac{22}{69}\phi$$

$$\delta_B = \vec{IB}.\phi = \left(\frac{78}{40}L\right)\phi$$

$$\delta_B = \vec{AB}.\theta_A = \left(\frac{43}{40}L\right)\theta_A$$

$$\therefore \theta_A = \frac{\delta_B}{\left(\frac{43}{40}L\right)} = \frac{\left(\frac{78}{40}L\right)\phi}{\left(\frac{43}{40}L\right)} = \frac{78}{43}\phi$$

WORK EQUATION

$$P \times \frac{12}{40}L \times \phi - W_1 \times 0.2L \times \theta_A - W_2\left(\frac{8}{40}L\right)\times\phi + W_3(0.5L)\times\theta_D = 0$$

But $W_1 = W_2 = W_3 = W$
Since area of each block is the same.

$$\frac{12}{40}PL\phi = 0.2WL\theta_A + \frac{8}{40}WL\phi - 0.5WL\theta_D$$

$$12PL\phi = 8WL\left(\frac{78}{43}\right)\phi + 8WL\phi - 20WL\left(\frac{22}{69}\right)\phi$$

$$\therefore P = \frac{1}{12}\left(\frac{78}{43}.8W + 8W - 20W.\frac{22}{69}\right)$$

$$= \frac{W}{12}\left(14.51 + 8 - 6.38\right)$$

$P = 1.344W$ Assume require factor of safety of 2

$$P_{allowed} = \frac{1.344W}{2} = 0.67W$$

This is an upper bound estimate of the load P.

4(c) $W_1' = \frac{W_1}{2} = \frac{W}{2}$ Pinnacle $= W_4$.

Additional weight of pinnacle must equal reduced weight of stone ①.

$\therefore W_{pinnacle} = W_4 = \frac{W_1}{2}$.

Area of pinnacle $= \frac{1}{2} \times 0.4L \times H = \underline{0.2HL}$

Wt of pinnacle $= 0.2HL \times \gamma_{stone} = W_4$ (per metre into page)

Original Weight of block $W_1 = W = 0.4L \times L \times \gamma_{stone} = 0.4L^2 \gamma_{stone}$

Require $W_4 = \frac{W_1}{2} \Rightarrow 0.2H\cancel{L}\cancel{\gamma_{stone}} = \frac{0.4}{2} L^{\cancel{2}} \cancel{\gamma_{stone}}$

$\therefore \underline{H = L}$. Pinnacle height must equal wall height.

This can be deduced by inspection. Area of pinnacle must equal half area of wall stone.

Examiner's comment:

This was the least popular question but most who attempted it did quite well. Part (a) was straight from the lecture notes and should have posed little problem although a few became a little confused with applying safety factors.

The second part (b), which accounted for 60% of the marks, was again a very straightforward masonry frame structure however the marks were somewhat polarised. Those who could correctly identify a valid collapse mechanism and, in particular, the appropriate instantaneous centre of rotation produced good results. Those who were unable to define a valid collapse mechanism got low scores as they could not really do anything without this correct starting point. Only a few could identify th4e correct solution to part (c) which one should be able to deduce directly without calculation.

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ENGINEERING TRIPOS PART IIB

2012 2.30 to 4

Q4-(b) Module 4D7 CONCRETE AND MASONRY STRUCTURES

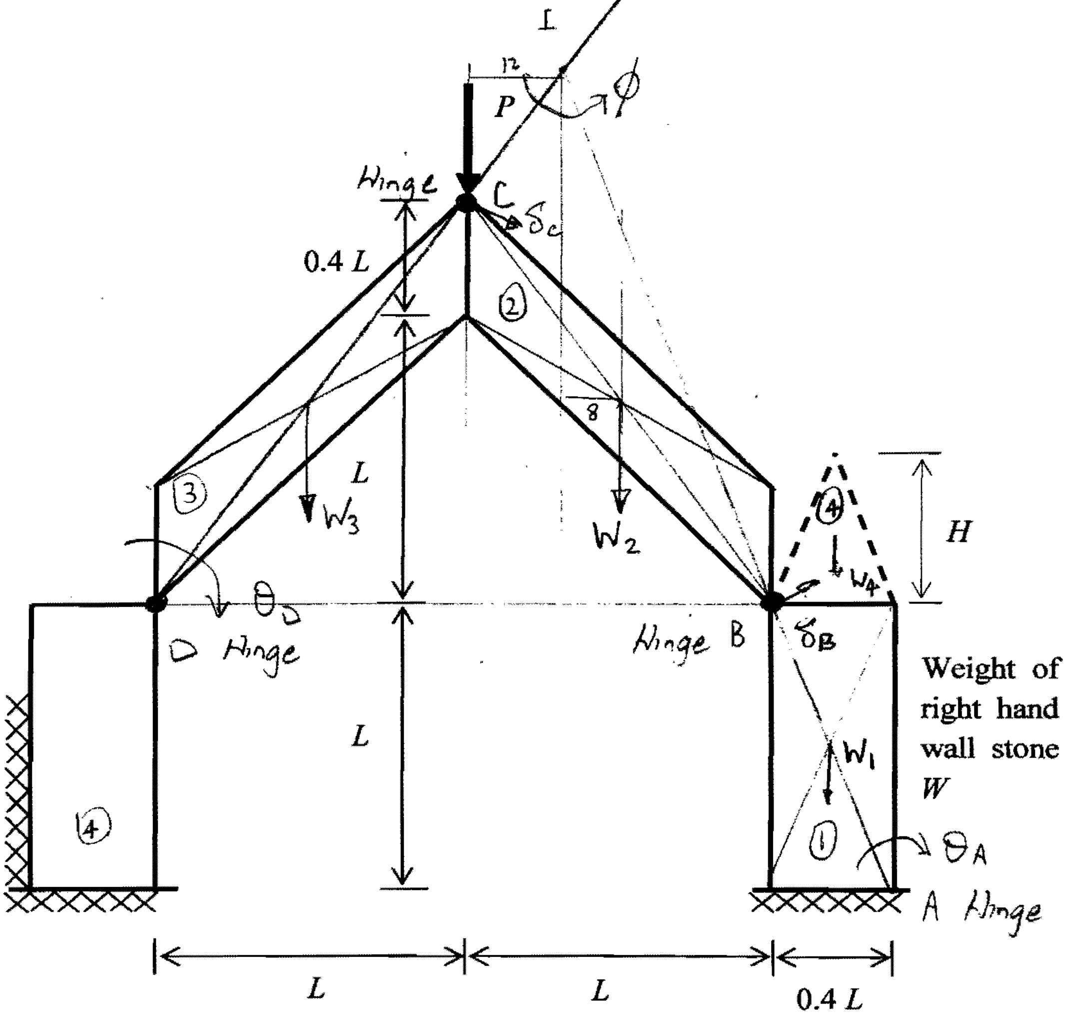

Fig. 3
Working sheet for Question 4 (may be handed in with your script)