ENGINEERING TRIPOS PART IIB 2012
4D8 PRESTRESSED CONCRETE **Dr C J Burgoyne**

Solutions

1.(a) Section Properties $A = 7\,m^2$ $I = 6\,m^4$

$y_t = -1.2\,m \Rightarrow Z_t = -6/1.2 = -5\,m^3$

$y_b = 2.7 - 1.2 = 1.5\,m \Rightarrow Z_b = -6/1.5 = +4\,m^3$

Eccentricity of tendons.

Mean depth of tendons = 0.2m below top fibre

$\therefore\ e = -1\,m$

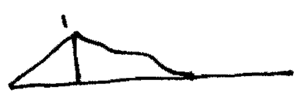
Top fibre stress $= \dfrac{P}{A} + \dfrac{Pe}{Z_t}$

$$= \frac{36000}{1000}\left(\frac{1}{7} + \frac{(-1)}{(-5)}\right) = \underline{\underline{12.3\text{ MPa}}}$$

Bottom fibre stress $= \dfrac{P}{A} + \dfrac{Pe}{Z_b} = \dfrac{36000}{1000}\left(\dfrac{1}{7} + \dfrac{(-1)}{4}\right) = \underline{\underline{-3.85\text{ MPa}}}$

(30%).

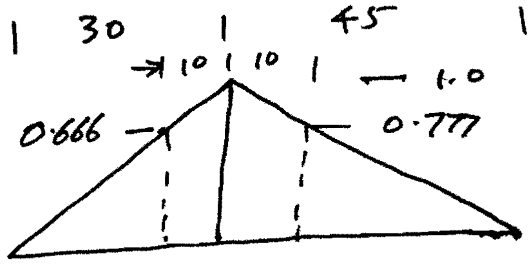
(b)
$$\begin{bmatrix} \int \beta_1^2 & \int \beta_1\beta_2 \\ \int \beta_1\beta_2 & \int \beta_2^2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = Pe_s \begin{bmatrix} \int \beta_1 \\ \int \beta_2 \end{bmatrix}$$

β_1 

β_2 

Bookwork to get
L.H.S. integrals

$P_{es}\int \beta_1$ Only need to consider area where tendons are



$\therefore \int \beta_1 = \left(\frac{1+0.666}{2}\right)10 + \left(\frac{1+0.777}{2}\right)10$

$= 17.22$

$\int \beta_2$ the same by symmetry

$$\frac{1}{6}\begin{bmatrix} 2(30+45) & 45 \\ 45 & 2(30+45) \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = 36000.(-1)\begin{bmatrix} 17.22 \\ 17.22 \end{bmatrix}$$

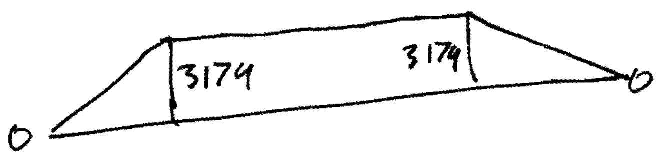
Could solve as simultaneous equations but note that symmetry means $Q_1 = Q_2$

$\therefore 150Q + 45Q = -36000.17.22 \times 6$

$\Rightarrow Q = -19070$ kNm

$\therefore$ Secondary moments are

[60%]



(c) Stresses in top fibre caused by secondary moments are $\frac{-19070}{-5.1000} = +3.8$ MPa

Bottom fibre $\frac{-19070}{+4.1000} = -4.8$ MPa [10%]

Examiner's comment:

Q1. Secondary Moments. Simplified version of Hammersmith Bridge Repair. Intended to be straightforward but required thinking. With constant eccentricity the integrals did not need Simpson's rule, and the symmetry meant that there should have been no need to solve simultaneous equations. The least popular question and only two got close to the right answer.

③

2.

(a) The range of moments that the beam must carry at mid-span is purely related to the live load, not the fixed dead load

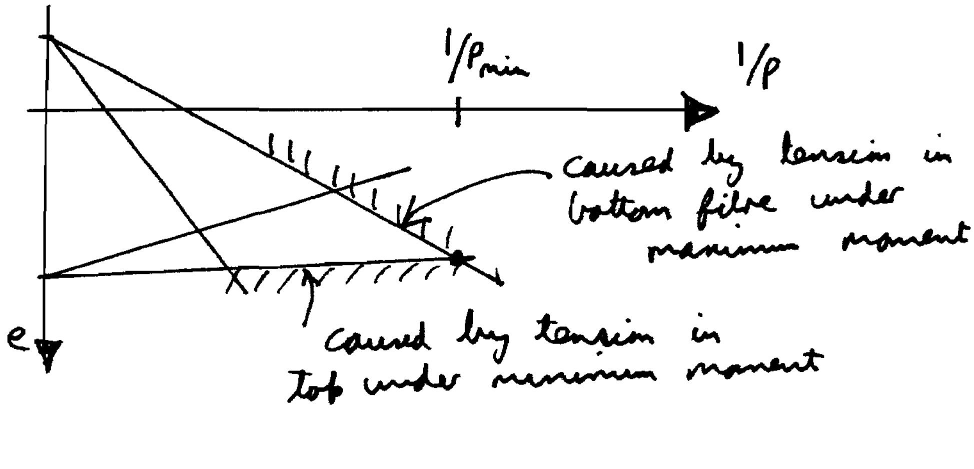
Thus $\frac{qL^2}{8} / f_c = \frac{bd^2}{6}$

But, $bd\rho = w$

$\therefore \quad \frac{6 q \rho L}{8 f_c} \left(\frac{L}{d}\right) = w$ ①

[25%]

(b) Magnel diagram



Consider the two stress limits which must apply at the $1/P_{min}$ condition

Top fibre, tension stresses, minimum moment

$$0 = \frac{P}{A} + \frac{Pe}{Z_t} - \frac{M}{Z_t} \qquad \text{N.B. } \underline{\underline{Z_t \ -ve}}$$

$$= \frac{P}{\cancel{b}\cancel{d}} - \frac{Pe\,6}{\cancel{b}d^{\cancel{2}}} + \frac{wL^2}{8}\cdot\frac{6}{\cancel{b}d^{\cancel{2}}} \qquad (2)$$

[10%]

Bottom fibre, tension stresses, maximum moment

$$0 = \frac{P}{A} + \frac{Pe}{Z_b} - \frac{M}{Z_b} \qquad Z_b \ +ve$$

$$= \frac{P}{\cancel{b}\cancel{d}} + \frac{Pe\,6}{\cancel{b}d^{\cancel{2}}} - \frac{(w+q)L^2}{8}\,\frac{6}{\cancel{b}d^{\cancel{2}}} \qquad (3)$$

[10%]

Solve (2) & (3) to find P & e.

Add: $\quad 0 = 2P - \frac{qL^2 6}{8d}$

$$\Rightarrow \quad \underline{\underline{P = \frac{3qL^2}{8d}}} \qquad (4)$$

[10%]

(4) ⇒ (2)

$$0 = \frac{3qL^2}{8d}\left(1 - \frac{6e}{d}\right) + \frac{wL^2\,6}{8d}$$

$$\Rightarrow \quad e = \left(\frac{6w}{3q} + 1\right)\frac{d}{6} \qquad (5)$$

(could substitute from (1) here)

[10%]

5

(c) Fix e as $0.4d$

From (5)

$$0.4d = \left(\frac{36 q \rho L}{8 f_c \cdot 3q}\left(\frac{L}{d}\right) + 1\right)\frac{d}{6}$$

$$\Rightarrow L = 0.933 \cdot \frac{f_c}{\rho}\left(\frac{d}{L}\right)$$

[25%]

This is L_{crit}

$f_c = 20\ N/mm^2$
$\rho = 24\ kN/m^3$
$L/d = 20$

$$\Rightarrow L_{crit} = \frac{0.933 \cdot 20}{24 \cdot 10^{-6} \cdot 20} = \underline{38.9\,m}$$

[10%]

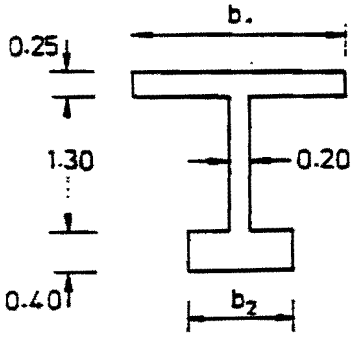
Examiner's comment:

Q2. Derivation of Critical Span. This was not a concept that had been covered in lectures, but the outline of the derivation was given and it was relatively straightforward for those candidates who understood the basic principles. The major error was to waste a huge amount of time writing down all the equations that governed the Magnel diagram, when all that was needed was a simple sketch and the recognition that it was the two tension limits that governed the limiting eccentricity. There were several completely correct solutions and several others that suffered from minor slips. As intended, it clearly separated those that understood the material from those who could merely put numbers into equations. Rather surprisingly, it was by far the more popular of the longer questions.

⑥

4D8/3/1

3.



Maximum bending moment due to uniformly distributed dead load

$$M_1 = \frac{\text{u.d.l.}\cdot\text{span}}{8} = \frac{50\cdot 25^2}{8} = 3906 \text{ kNm}$$

Maximum bending moment due to moving load (load at mid-span)

$$M_2 = \frac{\text{p.l.}\cdot\text{span}}{4} = \frac{500\cdot 25}{4} = 3125 \text{ kNm}$$

(a.i) Both the prestressing force P and its eccentricity e are constant, hence the section modulus has to be based on the overall moment range

$$Z = \frac{M_1 + M_2}{\Delta\sigma} = \frac{3906+3125}{20\cdot 10^3} = 0.352 \text{ m}^3$$

20%

(a.ii) The eccentricity is not constant, hence the section modulus is based on the maximum moment range at a section

$$Z = \frac{M_2}{\Delta\sigma} = \frac{3125}{20\cdot 10^3} = 0.156 \text{ m}^3$$

20%

(b) We choose b_1 and b_2 such that the top and bottom flanges have equal cross-sectional area A and, for simplicity, we neglect the web contribution to the second moment of area. The distance between the centroids of the top and bottom flanges is

$$\frac{0.25}{2} + 1.30 + \frac{0.40}{2} = 1.625 \text{ m}$$

We calculate the cross-sectional areas of the flanges from $A = \frac{Z}{1.625} = \frac{0.352}{1.625} = 0.217 \text{ m}^2$, hence:

$$b_1 = \frac{0.217}{0.25} = 0.86 \text{ m, say } 0.90 \text{ m}$$

$$b_2 = \frac{0.217}{0.40} = 0.54 \text{ m, say } 0.55 \text{ m}$$

30%

For these values of b_1 and b_2, the section properties are

$$\bullet A = 0.705 \text{ m}^2,\ Z_t = 0.372 \text{ m}^3,\ Z_b = 0.319 \text{ m}^3$$

Simplifications have been made when calculating these properties, for example it has been assumed that the centroid is in the middle of the web.

(c) To determine suitable values of P and e we note that the stress inequalities can all be expressed in terms of

$$f_t \leq \sigma = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \leq f_c$$

where Z is the elastic section modulus of the fibre in question.

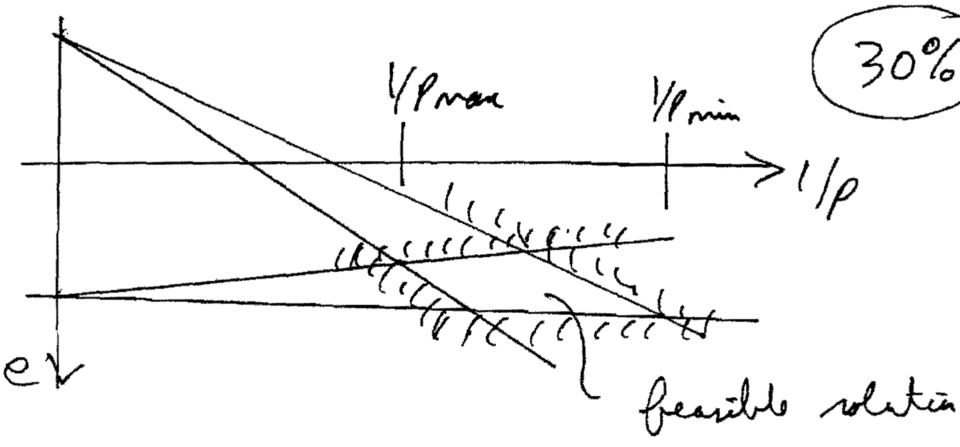
These can be rearranged into the form

$$e \lessgtr \frac{Z f}{P} - \frac{Z}{A} + \frac{M}{P}$$

which are straight lines on a plot of e vs $1/P$ (a Magnel diagram).

4 such lines define a feasible solution

30%

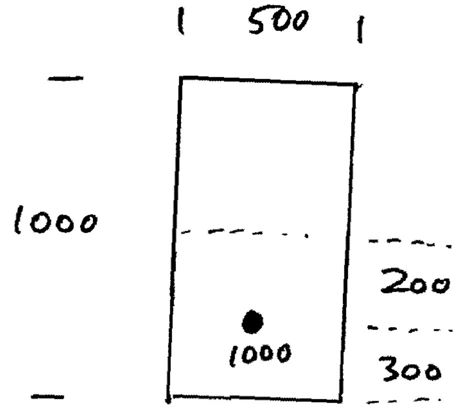


(8)

examiner's comment:

Q3. Section design. Tested the most basic assumptions of prestressed design. Very few picked up the important distinction that with the eccentricity fixed the same section has to work both at midspan and at the supports. Several had not picked up on the section modulus as the moment range divided by stress range.

4



Initial steel stress $= 800\ N/mm^2$

$\therefore \frac{800}{2200} = 1 - e^{-125\varepsilon}$

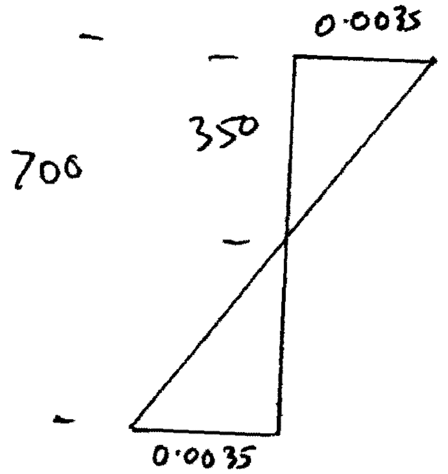
$\therefore \varepsilon_{pe} = 0.0036$

(a) Need iterative calculation

1st guess n.a depth = 350 mm (½ of effective depth)

(This value not critical - other guesses OK)

Limiting concrete strain $= 0.0035$



Steel strain $= 0.0036 + 0.0035 = 0.0071$

Steel stress $= 2200(1 - e^{-125 \cdot 0.0071})$
$= 1294\ N/mm^2$

$\therefore T = 1294 \times 1000 = 1294$ kN

$C = 350 \times 500 \times 20 = 3500$

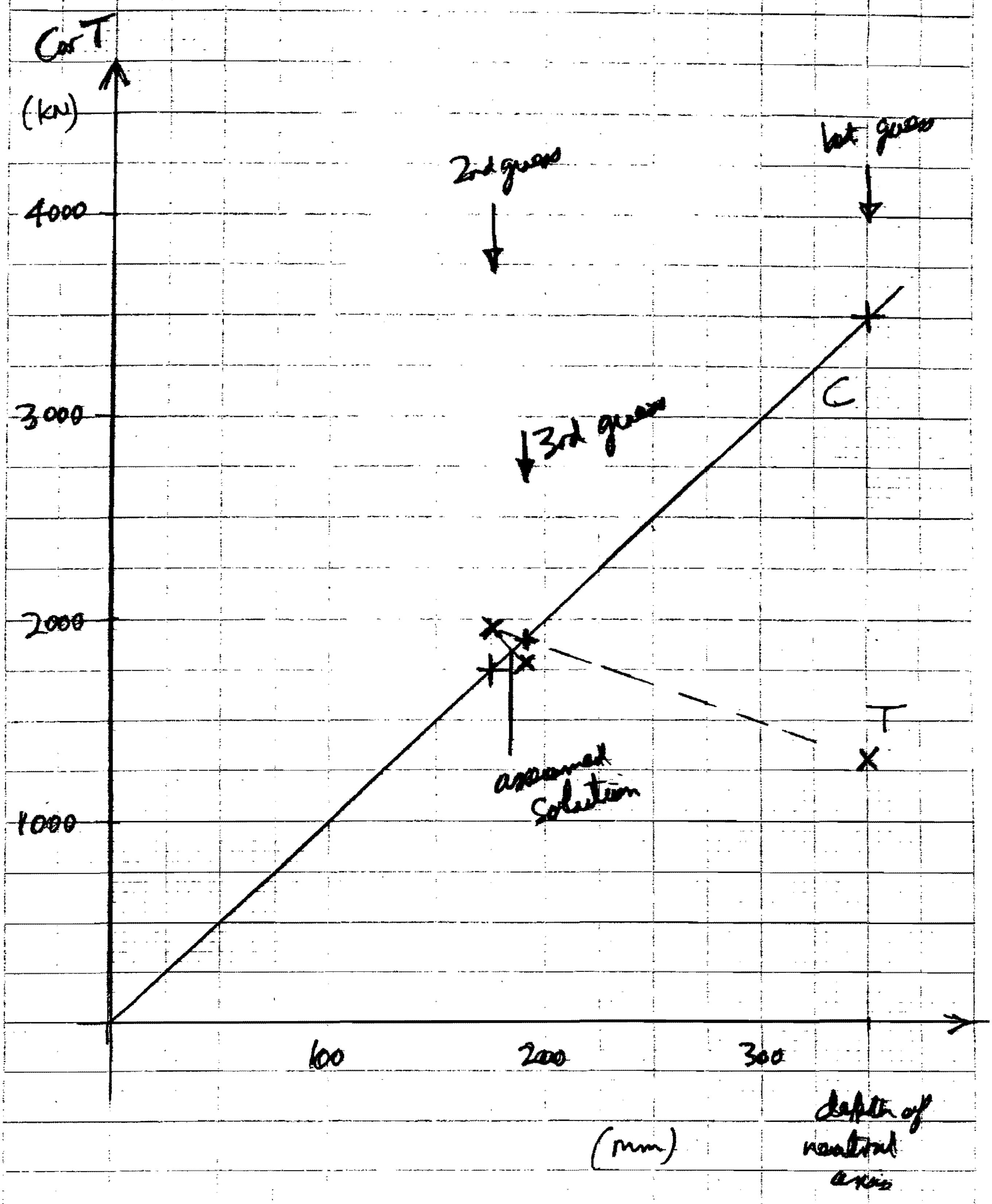
(C too high - raise n.a)

2nd guess n.a. depth = 175 mm

Steel strain $4 \times 0.0035 + 0.0036 = 0.0176$

Steel stress $= 1956\ N/mm^2$ $\therefore T = 1956$ kN

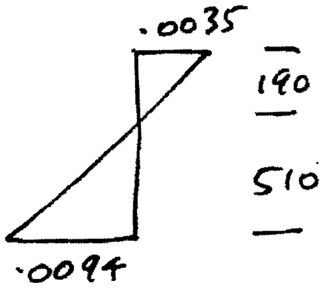
$C = 175 \times 500 \times 20 = 1750$ kN


C or T
(kN)
4000
3000
2000
1000
100
200
300
(mm)
depth of neutral axis
1st guess
2nd guess
3rd guess
C
T
assumed solution

11

3rd guess (from graph) or by reasonable guess

n.a. depth = 190 mm



Steel strain = 0.013

Steel stress = 1767 N/mm^2

∴ T = 1767 KN

C = 1900 KN

Take assumed solution from graph T = C = 1850 KN

neutral axis depth = 185 mm

∴ lever arm = $700 - \frac{185}{2} =$ 607 mm

∴ Moment = 1123 KNm

[75%]

(b) if steel strain limited to 0.01

∴ Steel stress = 1570 N/mm^2

∴ Steel force = 1570 KN

= compressive force

∴ depth of neutral axis = 157 mm

∴ Moment = $1570\left(700 - \frac{157}{2}\right) =$ 976 KNm

[25%]

Examiner's comment:

Q4. Ultimate Moment Calculation. Intended to be a straightforward question that all but the weakest candidates could do. A surprising number of candidates did not allow for the correct lever arm when taking moments.

5 (a) (i) Increasing the age of concrete reduces the amount of creep since the Young's Modulus of concrete is higher and the porosity of the concrete is reduced [5%]

(ii) Increasing the thickness of the concrete reduces creep because the water vapour has further to migrate, thus slowing down the rate at which it moves. However, it may not significantly affect the eventual creep since the water vapour will be squeezed out in the end. [5%]

(iii) Increasing the humidity of the surrounding air will reduce creep since it will lower the tendency of water to evaporate from the surface [5%]

(iv) Increasing the w/c ratio increases the rate of creep since it will increase the amount of voids and increase the amount of water that is available for movement. [5%]

General principles

1. Equilibrium applies before creep
2. Equilibrium applies after creep
3. Change in strain in steel = change in strain in concrete.

Equilibrium at start

Area of concrete	A_c	at	$f_{cu}/4$
Area of steel	A_s	at	$0.7 f_y$

Equilibrium gives $\dfrac{A_c f_{cu}}{4} = 0.7 f_y . A_s$

$$\therefore \frac{A_c}{A_s} = 2.8 \frac{f_y}{f_{cu}} \qquad [10\%]$$

Equilibrium at end

Steel stress not known – say λf_y

$$\frac{A_c f_{cu}}{6} = \lambda f_y . A_s \qquad [20\%]$$

$$\therefore \lambda = \frac{A_c}{A_s} . \frac{f_{cu}}{6 f_y} = \frac{2.8}{6} = 0.466$$

4D8/5/3

Initial strain in concrete = $\frac{f_{cu}}{4E_c}$

Final strain in concrete = $\frac{f_{cu}}{6\,E_c}(1+\phi) + 0.0005$

where ϕ = creep factor (here = 3)

$= \frac{4}{6}\frac{f_{cu}}{E_c} + 0.0005$

Difference $= \frac{f_{cu}}{E_c}\left(\frac{8-3}{12}\right) + 0.0005$

$= \frac{5}{12}\frac{f_{cu}}{E_c} + 0.0005$

[10%]

Change in strain in steel.

Initial strain = $\frac{0.7 f_y}{E_s}$

Relaxation causes stress to reduce by 2.5% without change in strain. Becomes $0.6825 f_y$

New stress = $0.4\dot{6}\dot{6} f_y$

$\therefore$ Strain change $= \frac{(0.6825 - 0.4\dot{6}\dot{6})f_y}{E_s}$

$= \frac{0.216 f_y}{E_s}$

Must equal $\frac{5}{12}\frac{f_{cu}}{E_c} + 0.0005$ [10%]

(15)

$$\therefore f_y = \frac{E_s}{E_c} \cdot f_{cu} \cdot 1.929 + 0.00231\, E_s$$

If $E_c = 9000 \sqrt[3]{f_{cu}}$ (MPa)

$$\therefore f_y = \frac{E_s}{E_c} f_{cu} \cdot 1.929 + 0.00231\, E_s$$

[10%]

$$= 42.9\,(f_{cu})^{2/3} + 462 \qquad \text{(MPa)}$$

Typical values (not asked for)

$f_{cu} = 20$ $f_y = 778$ MPa

$= 40$ $= 964$ MPa

$= 60$ $= 1119$ MPa

(c) Early attempts at prestressing used values much lower than this so they all failed.

[20%]

Examiner's comment:

Q5. Creep. The least popular of the short questions. The chatty points were both done reasonably well, but there was lack of clarity in the approaches to the calculation. Many candidates wrote down some of the principles (equilibrium, equality of strain changes), but few carried it through to a logical conclusion.