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(a) $t = 0.9$mm, 50 mm deep troughs ⇒ use DS6/c. Total depth is 100mm, and 5kN/m² are imposed onto a floor with a 'multi-span layout'.

Without props, the max. dist between sec. beams is 2.3m from DS6/c. The actual span is 3m; so props are needed (with a max span of 4.1m)

(b) Loads:

slab: $\overset{\text{concrete } \rho}{24} \times \left(0.05 + \frac{0.05}{2}\right) \times \overset{\text{span}}{3} = 5.4$ kN/m

50% of area below troughs is air

UB 457×152×52

UB: $52.3 \times 9.81 = 0.51$

services: 2.5×3m span $= 7.5$

(dead) 13.41

factor, ×1.35

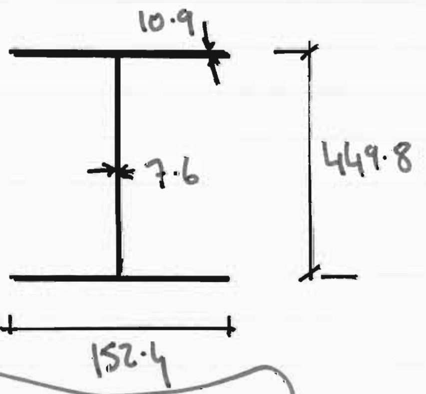
$= \underline{18.1 \text{ kN/m}}$

Imposed: $\underset{\text{factor}}{1.5} \times 5 \times \underset{\text{span}}{3} = \underline{22.5 \text{kN/m}}$

Total factored load $= \underline{40.6 \text{kN/m}}$

⇒ Applied bending moment (max) $= \frac{wL^2}{8}$

$= 40.6 \times 10^2/8 = \underline{507.5 \text{ kNm}}$



UB 457×152×52
S275

$A_s = 66.6$ cm² via S.D.B.

Compactness check: $\left(b/t\sqrt{\sigma_y/355} = \lambda\right)$

$\lambda_{flange} = \frac{(152.4 - 7.6)/2}{10.9}\sqrt{\frac{275}{355}}$ ← given

$= \underline{5.84 < 8}$, OK

$\lambda_{web} = \frac{449.8 - 2\times 10.9}{7.6}\sqrt{\frac{275}{355}}$

$= \underline{49.6 < 56}$, OK

(b) contd Effective width = minimum of $(\text{span}, \frac{\text{span}}{4}) = (3, \frac{10}{4})$

$\Rightarrow$ $\underline{b_e = 2.5m}$ Assume N.A. in concrete at depth of x_p. Axial equilibrium gives

$$A_s \sigma_y = 0.6 \, f_{cd} \cdot b_e \cdot x_p \;; \quad f_{cd} = 25 \text{ MPa}.$$

$$\Rightarrow 66.2 \times 10^2 \times 275 = 0.6 \times 25 \times 2500 \times x_p$$

$$\Rightarrow \underline{x_p = 48.8mm}$$

$\therefore$ OK with assumption and it lies just above trough uppermost line.

$$M_d = A_s \sigma_y \left[\frac{D}{2} + h_c - x_p/2 \right]$$

beam depth (D); total slab depth (h_c)

$$= 66.6 \times 10^2 \times 275 \left[\frac{0.4498}{2} + 0.1 - \frac{0.0488}{2} \right]$$

$$= \underline{550.4 \text{ kNm}}$$

$M_d > M_{applied}$, so the floor is adequate.

(c) 13 × 65mm studs in 25MPa concrete: strength, P_d, via DS6 = $\frac{42+47}{2}$ (interpolate) = $\underline{44.5kN}$

$$\# \text{ studs} > 2 \times \frac{A_s \sigma_y}{P_d} = 2 \times \frac{66.2 \times 10^2 \times 275}{44.5 \times 10^3} = 82.3$$

choose 83 or 84

Spacing requirements $\Rightarrow$ 10m/84 = every 119 mm: but troughs have a pitch of 150mm.

$\Rightarrow$ need to use pairs of studs placed every 150mm i.e.

m $\rightarrow$ 10/150 mm = 66.6 i.e. 67 pairs:

Check: each stud in pair has 80% P_d; i.e. each pair offers $1.6 P_d$ = 71.2kN strength $\Rightarrow$

P.T.O.

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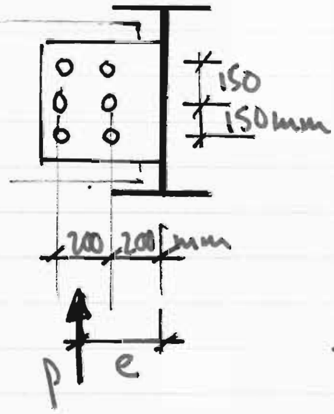
(c) contd $67 \text{ pairs} \times 71.2\text{kN} = \underline{4770.4\text{kN}}$, which is $> 2A_s\sigma_y$ $(= 3641\text{kN})$

(d)



$P = \frac{wL}{2} = \frac{40.6 \times 10}{2} = \underline{203\text{kN}}$. The eccentricity from primary beam is $e = 300\text{mm}$ ($1/2$-way bolt lines).

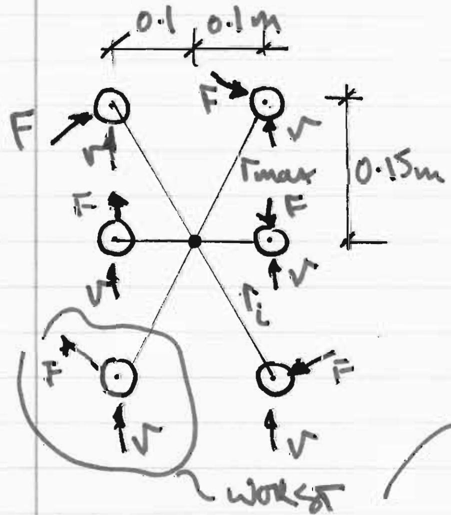
$\Rightarrow$ moment to be carried $= \underline{Pe = 60.9\text{kNm}}$

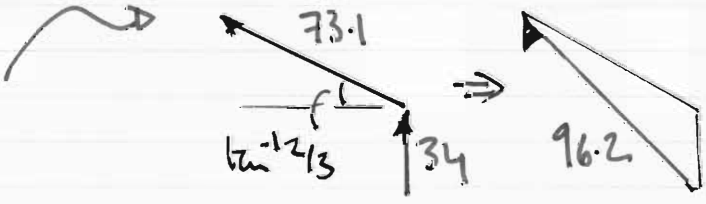
This creates a shear force, F, given by

$Pe / \left(\Sigma r_i^2 / r_{max}\right)$: $\Sigma r_i^2 = 4 \times [0.15^2 + 0.1^2] + 2 \times 0.1^2$

$r_{max} = [0.1^2 + 0.15^2]$

$\Rightarrow \underline{F = 73.1\text{kN}}$. $V = \frac{P}{6} = \underline{33.8\text{kN}}$





Total, max S.F. $= \underline{96.2\text{kN}}$ $\Rightarrow$ if using grade 8.8 the limit in shear for M24 is 125kN
M20 ~ 80kN

$\therefore$ Use M24 bolts

Comments: A popular question but with following errors: incorrect floor loads, wrong M_d formula for this slab configuration; wrong h_c in M_d formula. incorrect spacing calculation for studs. not being able to add the S.F. components on the bolts in a vector fashion.

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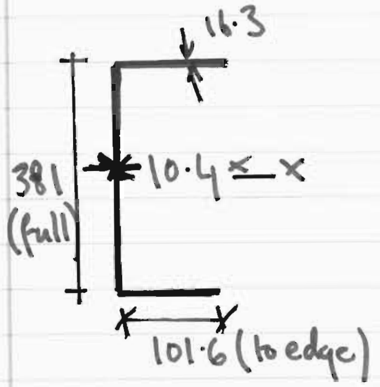
(a) channel, 381 × 102 × 55, S355 grade.



$A = 70.1\,cm^2$, $I_{xx} = 14870\,cm^4$.

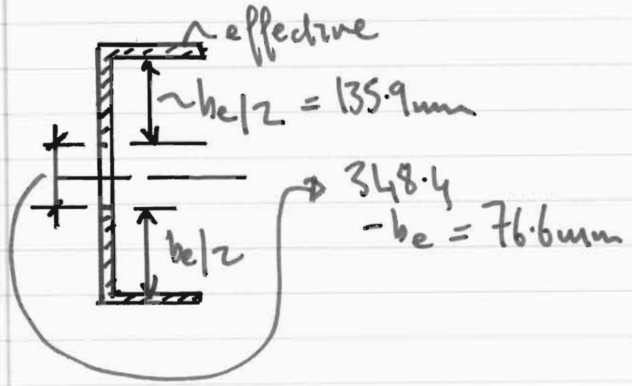
Compression: $\left(\frac{b}{t}\sqrt{\sigma_y/355}\right)$ $\lambda_{flange} = \frac{101.6-10.4}{16.3}\sqrt{\frac{355}{355}} = \underline{5.6 < 8}$, OK

$\lambda_{web} = \frac{381-2\times16.3}{10.4}\sqrt{\frac{355}{355}} = \underline{\underline{33.5}}$

$\lambda_{web} > 24$, ∴ non-compact in compression (but OK in bending, <56)

K_c via DS4 for $\lambda = \frac{\cancel{24}}{335} = 0.78 \Rightarrow b_e = K_c \times b$ insert depth

$b = 381 - 2\times16.3 = 348.4\,mm \Rightarrow b_e = 0.78 \times \cancel{384.4} = \underline{\underline{271.8\,mm}}$



$A_{eff} = 70.1\times10^2 - 76.6\times10.4$
$= \underline{6213\,mm^2}$

$I_{eff} = 14870\times10^4 - \frac{10.4\times76.6^3}{12}$
$= \underline{\underline{148.3\times10^6\,mm^4}}$

(b) $N_{pl} = A_{eff}\times\sigma_y = 6213\times355 = \underline{2.206\,MN}$

For major axis buckling $N_{el} = \pi^2 E I_{eff}/L^2$

$= \pi^2\times210\times10^9\times148.3\times10^{-6}/10^2 = \underline{\underline{3.074\,MN}}$

$\bar\lambda = \sqrt{\frac{N_{pl}}{N_{el}}} = \sqrt{\frac{2206}{3074}} = \underline{\underline{0.85}}$

DS2 informs that curve (c) in DS1 must be employed for a channel section

From DS1, for $\bar\lambda = 0.85$, $\chi \simeq \underline{0.63}$

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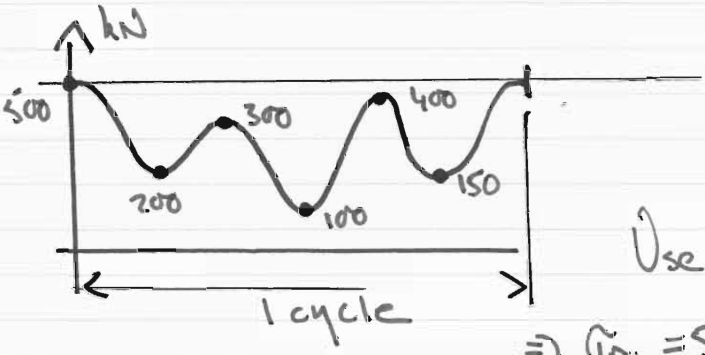
(b) contd Limiting axial force = $\chi N_M = 0.6 \times 2206 = 1.390\,MN$

This is larger than the 900kN applied, so the column can carry the load..

c)



$P_{r1} = 500 - 100 = 400\,kN$
$P_{r2} = 400 - 150 = 250\,kN$
$P_{r3} = 300 - 200 = 100\,kN$

Use actual area, $A = 70.1\,cm^2$

$\Rightarrow \sigma_{r1} = 57.1\,MPa$, $\sigma_{r2} = 35.7\,MPa$
$\sigma_{r3} = 14.3\,MPa$

For a class F weld; $m = 3$, $K_2 = 0.63 \times 10^{12}$, $\sigma_0 = 40\,MPa$

For $\sigma_r > \sigma_0$, $N\sigma_r^m = K_2 \Rightarrow N_1 = 3.384 \times 10^6$
$N_2 = 17.38 \times 10^6$

For $\sigma_r < \sigma_0$, $N\sigma_r^{m+2} = K_2\sigma_0^2 \Rightarrow N_3 = 1686 \times 10^6$

$\Rightarrow$ Miner's Law: $\Sigma N_T/N_i = 1$

$$\Rightarrow \frac{N_T}{3.384} + \frac{N_T}{17.38} + \frac{N_T}{1686} = 10^6$$

$$\Rightarrow N_T = 2.83 \times 10^6 \text{ cycles}$$

Comments: Popular & well done: but there were a spread of K_c values after improper/inaccurate reading of the graphs in datasheets: many interpreted the fatigue loading cycle as a stress instead of a force and used the effective area rather than the 'true' area for this calculation..

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(a) Perry - Robertson $(\sigma_y - \sigma)(\sigma_E - \sigma) = \eta \sigma \sigma_E$

$\div \sigma_y^2$ and define $\lambda^2 = \sigma_y / \sigma_E \Rightarrow (1-\bar{\sigma})\left(\frac{1}{\lambda^2} - \bar{\sigma}\right) = \eta \bar{\sigma} \cdot \frac{1}{\lambda^2}$

with $\bar{\sigma} = \sigma / \sigma_y$

Define $\phi = (1 + \eta + \lambda^2)/2$
and substitute into quadratic

$$\Rightarrow \quad \lambda^2 \bar{\sigma}^2 - 2\phi\bar{\sigma} + 1 = 0$$

$\Rightarrow$ roots $\bar{\sigma}_{1,2} = \left\{\phi \pm [\phi^2 - \lambda^2]^{1/2}\right\} / \lambda^2$

and their product $\bar{\sigma}_1 \bar{\sigma}_2 = 1/\lambda^2$ after multiplying out.
Therefore

$$\bar{\sigma}_{lowest} = \frac{1}{\lambda^2 \bar{\sigma}_{highest}} = \frac{1}{\lambda^2} \cdot \frac{\lambda^2}{[\phi + [\phi^2 - \lambda^2]^{1/2}]}$$

ie. $\bar{\sigma}_{lowest} = \sigma_{lowest} / \sigma_y = 1 / \left[\phi + [\phi^2 - \lambda^2]^{1/2}\right]$ (QED)

b) Rationale for P-R use in flexural buckling:
- uses 1st yield at extreme fibre as failure criterion
- assumes an initial imperfection
- it rounds or softens the transition between purely plastic yielding and elastic (perfect) buckling

observed in flex. buckling expts.

Shortcomings: no account of residual stresses; have to "fit" η to experimental data depending on section type and mode of manufacture.

For lateral torsional buckling:
- Rationale as above, but shortcomings are more sensitive since there are "two" imperfections with moving sideways and rotation; there are still residual stresses and the need to fit data with experiments for η.

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(c) $203 \times 203 \times 60$ UC, S275, $L = 2.8\,m$.

$$N_{pl} = A\,\sigma_y = \underbrace{(76.4 \times 10^2)}_{A,\ mm^2} \times 275\,N/mm^2 = \underline{\underline{2101\,kN}}$$

$$M_{pl} = Z_p \cdot \sigma_y = \underbrace{656 \times 10^{-6}}_{Z_{p,major},\ m^3} \times 275 \times 10^6\,N/m^2 = \underline{\underline{180.4\,kNm}}$$

$$\text{Web fraction, } a, = A_{web}/A_{sect} = \frac{\overbrace{9.4}^{t_{web}}(\overbrace{209.6}^{D} - 2 \times \overbrace{14.2}^{t_{flange}})}{\underbrace{76.4}_{A_{sect}}}$$

$$\Rightarrow \underline{\underline{a = 0.223}}$$

$$N_{cr} = N_{pl} \times a/2 = 2101 \times 0.1115 = \underline{234.2\,kN}$$

For purely elastic, axial buckling about both axes:

$$N_{euler,\ major} = \frac{\pi^2 E I_{xx}}{L^2} = \frac{\pi^2 \times (210 \times 10^9) \times (\overbrace{6125 \times 10^{-8}}^{I_{xx},\ m^4})}{2.8^2}$$

$$= \underline{\underline{16192\,kN}}$$

$$N_{euler,\ minor} = \frac{\pi^2 E I_{yy}}{L^2} = \frac{\pi^2 \times (210 \times 10^9) \times (2065 \times 10^{-8})}{2.8^2} = \underline{\underline{5459\,kN}}$$

For elastic-plastic interaction:

$$\text{major: } \lambda = \sqrt{\frac{N_{pl}}{N_{major}}} = \sqrt{\frac{2101}{16192}} = \underline{\underline{0.36}}$$

$$\text{minor: } \lambda = \sqrt{\frac{N_{pl}}{N_{minor}}} = \sqrt{\frac{2101}{5459}} = \underline{\underline{0.62}}$$

Using DS2: $h/b = \dfrac{209.6\ (D)}{205.6\ (B)} = 1.02 < 2$.

Since $t_{flange} = 14.2\,mm \leq 100\,mm$

$\Rightarrow$ major $\rightarrow$ $\underline{\underline{\text{use curve (b); minor (c) in DS1}}}$

$\Rightarrow$ $\underline{\underline{\chi = 0.94}}$ for major and $\underline{\underline{\chi = 0.78}}$ for minor

$$\Rightarrow N_{major} = 0.94\,N_{pl} = \underline{1975\,kN}$$
$$N_{minor} = 0.78\,N_{pl} = \underline{1639\,kN}$$

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(c) contd

$$M_{LT} = \underbrace{\frac{\pi}{L}\left[EI_{yy}GJ\right]^{1/2}}\cdot\overbrace{\left[1+\frac{\pi^2}{L^2}\cdot\frac{E\Gamma}{GJ}\right]^{1/2}}^{*}.$$

$$\frac{\pi}{2.8}\times\left[210\times10^9\times(2065\times10^{-8})\times81\times10^9\times\overbrace{(47.2\times10^{-8})}^{J,\,m^4}\right]^{1/2}$$

$$= \underline{\underline{456.9\,kN}}$$

$$* = \left[1+\frac{\pi^2}{2.8^2}\cdot\frac{210}{81}\times\frac{22.68}{47.2}\right]^{1/2} = 1.6026$$

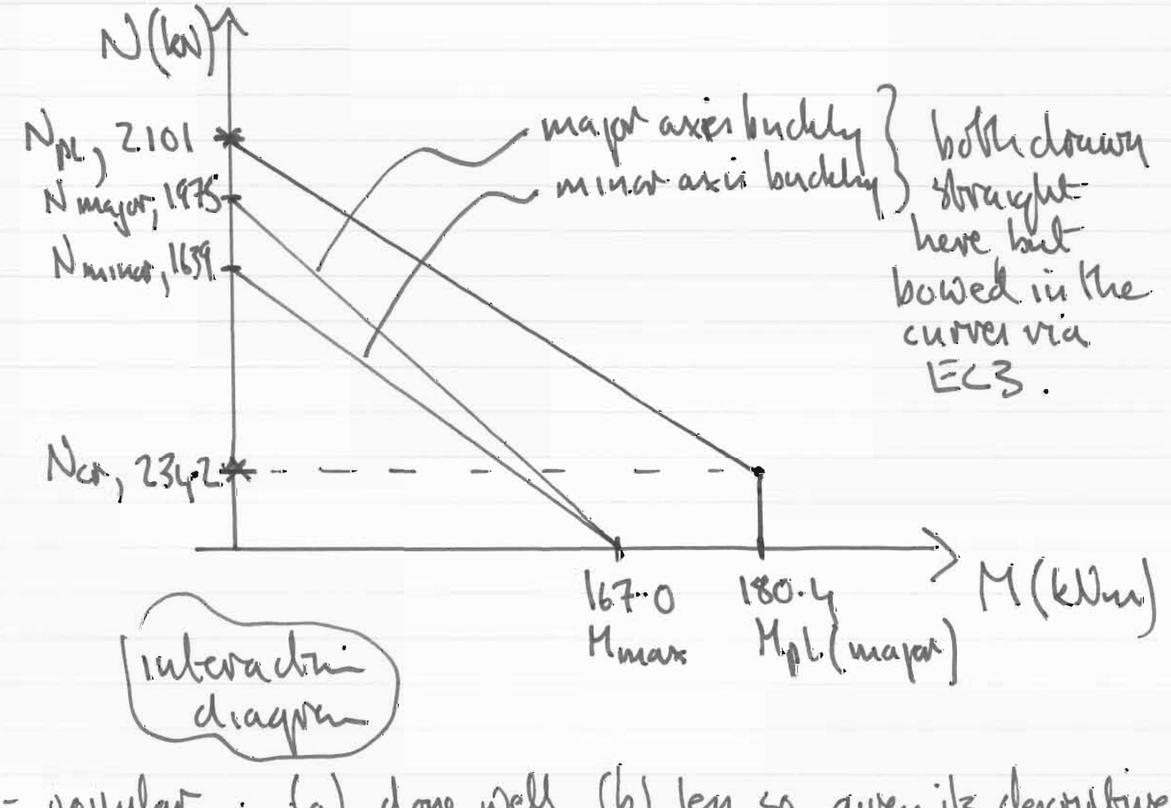
$I_{yy}D^2/4$

$$M_{LT} = 1.6026\times456.9 = \underline{\underline{732.2\,kNm}}$$

$C_{unequal} = 1$ (eq and opposite) $\lambda = \sqrt{\frac{M_{pl}}{M_{LT}}} = \sqrt{\frac{180.4}{732.2}} = \underline{\underline{0.5}}$

Since $h/b = 1.02 < 2 \Rightarrow$ curve (a) in DS1 $\Rightarrow \underline{\underline{\chi = 0.92}}$

$M_{max} = \chi M_{pl}$ (major axis) $= \underline{\underline{167.0\,kNm}}$



Comments: least popular: (a) done well, (b) less so given its descriptive nature! (c) done well but quite a few forgot to calculate all three curves.

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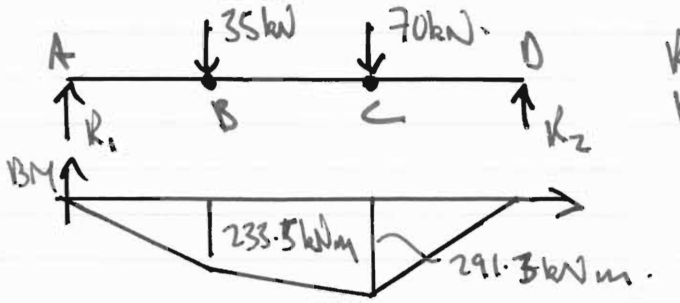
(a)



$R_1 = 46.7\text{kN}$
$R_2 = 58.3\text{kN}$

406×178×74 UB S275

$M_B = 46.7 \times 5 = \underline{233.5\text{ kNm}}$
$M_C = 58.3 \times 5 = \underline{291.5\text{ kNm}}$

Moment ratio in span BC (which we assume to be critical) is

$$\varphi = M_B/M_C = 0.8; \quad C_{unequal} = \max(0.6 + 0.4\varphi,\ 0.4)$$
$$(DS3) = (0.92,\ 0.4) = \underline{\underline{0.92}}$$

$$M_{LT}\ (DS3) = \underbrace{\frac{\pi}{L}\sqrt{E I_{yy} G J}}_{\text{basic}}\ \underbrace{\left[1 + \left(\frac{\pi}{L}\right)^2 \frac{E\Gamma}{GJ}\right]^{1/2}}_{\text{factor}}$$

$$M_{basic} = \frac{\pi}{5}\left[\underbrace{210\times10^9}_{E} \times \underbrace{1545\times10^{-8}}_{I_{yy},\,m^4} \times \underbrace{81\times10^9}_{G} \times \underbrace{62.8\times10^{-8}}_{J,\,m^4}\right]^{1/2}$$
$$= \underline{\underline{255.3\text{ kNm}}}$$

In factor, $$\Gamma = \frac{I_{yy} D^2}{4} = \frac{1545\times10^{-8} \times (\overbrace{0.4128}^{D \text{ via SDB}})^2}{4} = \underline{\underline{65.81\times10^{-8}\ m^6}}$$

$$\text{Factor} = \left[1 + \left(\frac{\pi}{5}\right)^2 \cdot \frac{210\times10^9 \times 65.81\times10^{-8}}{81\times10^9 \times 62.8\times10^{-8}}\right]^{1/2} = \underline{\underline{1.440}}$$

$$M_{LT} = 255.3\times10^3 \times 1.440 = \underline{367.5\text{ kNm}}$$

This is the elastic moment capacity in LTB for equal and opposite moments applied to the span: this is now used to find the actual capacity for uneven end moments above.

$$M_{pl} = Z_{pl} \times \sigma_y = \underbrace{1501\times10^3}_{\text{major } Z_p}\ mm^3 \times 275\ N/mm^2$$
$$= \underline{\underline{412.8\text{ kNm}}}$$

$$M_{cr} = M_{LT}/C_{unequal} = \frac{367.5}{0.92} = \underline{\underline{399.5\text{ kNm}}}$$

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(a) contd M_{cr} is the elastic capacity : the slenderness is

$$\lambda_{LT} = \sqrt{\frac{M_{pL}}{M_{cr}}} = \sqrt{\frac{412.8}{399.5}} = \underline{\underline{1.017}}$$

From DS2, $h/b = \frac{412.8}{179.5}$ (h being depth, b = flange total width)

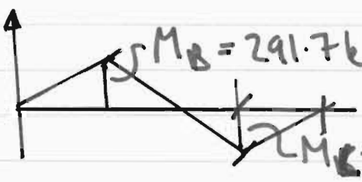
$= 2.3 > 2 \Rightarrow$ curve (b) via DS3

$\Rightarrow \underline{\chi \simeq 0.6}$ (DS1) $\Rightarrow M_{MAX}$ (elastic + plastic capacity)

$$= 0.6 \times \underset{\uparrow M_{pL}}{412.8} = \underline{247.7\,kNm}$$

This exceeds the moment M_c so the beam is NOT adequate for these loads.

(b) It can be shown $\underline{R_1 = -R_2 = -58.3\,kN}$



$\underbrace{\psi = 0, \quad C_{unequal} = 0.6}_{\text{end spans assumed to be critical.}}$

$$M_{cr} = \overset{\text{as before}}{M_{LT}} / C_{unequal} = \frac{367.5}{0.6} = \underline{612.5\,kNm}$$

$$\lambda_{LT} = \sqrt{\frac{M_{pL}\text{ as before}}{M_{cr}}} = \sqrt{\frac{412.8}{612.5}} = \underline{0.821}$$

Curve (b) again $\Rightarrow \underline{\chi \simeq 0.71}$ (DS1)

$$\Rightarrow M_{Max} = 0.71\,M_{pL} = \underline{\underline{293\,kNm}}$$

$M_{applied} = M_B$ or M_c as Maximum $= 291.7\,kNm$

So beam JUST adequate (but could be inadequate within margin of error in reading DS1, for χ)

Comments: Very popular & very well done. Some incorrect b.m. diagrams! Not acceptable at part IIB.