1

4F12 solutions (2012)

$$\left\{ \begin{array}{l} \cancel{I(\underline{x}+\underline{n}) \simeq I(\underline{x}) + \nabla I . \underline{n}} \\ \text{by Taylor Series Expansion} \end{array} \right.$$

1(a) $I(\underline{x}+\underline{n}) - I(\underline{x}) \simeq \nabla I . \underline{n} + O(\text{second})$

$$\therefore C(\underline{n}) \simeq \sum_W w(x)(\nabla I . \underline{n})^2 \propto \sum_W w(\underline{x}) \, I_n^2$$

where $I_n = \nabla I . \hat{\underline{n}}$ (4)

(b) $$C(\hat{\underline{n}}) = \sum_W w(x) \frac{(\underline{n}^T \nabla I^T)(\nabla I \underline{n})}{\underline{n}^T \underline{n}}$$

$$= \sum_W w(x) \frac{\underline{n}^T \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \underline{n}}{\underline{n}^T \underline{n}}$$

$$= \frac{\underline{n}^T A \, \underline{n}}{\underline{n}^T \underline{n}} \quad \text{where} \quad A = w(\underline{x}) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

"autocorrelation matrix"

$$= \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$$

where $\langle \; \rangle$ is the 2D smoothing by $w(\underline{x})$ = Gaussian blurr. $G_{\sigma_I}(\underline{x})$

$$w(x) * I_x^2 = \langle I_x^2 \rangle$$

(4)

(c) Must first smooth $S(x,y)$ and then differentiation, I_x and I_y.

$$S(x,y) = \sum_{-n}^{n} \sum_{-n}^{n} I(x-u, y-v) \, g_\sigma(u) \, g_\sigma(v)$$

$$I_x = S_x = \tfrac{1}{2}\left[S(x+1,y) - S(x-1,y)\right] \text{ etc.}$$ (4)

2

(d) Convolution with a larger σ_I for neighbourhood (4)
Convolve with 2D Gaussian $w(\underline{x}) = G_{\sigma_I}(\underline{x})$ and $\sigma_I > \sigma_D$
Separable exploited as 2, 1D convolutions — neighbourhood — derivatives

(e) Look at $\det A$ $(\lambda_1\lambda_2)$ and trace A $(\lambda_1+\lambda_2)$ since $\lambda_1 < C(\hat{\underline{n}}) < \lambda_2$
min dir $\underline{n}_1$ — max, dir $\underline{n}_2$

corners where $\dfrac{\det A}{1 + k \operatorname{trace} A} >$ threshold

(4)

Find maxima of $\det A - \alpha \operatorname{trace}^2 A = \lambda_1\lambda_2 - \alpha(\lambda_1+\lambda_2)^2$ $\alpha = 0.06$
[Harris-Stephens]

Examiner's comment:

Part (d) was incorrectly answered by most candidates - Difference between smoothing before differentiation (noise suppression) and the window size in the gaussian weighting function (scale of feature to be detected).

(2) (a)

(i) perspective $x = \frac{fX_c}{Z_c}$ in camera-coordinates of (X_c, Y_c, Z_c)

$y = \frac{fY_c}{Z_c}$ f is focal length.

In homogeneous coordinates

$$\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

where $x = \frac{sx}{s}$

$y = \frac{sy}{s}$

(ii) CCD scaling

$u = k_u x + u_0$

$v = k_v y + v_0$

(u_0, v_0) = principal pt

k_u pixels per unit length of f in x

k_v pixels per unit length of f in y

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(iii) Rigid-body motion

$$X_c = RX_w + T_w = \overset{\text{orientation}}{R}\, \underline{X}_w + \overset{\text{position}}{T}.$$

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} R & T \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Identify 11 parameters. $\underbrace{f, \overbrace{k_u, k_v, u_0, v_0}^{4}}_{\text{intrinsic}}$ $\underbrace{\overset{3}{R}, \overset{3}{T}}_{\text{extrinsic/external}} = 11.$

(8)

2(b)

2 eqns in unknown p_{ij}

$$0 = X_i\,(p_{11} - p_{31}u_i) + Y_i\,(p_{12} - p_{32}u_i) + Z_i\,(p_{13} - p_{33}u_i) + p_{14} - p_{34}u_i$$

$$0 = X_i\,(p_{21} - p_{31}v_i) + Y_i\,(p_{22} - p_{32}v_i) + Z_i\,(p_{23} - p_{33}v_i) + p_{24} - p_{34}v_i$$

Can be rewritten as

$$\overset{2n\times 12}{A}\;\overset{n\times 1}{p} = 0 \qquad \text{where } p = \overset{12\times 1}{\begin{pmatrix} p_{11} \\ \vdots \\ p_{34} \end{pmatrix}}$$

(3)

2(c) Non-coplanar, $N \geq 6$, span ΔZ + field of view. Good features to localize for accuracy + matching over views.

Estimate p_{ij} by least squares. Non-linear Optimisation to min S.S.E.
(Bundle adjustment)

Find p_{ij} which minimise

$$\sum_i \| \underbrace{x_i}_{\text{measurement}} - \underbrace{P X_i}_{\text{projection by model}} \|^2 .$$

(5)

2(d) Weak Perspective:

$$x = \frac{fX}{Z_0} \qquad \text{so a uniform scaling}$$

$$y = \frac{fY}{Z_0}$$

$$\therefore \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} & K & \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & Z_0 \end{bmatrix} \qquad \begin{bmatrix} u \\ v \end{bmatrix} = 2\times 4 \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}.$$

Calibrate with fewer pts (4) + linear equations. Least-squares is optimal in this case.

5

(3)(a)

$$\underline{u} = K \overset{3\times 4}{[I \mid 0]} \underset{\sim}{\tilde{X}} \quad \text{(world pt.)} \qquad \therefore \quad \underline{X} = K^{-1}\underline{u} \quad \text{where } \underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\underline{u}' = K' \overset{3\times 4}{[R \mid 0]} \underset{\sim}{\tilde{X}} \quad = K' R \, \underline{X}$$

$$\therefore \quad \underline{u}' = \underbrace{K' R K^{-1}}_{H \,=\, 3\times 3 \text{ transformation}} \underline{u}.$$

(4)

(b)(i) 4 (2)

(ii) RANSAC. Random sample

Estimate H parameters (homography t_{ij}).

Check for inliers by euclidean distance between $\underline{u}'$ and $H\underline{u}$

accept if ~~large~~ #.

(4)

(iii) Least-squares (SVD) $\quad A\underline{h} = 0$

normalize data to zero mean and variance 1

$$u' = \frac{h_{11}u + h_{12}v + h_{13}}{h_{31}u + h_{32}v + h_{33}}$$

$$v' = \frac{h_{21}u + h_{22}v + h_{23}}{h_{31}u + h_{32}v + h_{33}}$$

(4)

Write as linear in h_{ij} and stack up 2 eqⁿ per image pt.

3(c).

Consider a conic C such that rep by square, positive-definite matrix.

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

Under transformation $\underline{u}' = H \underline{u}$

$$\therefore \quad \underline{u} = H^{-1} \underline{u}'$$

$$\therefore \quad \underline{u}'^T \underbrace{H^{-T} C}\, H^{-1} \underline{u}' = 0$$

$$\underbrace{C' = H^{-T} C H^{-1}}$$

Conic becomes a conic.
For example if circle of radius a

$$u^2 + v^2 - a^2 = 0$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -a^2 \end{bmatrix}$$

$$[u \; v \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -a^2 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \quad u^2 + v^2 - a^2 = 0$$

$\therefore$ New conic will be $C' = H^{-T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -a^2 \end{bmatrix} H^{-1}$ (6).

Examiner's comment:
Part c - transformation of a conic - was poorly answered. Rest was well-attempted.

Q4

(a) $\underset{\text{left projection}}{\tilde{\underline{u}}} = \overset{3\times3}{K}\,\overset{3\times4}{[I\,|\,0]}\,\tilde{\underline{X}}$ where $\tilde{X} = \overset{4\times1}{\begin{pmatrix} \underline{X} \\ 1 \end{pmatrix}}$

$\underset{\text{right projection}}{\tilde{\underline{u}}'} = K'[R\,|\,T]\,\tilde{\underline{X}}$ where $\underline{X}' = R\underline{X} + T$

$\therefore P_L = K[I\,|\,0]$

$P_R = K'[R\,|\,T]$ (4)

(b) Each world pt, $\underline{X}$ projects to (u,v) and (u',v').
For each image we have 2 eqns in (X,Y,Z) unknowns

$\therefore$ 4 equations in 3 unknowns (X,Y,Z)

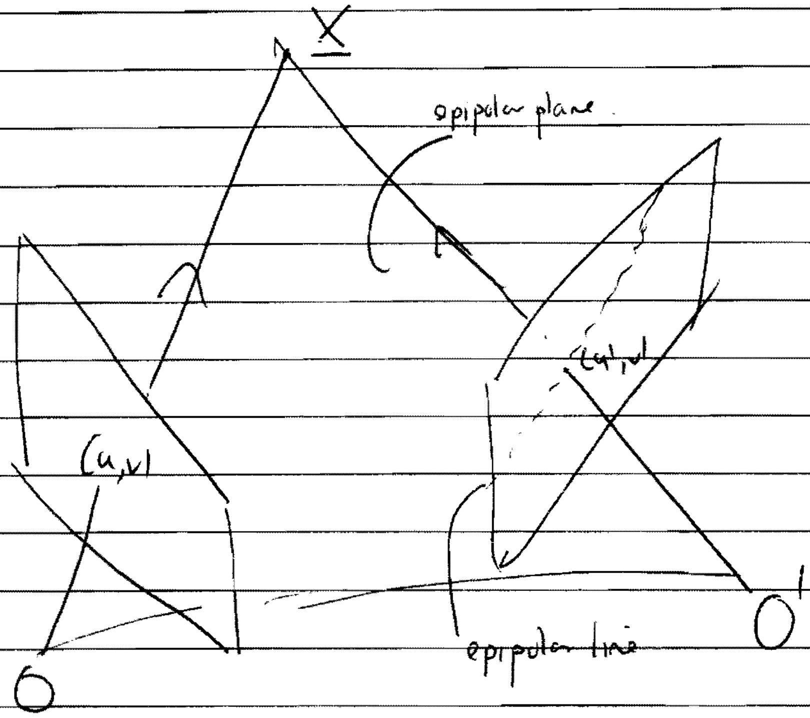
$$\overset{4\times3}{\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \underline{b}$$

where each row is equation of plane (coefficients) given by u, v, u', v'.

Solve by pseudo least-squares $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [A^TA]^{-1} A^T \underline{b}$

(6)

(c) 4 planes define 2 rays (1 for each view). These must intersect at 3D pt, $\underline{X}$



(c) Baseline and 2 rays are coplanar — in epipolar plane. $[\underline{X}, \underline{X}', \underline{T}] = 0$
Corresponding epipolar lines are intersection of plane and image planes.

(d) $$\underline{X}' = R\underline{X} + \underline{T}$$

$$\underline{T}_\wedge X' = T_\wedge R \underline{X}$$

$$\underline{X}' \cdot (\underline{T}_\wedge \underline{X}) = 0 = \underline{X}' [\underline{T}_\times R] \underline{X} = \underline{X}'^T \overset{E}{[T_\wedge R]} \underline{X}$$

Now $\underline{X}' \,//\, \underline{p}' \,//\, K'^{-1} \underline{u}'$ and $\underline{X} \,//\, \underline{p} \,//\, K^{-1} \underline{u}$

$$\therefore \quad \underline{\tilde{u}}'^T \underbrace{K'^{-T} [T_\times R] K^{-1}}_{3 \times 3 \text{ matrix}, F} \underline{\tilde{u}} = 0$$

(6)

9

Examiner's comment:

This question tested the candidates understanding of stereo vision. Poor understanding of triangulation (a) to recover depth and the b/g to the epipolar constraint (b). Rest was well answered.