ENGINEERING TRIPOS PART IIB

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Friday 27 April 2012 2.30 to 4

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Module 4A3

# TURBOMACHINERY 1

*Answer not more than **two** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments: Compressible Flow Data Book (38 Pages)*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

nra06

1 (a) For an axial flow turbine with repeating stages, derive the relationship

$$\psi = 2(1 - \Lambda - \phi \tan\alpha_1)$$

relating the stage loading coefficient $\psi$, the reaction $\Lambda$, the flow coefficient $\phi$ and the interstage swirl angle $\alpha_1$. [25%]

(b) A high pressure steam turbine is to have an overall enthalpy drop of $350\ \mathrm{kJ\,kg^{-1}}$. It is composed of repeating stages each with a mean blade speed $180\ \mathrm{m\,s^{-1}}$.

(i) One design features 50% reaction, a flow coefficient of 0.35 and no interstage swirl. Calculate the number of stages required to achieve the enthalpy drop. [10%]

(ii) Another design is proposed with 30% reaction, a flow coefficient of 0.3 and $-22°$ interstage swirl. Calculate the number of stages required to achieve the enthalpy drop in this case. [10%]

(c) The total-to-total efficiency of each stage can be estimated from

$$\eta_{tt} = 1 - 0.05\left(V_2^2 + V_{3rel}^2\right)/\Delta h_0$$

where $V_2$ is the absolute velocity at exit from the stator, $V_{3rel}$ is the relative velocity at exit from the rotor and $\Delta h_0$ is the change in specific total enthalpy across the stage. Draw the velocity triangles for each case described in part (b) in order to compare both the total-to-total and the total-to-static efficiencies of the two designs. [30%]

(d) It may be assumed that the polytropic efficiency of the expansion through the whole turbine is the same as the stage total-to-total efficiency calculated above. Using this assumption, calculate the stagnation pressure ratio and the total-to-total isentropic efficiency of the whole turbine for the 50% reaction design of part (b)(i). The turbine inlet stagnation temperature is 540 °C and steam may be treated as a perfect gas with $\gamma = 1.3$ and $c_p = 1800\ \mathrm{kJ\,kg^{-1}}$. [25%]

2 (a) A two-dimensional compressor cascade is tested with an inlet Mach number of 0.68 and an inlet flow angle of 42°. At this operating point the exit flow angle is 18°. The inlet stagnation pressure is 1 bar and the inlet stagnation temperature is 300 K. The stagnation pressure loss coefficient is given by

$$\frac{p_{01} - p_{02}}{p_{01} - p_1} = 0.035$$

where $p_{01}$ is the inlet stagnation pressure, $p_{02}$ is the exit stagnation pressure and $p_1$ is the inlet static pressure.

Determine the mass flow rate per unit frontal area at the cascade inlet, the stagnation pressure ratio across the cascade, the exit Mach number and the static pressure ratio across the cascade. [30%]

(b) The blade profiles in the cascade of part (a) are to be used in the first rotor of an axial compressor which has no absolute swirl at inlet. The rotational speed is such that the relative inlet Mach number and relative inlet flow angle are the same as in the cascade test. The absolute inlet stagnation temperature is 300 K. Show that at this operating condition, the blade speed at rotor inlet is approximately 154 $\mathrm{m\,s^{-1}}$ and that the relative inlet stagnation temperature is approximately 312 K. [25%]

(c) The stagnation pressure loss coefficient of the rotor in part (b) again equals 0.035 when evaluated in the relative frame of reference. The mean radius is constant through the rotor. Determine the absolute flow angle at rotor exit. Sketch the velocity triangles at inlet to and exit from the rotor. [25%]

(d) The rotor is part of a repeating stage and the mean radius is also constant through the stator. Determine the degree of reaction of the stage. [20%]

In all parts of the question, use $c_p = 1005\ \mathrm{J\,kg^{-1}K^{-1}}$, $\gamma = 1.4$, and $R = 287\ \mathrm{J\,kg^{-1}K^{-1}}$ for the air.

3 (a) Sketch the way in which the pressure ratio changes with the non-dimensional mass flow and the non-dimensional speed for a multistage high-speed axial compressor. Explain how and why the shape of the constant non-dimensional speed lines changes with the magnitude of the non-dimensional speed. [20%]

(b) For a multistage axial compressor, explain how and why the non-dimensional speed affects the axial location of stall. [20%]

(c) A compressor is tested with a fixed area nozzle attached to its exit duct. At the design operating conditions, the mass flow rate through the compressor is $80\,\mathrm{kg\,s^{-1}}$, and the stagnation pressure ratio is 12. Calculate the compressor pressure ratio at a mass flow of $50\,\mathrm{kg\,s^{-1}}$. You may assume that the compressor has a constant polytropic efficiency of 88%. [30%]

(d) Show that the working line of a single shaft turbo-jet can be approximated by

$$\frac{\dot{m}\sqrt{c_p\,(T_{03}-T_{02})}}{p_{03}} = constant$$

where $T_{02}$ is the stagnation temperature at the compressor inlet, $p_{03}$ and $T_{03}$ are the stagnation pressure and temperature at combustor inlet and $\dot{m}$ is the compressor mass flow rate. All of the shaft power extracted by the turbine is used to drive the compressor. Explain any approximations made and the validity of each. [30%]

**END OF PAPER**