ENGINEERING TRIPOS PART IIB

Monday 30 April 2012 2.30 to 4

Module 4A11

TURBOMACHINERY II

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

lx08

1 (a) Starting from the Meridional Streamline Curvature equation

$$V_m \sin\phi \frac{\partial V_m}{\partial m} + \frac{V_m^2}{R_m}\cos\phi - \frac{V_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{F_r}{\rho}$$

derive the Simple Radial Equlibrium (SRE) equation

$$V_x \frac{\partial V_x}{\partial r} + \frac{V_\theta}{r}\frac{\partial (rV_\theta)}{\partial r} = 0$$

Explain any assumptions used. [20%]

(b) A low speed, axial flow, single stage air turbine test rig has cylindrical casing and hub lines with a hub-to-tip ratio of 0.6 and a rotor tip diameter of 1.0 m. The flow is axial and uniform at the stage inlet and exit. The inlet velocity is $50.0\,\mathrm{ms}^{-1}$ and the flow leaving the stator has a free vortex design, with the tangential velocity at the hub being $75.0\,\mathrm{ms}^{-1}$. The stator aerodynamic loss is constant across the span, the density of the air is $1.2\,\mathrm{kgm}^{-3}$. The stage reaction at the midspan is 0.5 .

(i) Justify the validity of SRE in the axial gap between the stator and the rotor. Using the SRE equation, estimate the pressure and the axial velocity differences between the casing and hub in the axial gap between the stator and the rotor. [40%]

(ii) Using the information obtained in (i) above, or otherwise, estimate the stage reaction at the hub and at the casing respectively. [20%]

(c) It is known that a free vortex design can lead to a low stage reaction at the hub. Explain why this is the case and comment on the associated challenges for the design of the rotor hub section. How can stator lean be used to increase stage reaction at the hub? [20%]

2 (a) Comment on the benefits of high-speed turbomachinery. Explain why it is also desirable to design turbomachinery blade profiles that avoid unnecessary accelerations and keep isentropic surface velocities as low as possible. [20%]

(b) Two 2-dimensional turbine blade sections are designed to perform the same aerodynamic duty, i.e., to have the same aerodynamic loading. One is designed to be front-loaded and the other is rear-loaded. The loading pattern can be approximated as trianglular as shown by the isentropic surface velocity distributions in Fig. 1, where V_1 and V_2 are the blade inlet and exit velocities respectively. The blade axial chord is C_x. Gas enters the blade section axially at $30.0\,\mathrm{ms}^{-1}$ and at the exit it has a swirl angle of $70.53°$ measured from the axial direction. Estimate the ratio of the lost work due to the surface shear stress on the front-loaded blade as compared to the rear-loaded blade. [50%]

(c) Comment, using sketches of surface velocity distributions where appropriate, on the likely tolerance to incidence of the front-loaded and rear-loaded blades shown in Fig. 1. Describe, with reasoning, how three-dimensional blade design can be used to increase incidence tolerance in the endwall regions. [30%]

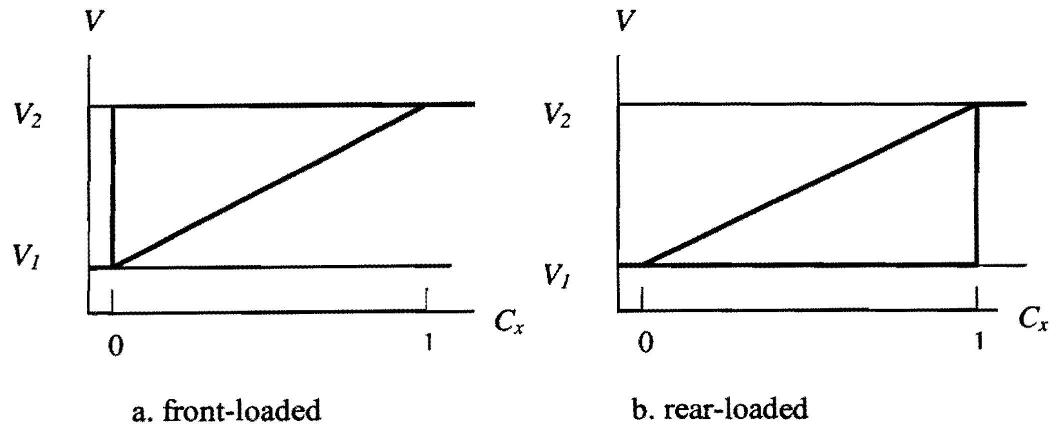

Fig. 1

(TURN OVER

3 (a) Consider the elementary tip clearance mixing loss model

$$Tds = V_s^2\left(1-\frac{V_p}{V_s}\right)\frac{d\dot{m}_j}{\dot{m}}$$

where V_s and V_p are the free stream velocities close to the suction and pressure surfaces near the tip section, $d\dot{m}_j$ the elementary mass flow rate across the blade tip over a small chord distance dz, and $\dot{m}$ the total mass flow rate through the blade passage. Show that the loss coefficient based on the entropy produced due to the mixing of the tip leakage flow with the main blade passage flow of an unshrouded turbine rotor blade can be estimated as:

$$\zeta = \frac{2C_d gC}{hp\cos\alpha_2}\int_0^1\left(\frac{V_s}{V_2}\right)^3\left(1-\frac{V_p}{V_s}\right)\sqrt{\left(1-\frac{V_p^2}{V_s^2}\right)}\,d\left(\frac{z}{C}\right)$$

where z is the chordwise direction, g the tip clearance gap, h the blade height, p the blade pitch, C the blade tip section chord, α_2 and V_2 are the exit flow angle and velocity respectively and C_d the discharge coefficient. State the assumptions involved in the derivation. [30%]

(b) (i) Describe the three-dimensional aspects of the flowfield that are found at the outlet of a typical turbine blade row without tip leakage. Compare these flow features with those expected at the outlet of a compressor blade row without tip leakage. Use sketches of relative stagnation pressure contours on a plane immediately downstream of the trailing edge to aid your descriptions. [25%]

(ii) How do these flowfields change when the blade rows are unshrouded rotors with a typical tip clearance? [15%]

(iii) Explain how a compressor blade with a small tip clearance may have a lower loss than one with zero tip clearance. [15%]

(iv) State, with reasons, how the shape of an unshrouded turbine rotor blade could be modified to reduce the tip leakage loss. [15%]

END OF PAPER