ENGINEERING TRIPOS PART IIB

Friday 27 April 2012 9 to 10.30

Module 4A15

AEROACOUSTICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A15 data sheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

aa05

4A15, 2012, Answers

Q1
(a) –
(b) –
(c) –

Q2
(a) –
(b) –

Q3
(a) –
(b) –
(c) –

Q4
(a) Porosity = 0.1
(b) Hole diameter = 0.0177 m.

1 A source S of spherical waves of angular frequency ω is located at the centre of a helium-filled spherical balloon of radius a as shown in Fig. 1. The balloon is surrounded by air. The values of density and speed of sound are respectively, ρ_1 and c_1 inside the balloon and ρ_2 and c_2 outside the balloon. The incident, reflected, and transmitted waves may be represented respectively by

$$p_i'(r,t) = \frac{A}{r}\exp[\mathrm{i}\omega(t-(r-a)/c_1)]$$
$$p_r'(r,t) = \frac{B}{r}\exp[\mathrm{i}\omega(t+(r-a)/c_1)]$$
$$p_t'(r,t) = \frac{C}{r}\exp[\mathrm{i}\omega(t-(r-a)/c_2)]$$

where r denotes the radial distance and t the time.

(a) Show that the corresponding radial velocity fields are given by

$$v_i'(r,t) = \frac{1-\mathrm{i}c_1/(\omega r)}{Z_1}p_i'(r,t)$$

$$v_r'(r,t) = -\frac{1+\mathrm{i}c_1/(\omega r)}{Z_1}p_r'(r,t)$$

$$v_t'(r,t) = \frac{1-\mathrm{i}c_2/(\omega r)}{Z_2}p_t'(r,t)$$

where $Z_1 = \rho_1 c_1$ and $Z_2 = \rho_2 c_2$. [20%]

(b) By applying the appropriate boundary conditions on the surface of the balloon, show that the pressure reflection coefficient is given by

$$R = \frac{p_r'(a,t)}{p_i'(a,t)} = \frac{(1-Z_1/Z_2)-\mathrm{i}\alpha_1(1-\rho_1/\rho_2)}{(1+Z_1/Z_2)+\mathrm{i}\alpha_1(1-\rho_1/\rho_2)}$$

where $\alpha_1 = c_1/(\omega a)$. [60%]

(c) Use this result to show that for a compact balloon the transmitted pressure is small in comparison with p_i' and p_r' on $r=a$. [20%]

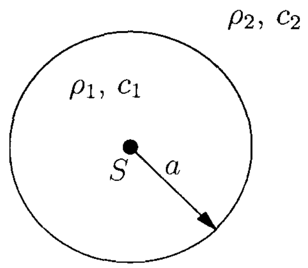

Fig. 1

2 Consider a low Mach-number, isentropic cold turbulent jet.

(a) By finding the sound radiated from a single compact eddy, show that the acoustic far-field density from the eddy scales as

$$\rho' \sim \rho_0 \frac{x_i}{x} \frac{x_j}{x} \frac{l}{x} m^4$$

where ρ_0 is the ambient density, x is the distance from the source to the observer, x_i represents the component of $\mathbf{x}$ in the i-direction, l is the length scale of the eddy, $m = u'/c_0$, u' is the velocity scale of the eddy and c_0 represents the speed of sound. [60%]

Hint: $$\frac{\partial^2}{\partial x_i \partial x_j} \int T_{ij}\left(\mathbf{y}, t - \frac{x}{c_0}\right) \mathrm{d}\mathbf{y} = \frac{1}{c_0^2}\left(\frac{x_i x_j}{x^2}\right) \frac{\partial^2}{\partial t^2} \int T_{ij}\left(\mathbf{y}, t - \frac{x}{c_0}\right) \mathrm{d}\mathbf{y}$$

(b) Use this result to deduce an expression for the acoustic power radiated by the jet (Lighthill's eighth power law). [40%]

3 The acoustic pressure for a plane sound wave with an angular frequency ω and wavenumber k_0 is given by

$$p' = p_i \exp(\mathrm{i}\omega t - \mathrm{i}k_0 y)$$

The sound wave is incident on to a thin rigid plate, which is between $\{-L < x < L, y = 0\}$, with $k_0 L \gg 1$.

(a) Calculate the acoustic pressure along the line $x = 0$, $y > 0$. [40%]

(b) Sketch a graph of the variation of the acoustic pressure with x along the lines $y = -H$ and $y = +H$, where $H \gg L$. (detailed calculations are not required) [40%]

(c) Explain briefly how your answer to (b) would change if the thin rigid plate were replaced by a rigid circular cylinder of radius L. [20%]

4 (a) A Helmholtz resonator has a bulb of volume V, a neck of cross-sectional area A and effective length l as shown in Fig. 2. Viscous and flow separation effects cause a pressure drop of $\rho_0 c_0 \alpha u_1'(t)$ across the neck of the Helmholtz resonator, where ρ_0 is the mean density, c_0 is the speed of sound, $u_1'(t)$ is the unsteady air velocity into the neck of the Helmholtz resonator and α is a constant. Show that for perturbations of frequency ω, $u_1'(t)$ is related to $p_1'(t)$ the pressure perturbation at the neck of the Helmholtz resonator by

$$p_1'(t) = \rho_0 u_1'(t) \left(\frac{c_0^2 A}{V i\omega} + l i\omega + c_0 \alpha \right)$$

[40%]

(b) An aeroengine acoustic liner has circular holes of radius a, porosity (i.e. open area ratio) β and a honeycomb structure with cavity depth d as shown in Fig. 3. Each cavity within the honeycomb can be treated as a Helmholtz resonator as modelled by the above equation with $\alpha = 0.1$. The walls of the honeycomb are rigid but very thin and the effective neck length of each hole is $1.2a$. If $d = 0.05$ m, what porosity and hole radius would you choose to absorb all the energy in a normally incident plane sound wave of frequency 750 Hz? Take $\rho_0 = 1.2$ kg m^{-3} and $c_0 = 343$ ms^{-1}. [60%]

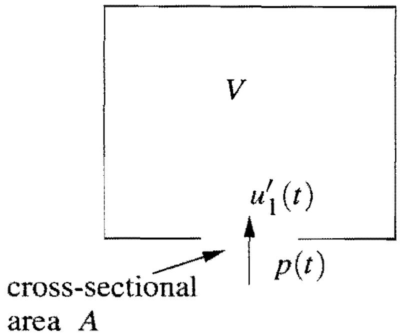

Fig. 2

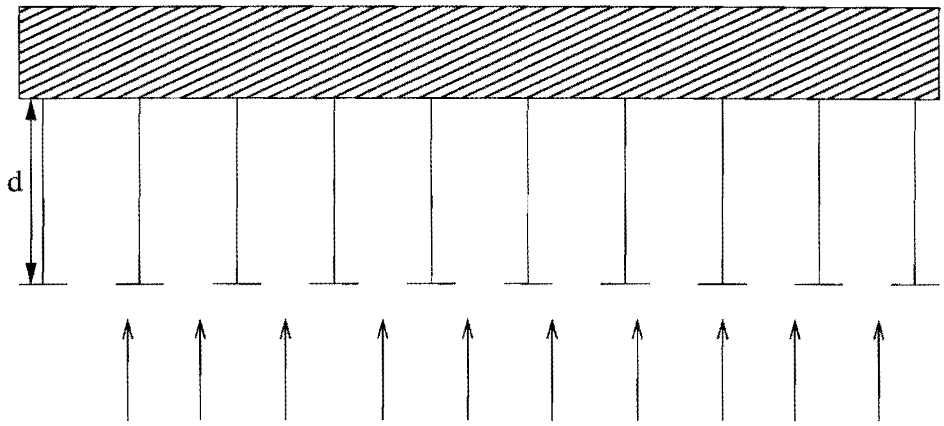

Fig. 3

END OF PAPER

4A15, 2012, Answers

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