ENGINEERING TRIPOS PART IIB

Monday 23 April 2012 9 to 10:30

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator

cd04

1 (a) Explain, with the aid of three examples, how it became clear that classical, Newtonian Mechanics was flawed and how this led to the development of Quantum Mechanics. Your answer should include a discussion of the key differences between both frameworks.

[40%]

(b) Using the correspondence principle, deduce the time-dependent and then the time-independent Schrödinger equations. In what way is the time-dependent equation different to the conventional wave-equation, and how is this manifest in the solutions? What is the physical significance of the solutions of the Schrödinger equation, and how do we use them to describe real systems?

[60%]

2 (a) Describe, with the aid of energy diagrams, the basic principle of operation of the Scanning Tunnelling Microscope (STM). [25%]

(b) Derive an *approximate* formula for the variation of tunnel current with voltage and tip-sample distance in an STM. Using this formula, calculate the proportion by which the current will change if the tip-sample distance is varied by 0.01 nm, assuming the tip and sample both have a work-function of 4 eV.

[30%]

(c) Taking the result of part (b) into account, what are the main design considerations that need to be taken in to account when constructing an STM? Discuss the requirements on stability of an STM and ways in which these may be achieved. [20%]

(c) How may STM be used to measure the variation with energy of the density of electronic states, ρ, of a sample? Derive an approximate formula for ρ in terms of the tunnel current, I and bias voltage, V. [25%]

3 (a) Describe, with the aid of diagrams, the basic principle of operation of the Atomic Force Microscope (AFM). Include a description of the three modes of operation that are most commonly used, and describe how each mode works and when it might be used. [40%]

(b) The major limitations of the AFM are (i) the time required to take an image and (ii) the maximum achievable scan area. Discuss how these issues are being addressed.

[30%]

(c) The thermal energy present in all systems at temperature T is $\frac{1}{2}k_B T$ per degree of freedom, where k_B is Boltzmann's constant. Calculate the amount by which an AFM cantilever of stiffness 0.001 N/m will be vibrating normal to the surface when in air at room temperature due to this thermal energy. What would happen if the cantilever had a stiffness of 50 N/m? Given this result, explain why there are still many cases when soft cantilevers are used in AFM experiments. [30%]

4 (a) The band-gap of GaAs is 1.45 eV, and of $Ga_{1-x}Al_xAs$ is (1.424 + 1.247x) eV. For the case of 70% substitution of Al atoms for Ga atoms, sketch the potential as seen by an electron in a 25 nm wide quantum well formed using these materials, indicating which material is used, and where. [20%]

(b) Assuming that the electron and hole effective masses in both materials are 0.067 and 0.45 times the free-electron mass, respectively, estimate the energy of the electron and hole ground states in the well. Which assumptions are used in arriving at your estimate, and at which point would they start to break down? [50%]

(c) Sketch the wave-function and the probability density for an electron in the ground state and first excited state of this quantum well. Determine the value of the characteristic decay length of an electron outside the well, assuming the electron is in the ground state. Discuss how to determine the probability of finding the electron outside the well. [30%]

(TURN OVER

5 Electrons at an energy E are incident from the left on a one-dimensional potential step of height V, which is 1 eV less than E, as shown below in Fig. 1.

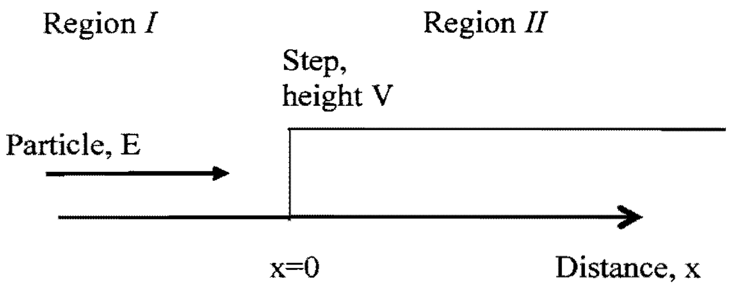

Fig. 1

(a) Using Schrödinger's time-independent equation, derive the single-particle wave-functions for regions *I* and *II*. Hence calculate the Transmission coefficient, T, for this potential step. [40%]

(b) Now consider the case where the potential step does not extend to infinity, but only to a distance d, as shown below in Fig. 2. Derive the wave-functions and sketch the probability densities in the three regions. Calculate the transmission probability for the case where d = 1 nm, V = 4 eV, and the electron's effective mass is the free electron mass. [60%]

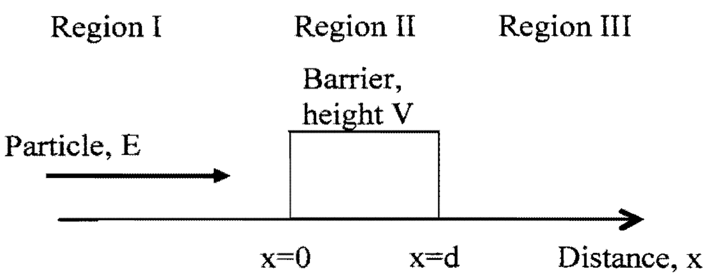

Fig. 2

END OF PAPER

Engineering Tripos, Part IIB 2012

Paper 4B5, Nanotechnology

Numerical Solutions

2(b) 0.82
3(c) 2.04 nm, 9 pm
4(b) 8.9 meV, 1.3 meV
5(b) T = 0.58