ENGINEERING TRIPOS PART IIB

Thursday 26 April 2012 2.30 to 4.00

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C6 Advanced Linear Vibration data sheet (10 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

JW04

1 Figure 1(a) shows a mass on a spring with damping. An instrumented hammer fitted with a piezo force transducer is used to provide force excitation, and motion is measured with a piezo accelerometer. Two charge amplifiers convert charge to voltage and two low-pass filters are set to reject data above 75 Hz. The signals are digitised by a computer at a sampling rate of 1000 Hz.

(a) The dotted lines in Figs. 1(b) and 1(c) show the force and acceleration data as measured by the computer. The solid lines show the data as observed on an oscilloscope at the output of the charge amplifiers. List the differences between the measured and observed data and explain how they may have arisen. [40%]

(b) The transfer function "acceleration/force" is computed and shown as a dotted line in Fig. 1(d) while the expected theoretical transfer function is shown as a solid line. Explain the differences between computed and ideal transfer functions. [30%]

(c) The mass of the system under test is 1 kg. Estimate the stiffness and damping. [30%]

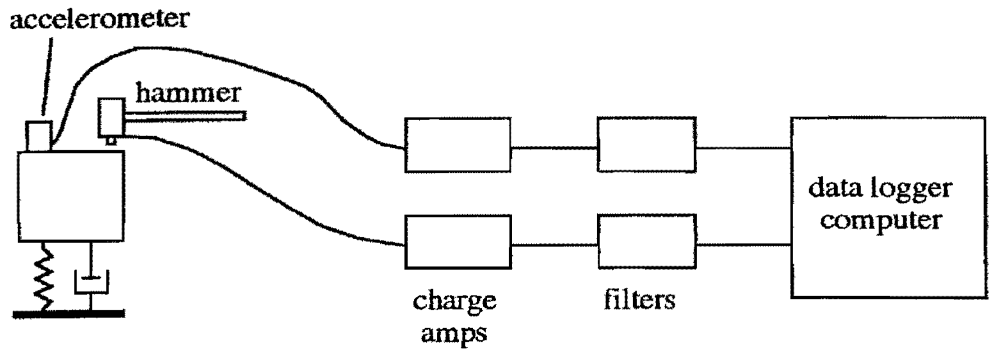

Fig. 1(a)

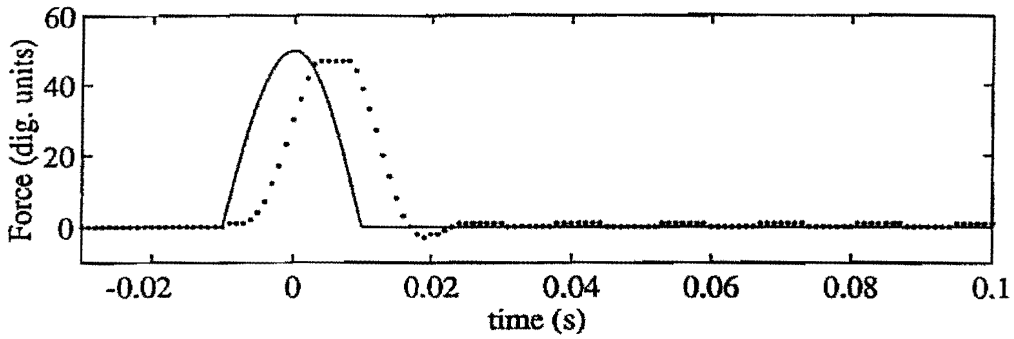

Fig. 1(b)

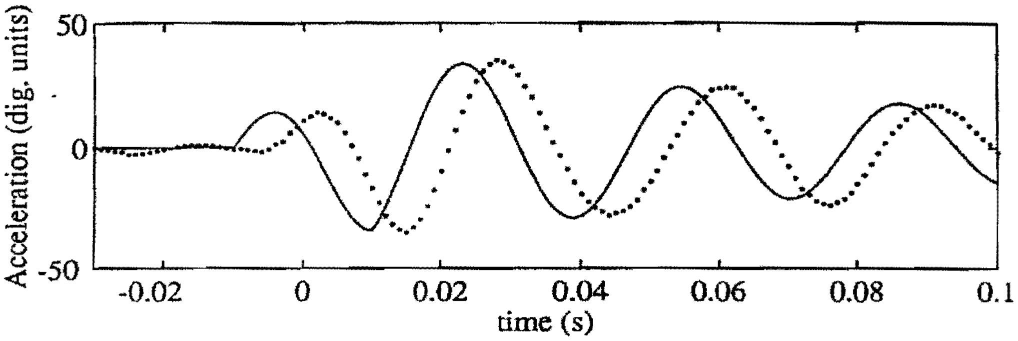

Fig. 1(c)

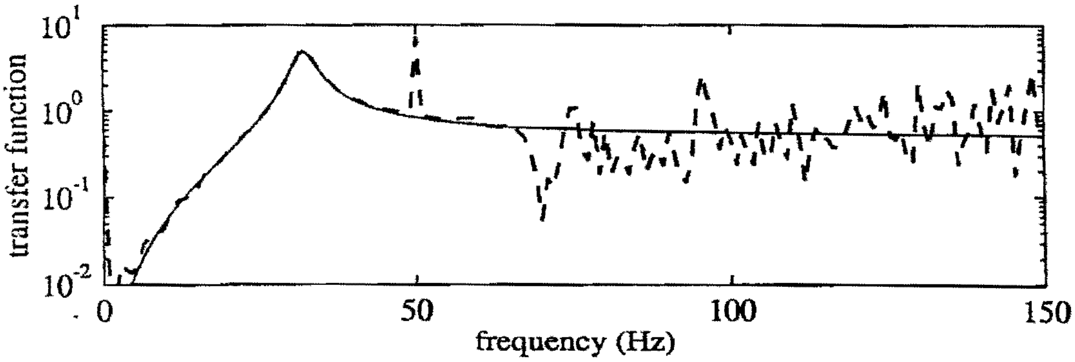

Fig. 1(d)

2 The differential equation of motion of a rectangular plate has the form

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+m\frac{\partial^2 w}{\partial t^2}=0$$

where D is the flexural rigidity, m is the mass per unit area, and $w(x,y,t)$ is the out-of-plane displacement at the point (x,y). The plate is of length L_1 in the x-direction and L_2 in the y-direction.

(a) Show that a separable solution in the form

$$w(x,y,t)=X(x)Y(y)e^{i\omega t}$$

is possible only if *at least one* of the following equations is satisfied

$$\frac{X''}{X}=a_1\,, \qquad \frac{Y''}{Y}=a_2$$

where a_1 and a_2 are constants. [25%]

(b) Assuming that $X''/X=-k_1^2$, where k_1 is a real constant, derive the differential equation that governs Y. [25%]

(c) If the plate has simply supported boundary conditions such that

$$X(0)=X''(0)=X(L_1)=X''(L_1)=Y(0)=Y''(0)=Y(L_2)=Y''(L_2)=0$$

show that a solution of the form

$$X=\sin\left(\frac{p\pi x}{L_1}\right), \qquad Y=\sin\left(\frac{q\pi y}{L_2}\right)$$

can satisfy the differential equation and boundary conditions for both X and Y. Derive an expression for the natural frequencies of the plate. [25%]

(d) Describe how the natural frequencies would be affected by a mass placed at the centre of the plate. [25%]

3 The transfer function of displacement per unit force (receptance) at a point A on an undamped three degree of freedom system is written as

$$Y_1(\omega) = \sum_{n=1}^{3} \frac{a_n^2}{\omega_n^2 - \omega^2}$$

where a_n are constants and ω_n is the nth natural frequency. Similarly the receptance at a point B on an undamped two degree of freedom system is written as

$$Y_2(\omega) = \sum_{n=1}^{2} \frac{b_n^2}{\alpha_n^2 - \omega^2}$$

where b_n are constants and α_n is the nth natural frequency.

(a) The two systems described above are joined by rigidly connecting point A to point B. Show that the natural frequencies of the coupled system are given by the roots of the equation

$$\frac{1}{Y_1(\omega)} + \frac{1}{Y_2(\omega)} = 0$$ [15%]

(b) If $\omega_1 < \alpha_1 < \omega_2 < \alpha_2 < \omega_3$ demonstrate by graphical means that the natural frequencies of the coupled system conform to the interlacing theorem. [30%]

(c) Repeat part (b) for the case $\omega_1 < \alpha_1 < \omega_2 < \omega_3 < \alpha_2$. [25%]

(d) A mass M is now attached to the coupling point. Show how the previous analysis can be extended to this case, taking $\omega_1 < \alpha_1 < \omega_2 < \alpha_2 < \omega_3$ as in part (b). [30%]

4 A simply supported beam has length L, bending rigidity EI and mass per unit length m. Rotational springs of stiffness λ Nm rad^{-1} are added to each end of the beam. In the absence of the springs, the nth mode of vibration of the beam has the mode shape

$$w(x) = \sin\left(\frac{n\pi x}{L}\right)$$

where w is the lateral displacement and x is measured from one end of the beam. The rotational springs are sufficiently weak that they do not change the mode shape very much.

(a) The presence of damping is modelled by using the correspondence principle, so that the Young's modulus E and the spring stiffness λ are replaced by complex values $E(1+i\eta)$ and $\lambda(1+i\gamma)$. By using Rayleigh's principle, derive approximate expressions for the natural frequency and damping ratio associated with mode n of the beam. Sketch the damping ratio as a function of n for the cases (i) $\eta = \gamma$; (ii) $\eta << \gamma$. [35%]

(b) The rotational springs are now removed, and the beam is subjected to a uniform tension P. By combining the expressions given in the data sheet for the potential energies arising from bending and tension effects, derive approximate expressions for the new natural frequencies and damping ratios. [30%]

(c) It is observed that vibration damping in an aircraft fuselage appears to reduce when the fuselage is pressurised. Explain this effect. Describe the possible sources of damping in a structure of this type, and suggest ways in which damping might be added were excessive vibration to be a problem. [35%]

END OF PAPER