ENGINEERING TRIPOS PART IIB

Tuesday 1 May 2012 2.30 to 4.00

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

RSL 03

1 Two components of an aircraft structure are held in contact by a force N_0 acting normal to the contact surface. During flight, random excitation of the aircraft produces an additional, random, normal force $N(t)$ and a random lateral force $S(t)$. The force $N(t)$ is small compared to N_0, so the probability that $N_0 + N(t) < 0$ is negligible. The coefficient of friction between the two components is μ.

(a) Show that sliding between the two components will occur if

$$S(t) - \mu N(t) > \mu N_0 \qquad \text{or} \qquad S(t) + \mu N(t) < -\mu N_0$$ [15%]

(b) The forces $S(t)$ and $N(t)$ are statistically independent and Gaussian, with zero mean and mean squared values $\sigma_S^2 = \sigma_a^2$, $\sigma_N^2 = \alpha\sigma_a^2$, $\sigma_{\dot{S}}^2 = \sigma_b^2$, and $\sigma_{\dot{N}}^2 = \beta\sigma_b^2$. By defining $z_1(t) = S(t) - \mu N(t)$ and $z_2(t) = S(t) + \mu N(t)$, and considering appropriate crossing rates of $z_1(t)$ and $z_2(t)$, show that the mean rate at which sliding will initiate is given approximately by

$$\nu = \frac{1}{\pi}\left(\frac{1+\mu^2\beta}{1+\mu^2\alpha}\right)^{1/2}\left(\frac{\sigma_b}{\sigma_a}\right)\exp\left\{-\left(\frac{\mu^2 N_0^2}{2(1+\mu^2\alpha)\sigma_a^2}\right)\right\}$$

Carefully explain the assumptions which underlie this result. [40%]

(c) If instead of being statistically independent, $S(t)$ and $N(t)$ are fully correlated with $N(t) = \gamma S(t)$ and $\sigma_S^2 = \sigma_a^2$, $\sigma_{\dot{S}}^2 = \sigma_b^2$, find the new mean rate at which sliding will initiate. [45%]

2 The transmission of sound into the fairing of a launch vehicle is to be studied by using a two degree of freedom model of the form

$$\ddot{x} + 2\beta_n\omega_n\dot{x} + \omega_n^2 x = F(t)$$

$$\ddot{p} + 2\beta_a\omega_a\dot{p} + \omega_a^2 p = \alpha\ddot{x}(t)$$

Here α is a constant and $x(t)$ and $p(t)$ are generalized coordinates representing respectively the structural response of the fairing and the interior pressure. The terms ω_n and ω_a are the structural and acoustic natural frequencies, and β_n and β_a are the corresponding damping ratios. The force $F(t)$ is the random excitation arising from acoustic noise; the single sided spectrum of $F(t)$ can be approximated as white noise with spectral density S_0.

(a) Derive an expression for the spectrum of the structural response, $S_{xx}(\omega)$. [15%]

(b) Derive an expression for the spectrum of the acoustic response, $S_{pp}(\omega)$. [20%]

(c) If the structural damping ratio β_n is large and the acoustic damping ratio β_a is small, sketch $S_{xx}(\omega)$ and $S_{pp}(\omega)$. By making a suitable white noise approximation, show that the mean squared pressure can be estimated as

$$\sigma_p^2 = \frac{\pi\alpha^2 S_0\omega_a}{4\beta_a}\left[(\omega_n^2 - \omega_a^2)^2 + (2\beta_n\omega_n\omega_a)^2\right]^{-1}$$ [25%]

(d) Repeat part (c) for the case in which the structural damping ratio β_n is small and the acoustic damping ratio β_a is large, giving a revised approximation for the mean squared pressure. [20%]

(e) Derive an expression for σ_p^2 for the case in which both β_a and β_n are very small, assuming that the acoustic and structural natural frequencies are well separated. [20%]

(TURN OVER

3 An undamped vibratory system of unit mass has a nonlinear spring with the force-displacement relation shown in Fig. 1. The slope of the relation is s for $|x| < b$.

(a) For a sinusoidal displacement of amplitude α, with $\alpha > b$, sketch the waveform of the force in the spring. [20%]

(b) Determine the Describing Function associated with the spring. [40%]

(c) If the system is now driven by a harmonic force $f \cos \omega t$, determine an approximate relation between f, ω, and the response amplitude α. [20%]

(d) Sketch the approximate frequency response function for the system, and qualitatively describe the key features of the response. [20%]

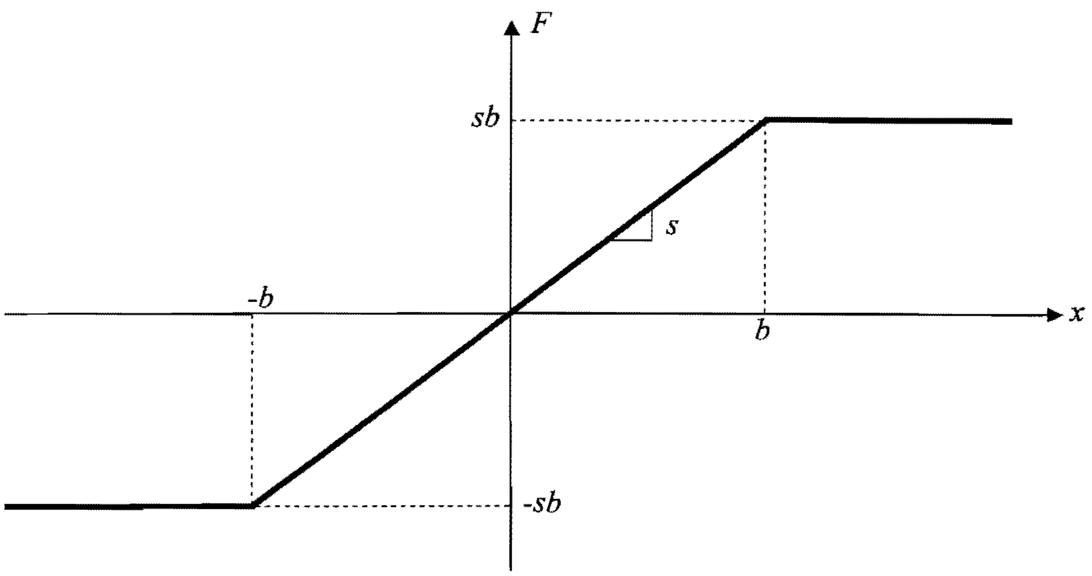

Fig. 1

4 The nonlinear vibrations of a damped system are described by the equation

$$\ddot{x} + 2\zeta\dot{x} - \alpha x + x^3 = 0$$

where $\zeta > 0$ is the damping factor and α is a parameter that can be either positive or negative.

(a) Identify the singular points of the system. [20%]

(b) Determine the type and stability of each singular point. [30%]

(c) Sketch the behaviour of the system in the phase plane for $\alpha > \zeta^2 / 2$. [30%]

(d) Sketch the bifurcation diagram associated with varying the parameter α. [20%]

END OF PAPER

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ANSWERS

1. (c) $\nu = \left(\frac{1}{2\pi}\right)\left(\frac{\sigma_a}{\sigma_b}\right)\left\{\exp\left[-\frac{(\mu N_0)^2}{2(1-\mu\gamma)^2\sigma_a^2}\right]+\exp\left[-\frac{(\mu N_0)^2}{2(1+\mu\gamma)^2\sigma_a^2}\right]\right\}$

2. (a) $S_{xx}(\omega)=\frac{S_0}{(\omega_n^2-\omega^2)^2+(2\beta_n\omega_n\omega)^2}$

(b) $S_{pp}(\omega)=\frac{\alpha^2\omega^4 S_{xx}(\omega)}{(\omega_a^2-\omega^2)^2+(2\beta_a\omega_a\omega)^2}$

(d) $\sigma_p^2=\left(\frac{\pi\alpha^2\omega_n S_0}{4\beta_n}\right)\left[(\omega_n^2-\omega_a^2)^2+(2\beta_a\omega_a\omega_n)^2\right]^{-1}$

(e) $\sigma_p^2=\left(\frac{\pi\alpha^2 S_0}{4}\right)\left(\frac{\omega_a}{\beta_a}+\frac{\omega_n}{\beta_n}\right)\left(\frac{1}{\omega_n^2-\omega_a^2}\right)^2$

3. (b) $DF=\frac{2s}{\pi}\left[\frac{\pi}{2}-\cos^{-1}\left(\frac{b}{\alpha}\right)+\left(\frac{b}{\alpha}\right)\sqrt{1-\frac{b^2}{\alpha^2}}\right]$ (for $\alpha > b$)

(c) $-m\omega^2\alpha+\frac{2s\alpha}{\pi}\left[\frac{\pi}{2}-\cos^{-1}\left(\frac{b}{\alpha}\right)+\left(\frac{b}{\alpha}\right)\sqrt{1-\frac{b^2}{\alpha^2}}\right]=f$ (for $\alpha > b$)

4. (a) $\dot{x}=x=0$, and for $\alpha>0$ the two additional points $\dot{x}=0,\ x=\pm\sqrt{\alpha}$

(b) For the point $\dot{x}=x=0$:

$\alpha>0$	saddle point
$-\zeta^2<\alpha<0$	stable node
$\alpha<-\zeta^2$	stable focus

For the points $\dot{x}=0,\ x=\pm\sqrt{\alpha}$ (note that $\alpha>0$):

$\alpha<\zeta^2/2$	stable node
$\alpha>\zeta^2/2$	stable focus