ENGINEERING TRIPOS PART IIB

Tuesday 8 May 2012 9.00 to 10.30

Module 4C8

APPLICATIONS OF DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

dc03

1 (a) Show that for motion along a straight track at steady speed u, the creep forces on a rigid railway wheelset result in a total lateral force Y of

$$Y = 2C\left(\frac{\dot{y}}{u} + \theta\right)$$

and a total yawing moment of

$$N = 2dC\left(\frac{\varepsilon y}{r} - \frac{d\dot{\theta}}{u}\right),$$

where y is the lateral tracking error of the wheelset; θ is the yaw angle of the wheelset; $2d$ is the track gauge; r is the rolling radius of each wheel when $y = \theta = 0$; ε is the effective conicity of the wheelset; and C is the creep coefficient relating the lateral and longitudinal creep velocities to the corresponding creep forces. [40%]

List your assumptions. [10%]

(b) Figure 1 shows a railway 'trailer' that is towed behind a train. The trailer is freely pinned to the towing vehicle at P, and is supported by a wheelset with stiff bearings. The wheelset is distance ℓ aft of P and the yaw moment of inertia of the trailer about P is I_P. Point P is constrained to move along the centre-line of the track. Derive an equation of motion for the trailer in terms of the small articulation angle ϕ and determine the conditions for which the motion is stable. [30%]

Quote the stability criterion for a free wheelset that rolls without slip along a straight track, and compare this with your answer for the trailer. Explain the differences. [20%]

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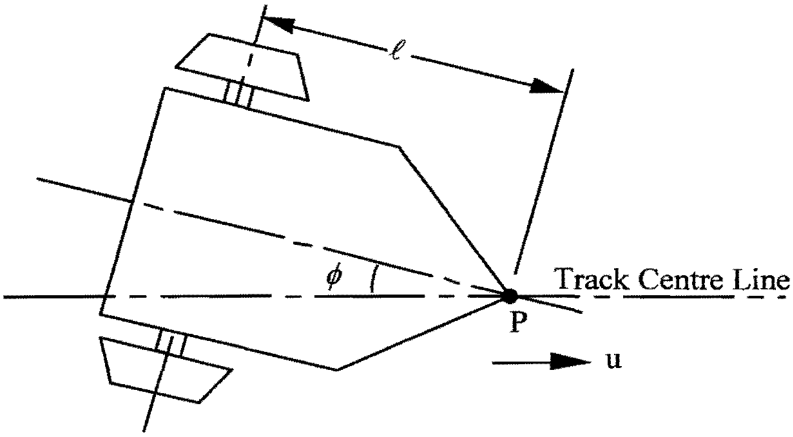

Fig. 1

(TURN OVER

2 Figure 2 shows a car, idealised as a two–wheeled vehicle, in which both the front and rear wheels steer. It has constant forward speed u, and has freedom to sideslip and yaw. The car is designed so that the steer angle γ of the rear wheels is proportional to the steer angle δ of the front wheels, with $\gamma = K\,\delta$. The vehicle has total mass m and yaw moment of inertia I about its centre of mass G. The front and rear tyres both have lateral creep coefficient C, and are located distances a and b from G.

(a) Derive the equations of motion of the vehicle in a moving coordinate frame fixed to the vehicle. [40%]

(b) Hence show that the steady state steer angle δ during a turn of constant radius R satisfies the equation

$$\delta = \frac{L}{R}\left[\frac{1-\dfrac{sm}{L^2C}u^2}{1-K}\right]$$

where $L = a + b$, and $s = a - b$. [20%]

(c) Sketch the relationship between δ and u for various combinations of K and s. Comment on the effect of K on the stability of the vehicle. Explain the terms 'understeer' and 'oversteer' in the context of the steadily-turning vehicle, and state with reasons whether you would choose K to be positive or negative for the best handling in steady turns. [40%]

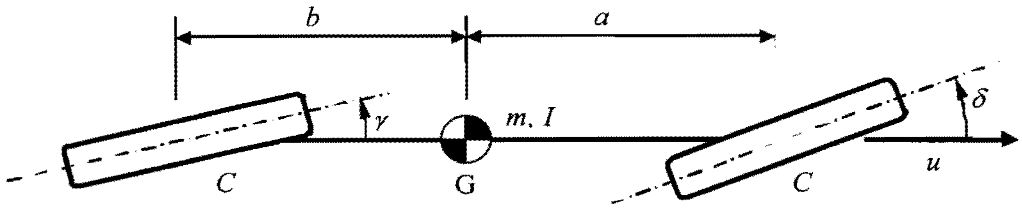

Fig. 2

3 (a) Define the terms *mean anomaly*, *mean motion*, *eccentric anomaly* and *true anomaly*, using diagrams where necessary. Derive an expression that relates the mean anomaly to the eccentric anomaly. [30%]

(b) A navigational receiver downloads the following ephemeris data from a GPS satellite that it is tracking:

ephemeris reference time: 560,670.0 seconds
mean anomaly at reference time: 0.1703 radians
eccentricity: 0.0210
semi-major axis: 26562.0 km
right ascension of ascending node at reference time: 0.2875 radians
inclination angle: 0.9599 radians (or 55.0°)
argument of perigee: 0.2574 radians
rate of change of right ascension: 7.83×10^{-9} radians/sec
all other corrections: negligible

The device then receives a positioning signal that was transmitted from the satellite at time 3,001.0 seconds.

(i) Explain why it is sometimes necessary to add 604,800.0 seconds to the signal transmission time, when computing the position of the satellite.

(ii) Find the mean motion of the satellite, and hence show that its mean anomaly at the time when it transmitted the positioning signal was 0.7607 radians.

(iii) Show that the eccentric anomaly of the satellite is 0.7754 radians, and hence calculate its true anomaly.

(iv) Calculate the distance of the satellite from the centre of the Earth and from the equatorial plane at transmission time.

(v) Find the right ascension of the satellite's ascending node at the signal transmission time, and explain the principal reason why this varies with time. How big an error in the calculation of the satellite's position would be caused if the variation was ignored in this case? [70%]

4 (a) With the aid of suitable diagrams, show that the gravitational potential of a hollow spherical shell of mass M is:

(i) the same at all points inside the shell, and

(ii) the same as that of a point mass M positioned at the centre of the shell, for all points outside the shell. [40%]

(b) An interstellar dust cloud is spherically symmetric, with a core region of radius a having uniform density ρ_0, and an outer region whose density varies with radius r according to:

$$\rho(r) = \rho_0 \left(\frac{a}{r} \right)^4 ,$$

extending to infinity.

(i) Using the data on potential energy on the data sheet, show that the gravitational potential in the region $r > a$ is given by:

$$U(r) = 4\pi\rho_0 G a^2 \left(\frac{4a}{3r} - \frac{a^2}{2r^2} \right).$$

(ii) An asteroid orbits within the dust cloud, with an orbit which never enters the core region. Write down an equation governing the shape of the orbit, in terms of the variables $u = 1/r$ and θ, where r and θ are the usual polar co-ordinates, and find a general solution to your equation. [60%]

END OF PAPER